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FRONTISPIECE.

THE ELECTRICAL TRANSMISSION OF ENERGY.

A MANUAL FOR THE DESIGN OF ELECTRICAL CIRCUITS.

BY

ARTHUR VAUGHAN ABBOTT, C.E.,

MEMBER AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS, MEMBER AMERICAN INSTITUTE OF MINING ENGINEERS, MEMBER AMERICAN SOCIETY OF CIVIL ENGINEERS, MEMBER AMERICAN SOCIETY OF MECHANICAL ENGINEERS, ETC.

WITH TEN FOLDING PLATES.

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J. Harold Libbey.

PREFACE.

IT has been tritely remarked that "There is nothing new under the sun." In view of this sapient aphorism the reader will not expect to find much that is strange or remarkable in the present volume. Books, however, are something like kaleidoscopes, in which ideas, like the bits of colored glass, resolve themselves into innumerable stellate forms, presenting to the inspector picture after picture, each of seemingly different origin from the preceding ones. While investigating a subject, it has been the custom of the author to obtain all the works by different writers on the question under consideration, and to read them successively; thereby viewing the matter from a number of different standpoints. He has found this an exceedingly valuable way of acquiring information, and remembers with the liveliest sense of gratitude the various expositors from whose differing horizons he has scanned the landscape of complicated topics.

The present volume has been prepared chiefly from the aspect of the author's experience, and is an endeavor to collect and arrange in an accessible and convenient form the data necessary to the scientific designing and proportioning of Electrical Circuits. No attempt has been made to describe any Central Station machinery; for the scope of the volume would not permit of an extension beyond the material relevant to the "Transmission of Energy," so aptly and untranslatably termed by the French "*Canalisation.*"

The opening chapters are devoted to an outline of Circuits, and to an annunciation of the principles and laws governing Conductors and Insulators. This is followed by a discussion of the methods of constructing Aerial Lines and description of Underground Conduits.

and Conductors. Succeeding these, a chapter is devoted to Testing Instruments, and one to the Methods of Measuring and Inspecting Lines, and of determining and remedying any faults that may be found to exist. In Chapters VII. and VIII., the laws of Continuous and Alternating Circuits are exhibited. Subsequently, distribution proper is treated in three chapters, under the heads of "Series Distribution," "Parallel Distribution," and "Miscellaneous Methods." In the concluding chapter, a rough approximation is given for obtaining the cost of Circuits and cost of the production of Electrical Energy.

It has been the attempt of the author to herein collate such methods of Circuit Construction, in connection with tabulated data, as have been sanctioned by the best practice, both in this country and in Europe. No attempt has been made to render the volume an encyclopedia; and, therefore, all matter obsolete or antiquated has been rejected, and only such is presented as seems to be fully warranted by the present state of the art. Wherever possible, the lines along which future practice is likely to lie have been indicated. The chapters on measuring instruments and methods of testing have been carefully abridged to include only such information as is valuable to the practicing engineer, laboratory appurtenances and methods being entirely eliminated. In a large proportion of the methods of measurement, the simple literal formulæ for the solution of the problem in question are given, without any attempt at the necessary demonstration of the truth of the same. Inasmuch, however, as nearly all such formulæ are directly derivable from the laws of Ohm and Kirchhoff, involving only algebraic processes, the reader can easily deduce the equations for himself. For a more complete exposition of the methods of measurement, the reader is referred to the works of Hospitalier, Gerard, Weiller & Vivarez, Kempe, and Monroe & Jameson. In the chapters on Distribution, sufficient importance is attached to the subject to give the mathematical discussion in full, involving, however, only the simplest applications of the calculus. Wherever practicable, liberal use of illustrations has been made; for

ocular demonstration is always much clearer and more concise than any verbal description.

To the works of Picou, Hospitalier, Cadiat, Gerard & Weiller, Kempe, Thomson, Kennelly, Ayrton, Perry, Preece, and Heaviside, and to the "Transactions of the Electrical Engineers," the *London Electrician*, the *Electrical World*, the *Electrical Engineer*, and *La Lumière Électrique*, also the *Street Railway Journal*, the author has long been indebted for information that has happily led to the successful construction of many transmission plants, and which, passed through the sieve of experience, is here presented to the public; and for benefits thus derived, he has long wished for an opportunity to gratefully acknowledge his obligation. Acknowledgment is particularly due to Mr. F. J. Dommerque, for aid in the preparation of many of the tables, and in verification of the proof-sheets. Convinced, from the standpoint of experience, of the utility of the information, the author trusts that the electrical section of the engineering profession may find the present presentation of value in practical construction.

CHICAGO, ILL., *Jan.* 15, 1895.

PREFACE TO SECOND EDITION.

IN the three years which have elapsed since the publication of the first edition of *The Electrical Transmission of Energy*, the extension of practical applications of electricity has even far exceeded the most sanguine prophesies of growth; but during this time progress has been chiefly along commercial lines rather than those of invention. Few new or startling ideas may be chronicled, but, through the furnace of practical experience, that which was good has been refined and freed from the dross of theory, — has settled into secure and reliable commercial forms. The Author, therefore, has little to add or change in such portions of this work as are purely theoretical, but details of practice have received careful revision.

It was with much apprehension that the Author watched the reception of the first edition, but its appearance developed so many hitherto unknown friends, and even its most severe critics were withal so just and kindly, that a feeling of gratitude soon displaced fear; and to all who have aided, either with welcome words of commendation, or the more valuable though perhaps less pleasing phrases of criticism, the Author here returns his most sincere acknowledgments.

CHICAGO, *Nov.* 1, 1898.

PREFACE TO FOURTH EDITION.

In the half-decade that has elapsed since the second edition of *The Electrical Transmission of Energy* appeared, the practice of transmitting energy by means of electricity has extended by leaps and bounds until to-day one can hardly gaze from the window of a railway train without encountering the pyramidal cross-arms of a transmission circuit.

Subjected to the alchemy of many minds, the cruder practice of a decade ago has undergone a refining and clarifying process that has purged away various errors, and has reduced the practice of building transmission lines to an approximately uniform standard. To correspond to more modern methods much that was more or less tentative in the former editions has been omitted, and its place filled by matter relative to transmission lines as they now exist. The electric railway, always progressive, having rebuilt the urban systems of the continent, stretches interurbanwise, and is fast linking together the large cities in a network that will soon rival the steam lines. So the chapter on electric railway circuits has been completely remodeled. To the paragraphs on the methods of constructing alternating current lines, enough has been added to enable the student to deal with the problems arising in the transmission line in the light of the practice of to-day; and finally, the section relating to cost of plant and cost of producing electricity has been thoroughly modernized.

Thus rehabilitated, the fourth edition is presented to the public. The author trusts that its former friends may perceive the effects of many kindly and valuable suggestions, and that thus improved and enlarged *The Electrical Transmission of Energy* may win for itself an ever-widening circle of readers.

ARTHUR VAUGHAN ABBOTT.

NEW YORK, *August*, 1904.

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TABLE OF SYMBOLS.

A	Ammeter, amperes.
a	Coefficient in temperature equation for dielectric.
α	Coefficient in temperature equation, also angular measure.
a, b, d, and x	Resistances in the arms of a Wheatstone Bridge.
β	Coefficient in temperature equation, also angular measure.
C	Condenser or capacity, and radiation coefficient.
D, d, d', d'', etc.	Deflection on any scale instrument, or diameter.
d_c	Rate of depreciation charged on cost of conduits.
d_l	Rate of depreciation charged on cost of line.
d_s	Rate of depreciation charged on cost of station.
E	Primary electro-motive force or battery.
e	Electro-motive force at any secondary point.
F	Number of hours per annum of operation; also galvanometer figure of merit.
G	Galvanometer and galvanometer resistance.
g and g'	The two halves of a differential galvanometer.
H	Heat units (gramme, degree).
H_c	Heat units lost by convection.
H_r	Heat units lost by radiation.
I, I', I''	(and i, i', i'', etc.) Currents in amperes, also rate of interest.
K and k	Key and coefficient of radiation per unit of surface.
K	Cost of producing energy per watt or $K.W.$
K'	Cost of station machinery per watt or $K.W.$ of output.
L and l	Length.
m	Multiplying power of shunt.
Q	Quantity of electricity in coulombs.
R, R', R''	Resistance unknown.
r, r', r''	Resistance known.
R_t	Resistance at temperature $t°$ C.
R_0	Resistance at temperature 0° C.
ρ	Resistance specific.
S	Shunt, area of cross-section, and crushing strength.
T	Time in seconds, also temperature, tension in pounds.
t	Conductor temperature in degrees C.
θ	Temperature of air in degrees C.
U'	Charge for interest and depreciation on line.
U''	Energy lost in transmission in line.
$u_0 - u'$	Drop on line.
V	Voltmeter and voltage.

mV . . . . Milli-meter voltmeter.
W and w . . . . Energy in watts.
W_c . . . . Energy in watts lost by convection.
W_r . . . . Energy in watts lost by radiation.
Z . . . . Annual cost to produce W watts.
f . . . . Deflection.
e . . . . Electro-motive force at any given instant.
i . . . . Current at the same instant.
f. . . . . Number of magnetic lines cut, or in field at any given instant
E . . . . The maximum values of the above quantities.
I. . . . .
F . . . .
$\mathfrak{E}$ . . . . The mean value of the above quantities.
$\mathfrak{I}$. . . . .
$\mathfrak{F}$ . . . .
L, L', etc. . . . . Coefficients of inductance.
M . . . . Coefficients of mutual inductance.
T . . . . Time of one complete period.
a . . . . Amplitude of harmonic motion.
θ . . . . Angle of epoch.
ϕ . . . . Angle described in time t or dt.
n . . . . The frequency or number of periods per second.
ω . . . . $2\pi n$.
m . . . . Strength of a magnetic pole.
d . . . . Distance.
B. . . . . The total induction, or induction per unit of area.
H . . . . Magnetizing force.
J . . . . Impedance.
$\mathfrak{J}$ . . . . Impedance Factor.
$E.M.F.$ . . . . . Electro-motive force.
c.m. . . . . Circular mils.
s.m. . . . . Square mils.
Σ . . . . Sign for summation.

When the symbols are applied to different circuits sub-letters are used to denote the corresponding value for each circuit.

THE

ELECTRICAL TRANSMISSION OF ENERGY.

CHAPTER I.

INTRODUCTION.

ELECTRICAL DISTRIBUTION.

Art. 1. Distribution in General. — The distribution of Electricity comprises a study of the appropriate methods for supplying Electrical Energy, generated by one or more sources, to a number of receiving mechanisms, or translating devices, the quantity given to each one being properly proportioned to its needs; the investigation of the conditions for accomplishing this distribution in the most exact and economical manner; and finally an examination of the means whereby distributing plants may be rendered permanent, durable, and secure.

The methods of distribution are chiefly controlled by the way in which it is considered advisable to arrange the receiving mechanisms. This arrangement of the receivers is indicated by the service which they are called upon to perform, and being involved in the design of the plant in question, must be settled in each particular case for itself.

Three principal methods are common for the arrangement of the receiving mechanisms; they may be arranged in *Series*, in *Parallel*, or by a *Combination of the two previous methods*. It is also frequently advisable to employ, between the generators and the receivers, intermediate or auxiliary contrivances, such as accumulators, transformers, or the like, the use of which gives rise to the various problems in indirect distribution.

2. Distribution in Series. — Under this method all of the receivers are placed one after the other in succession upon a single conductor extending throughout the entire circuit from pole to pole of the generator. This method is illustrated in Fig. 1.

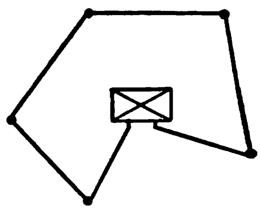


Fig. 1. Diagram of a simple Series Circuit.

3. Distribution in Parallel. — In this system one or more pairs of conductors, running parallel to each other, are arranged, extending to the limits of the circuit. Each receiver is connected across one of the pairs of mains, thus forming a circuit which is independent of that of every other receiver. See Fig. 2.

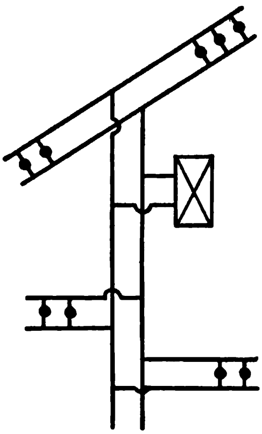


Fig. 2. Diagram of a simple Parallel Circuit.

4. Mixed Systems. — A combination of the two preceding methods is a natural consequence, giving rise to designs as exemplified in Fig. 3, some of the receivers being placed in parallel, as previously indicated, while others may be placed in series and joined across the mains from the generator, each series circuit being arranged in parallel to all the other series circuits, thus uniting in one both

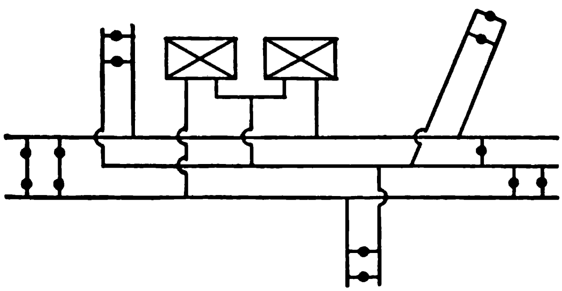


Fig. 3. Diagram of a Series-parallel Circuit.

systems. Obviously, to unite the generators in series and then to place them across the mains in parallel, in a manner similar to the arrangement of the receivers, readily followed; giving rise to the now

famous three, five, and seven wire systems now used for direct current distributions of magnitude.

5. Indirect Distribution. — Finally, if between the generator and the receiver intermediate contrivances for transformation or accumulation of Electrical Energy are employed, the arrangement of the circuits between the generator and the accumulator, or transformers, and between the latter and the receivers, may be entirely different. For example, Fig. 4 shows in outline the combination of a lamp circuit fed by accumulators charged from a generator; the accumulators are in series across the mains of the generator, while the lamps are placed in parallel across two halves of the battery of accumulators.

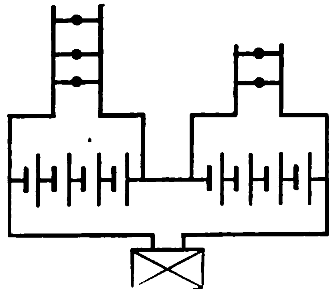


Fig. 4. Diagram of Indirect Distribution.

The various methods here outlined, together with the ramifications and modifications practically found to be of advantage, will be successively considered, proceeding from the simple to the more complicated forms. Previously, however, it is advisable to examine the characteristics of the materials adapted to the construction of electric circuits; and to study such methods of construction as the present state of the art indicates as advisable. It is also desirable to become sufficiently familiar with electrical instruments and methods of measuring to enable the practitioner to examine into and determine the performance of a transmission plant, and to remedy any defects or faults that may be revealed.

CHAPTER II.

THE PROPERTIES OF WIRE.

Art. 6. Every present system for the distribution of energy by electricity comprises, as its most important constituent, a circuit formed of some substance which is a good conductor of electricity, and which, connecting the generators and various receivers, conveys to each its appropriate supply. Inasmuch as the metals are the best conductors, they are universally selected to form at least a part of the conducting circuit, and for this purpose are most conveniently employed in the form of bands, or rods of small dimensions usually termed wire.

7. Wire Manufacture. — While a detailed description of the process of wire manufacture is foreign to the scope of this work, it is advisable to outline it sufficiently to enable a proper design for the line circuit to be made. So far as the distribution of electrical energy is concerned, but four kinds of wire have any commercial importance,—iron, copper, aluminum, and the various forms of bronze. The metal to be formed into wire is cast, or rolled, into masses about six inches square by three or four feet in length, technically termed "Blooms;" and each bloom is then, by succeeding passes through rolls, reduced to a long and slender rod about half an inch in diameter, and approximately of a circular section. Wire smaller than this size it has been found impracticable to roll. For the lesser diameters recourse is had to the process of wire drawing, which consists in pulling the rolled rod through constantly decreasing holes in a series of hardened steel or agate plates. Thus the rod is given an exact circular cross-section, and by repeated passes through the dies may be reduced in diameter to any desired extent.

8. Hard-Drawing. — Pulling the metal through the die produces a change in its molecular structure, whereby the rod becomes considerably compressed and hardened, its tensile strength being markedly augmented, with a corresponding diminution in elasticity. This effect, which seems to be analogous to the process of tempering in

steel, becomes of great importance in increasing the strength of the material forming the wire. Steel having a tensile strength of 60,000 lbs. to 80,000 lbs. per square inch in the bloom, may by this so-called method of "hard-drawing" have its tenacity raised to 300,000 or 350,000 lbs. per square inch. This extraordinary increase is, however, only found in wire of very small diameter. The effect of hard-

TABLE No. 1.

Physical Properties of Galvanised Iron and Steel Wire.

Numbers, B. W. G	Diameters in Mils.	Weights, Pounds.		Breaking Weights, Pounds.		Resistance per Mile in Ohms.		
		1000 Feet.	One Mile.	Iron.	Steel.	E. B. B.	B. B.	Steel.
0	340	304	1607	4821	9079	2.93	3.42	4.05
1	300	237	1251	3753	7068	3.76	4.4	5.2
2	284	212	1121	3363	6335	4.19	4.91	5.8
3	259	177	932	2796	5268	5.04	5.9	6.97
4	238	149	787	2361	4449	5.97	6.99	8.26
5	220	127	673	2019	3801	6.99	8.18	9.66
6	203	109	573	1719	3237	8.21	9.6	11.35
7	180	85	450	1350	2545	10.44	12.21	14.43
8	165	72	378	1134	2138	12.42	14.53	17.18
9	148	58	305	915	1720	15.44	18.06	21.35
10	134	47	250	750	1410	18.83	22.04	26.04
11	120	38	200	600	1131	23.48	27.48	32.47
12	109	31	165	495	933	28.46	33.3	39.36
13	95	24	125	375	709	37.47	43.85	51.82
14	83	18	96	288	541	49.08	57.44	67.88
15	72	13.7	72	216	407	65.23	76.33	90.21
16	65	11.1	59	177	332	80.03	93.66	110.7
17	58	8.9	47	141	264	100.5	120.4	139.0
18	49	6.3	33	99	189	140.8	164.8	194.8

drawing seems to be chiefly confined to a very thin layer or skin on the surface of the wire; so if any mechanical abrasion occurs to the surface, such as cutting or nicking, sufficient to destroy the integrity of this skin, the entire effect of the drawing will be lost. For this reason great care must be exercised in the erection of hard-drawn wire to prevent this destruction of the exterior. The same effect may be produced by annealing. In TABLE No. 1 the physical charac-

teristics of the trade varieties of iron and steel wire of the more common sizes are given. Unfortunately the process of hard-drawing reduces the electrical conductivity of copper wire from 2 to 4 per cent; yet the advantages to be derived from increased tenacity more than counterbalance this loss. Attempts have been made to manufacture all but the smallest sizes of wire entirely by rolling; and while the results thus far obtained point toward a very successful accomplishment of this process, rolled wire is not as yet of common commercial occurrence. Curiously, in wire thus manufactured, the hardening of the metals by the rolls seems to extend entirely through the wire, and not to be confined to a superficial skin. In order to make good wire, it is necessary that the blooms from which the rods are rolled should be sound, and free from all slivers, gas bubbles, cold shuts, or other imperfections; for, during the passage of the metal through the rolls and dies, all flaws in the blooms are simply elongated without being eradicated, tending to make the finished wire imperfect and difficult for the linemen to handle, as slivers or checks on the surface of the wire are likely to severely cut or injure the hands of the workmen, and make the process of stringing not only disagreeable but positively dangerous.

9. **Wire Gauges.** — Until recently an enormous amount of confusion existed as to the terminology applied to the different sizes of wire; and, indeed, in many instances the same trade name was applied by different manufacturers to wire of widely varying diameters. Even in 1883 a number of different wire gauges existed in Europe, and at least three different standards were in vogue in this country. The mistakes arising from the confusion of the gauges became so important that the iron and steel manufacturers met with a view of discussing this question, and of settling upon some universal standard to be adopted by all of the trade. Joint meetings of the Iron and Steel Institute of Great Britain, and of the American Institute of Mining Engineers, resulted in the establishment in England of the Imperial Standard Wire Gauge, and of the adoption in this country of the Brown & Sharp Gauge. On the Continent gauge numbers are rarely used, all wire work being measured in millimeters and decimal fractions thereof. In TABLE NO. 2 will be found a comparison between the various standard wire gauges now employed, together with the nearest corresponding number of millimeters,

TABLE NO. 2.—GIVING RELATIONS BETWEEN

Imperial Standard Wire Gauge.
Brown and Sharpe's Wire Gauge.
Birmingham or Stubbs Wire Gauge.
Washburn and Moen's Wire Gauge.
Trenton Wire Gauge.
Old English Wire Gauge.

Gauge No.	Diameter in Ten-Thousandths of an Inch.						Diam. in mms.
	I. S. W. G.	B. and S. W. G.	B.or S.W.G.	W. and M. W. G.	Trenton W. G.	Old Eng.	
7-0	5000	. . .	. . .	. . .	. . .	. . .	12.70
6-0	4640	. . .	. . .	4600	. . .	. . .	11.78
5-0	4320	. . .	. . .	4300	4500	. . .	10.97
4-0	4000	4600	4540	3930	4000	. . .	10.16
3-0	3720	4096	4250	3620	3600	. . .	9.45
2-0	3480	3648	3800	3310	3300	. . .	8.84
1-0	3240	3249	3400	3070	3050	. . .	8.23
1	3000	2893	3000	2830	2850	. . .	7.62
2	2760	2576	2840	2630	2650	. . .	7.01
3	2520	2294	2590	2440	2450	. . .	6.40
4	2320	2043	2380	2250	2250	. . .	5.89
5	2120	1819	2200	2070	2050	. . .	5.38
6	1920	1620	2030	1920	1900	. . .	4.88
7	1760	1443	1800	1770	1750	. . .	4.47
8	1600	1285	1650	1620	1600	. . .	4.06
9	1440	1144	1480	1480	1450	. . .	3.66
10	1280	1019	1340	1350	1300	. . .	3.25
11	1160	907	1200	1200	1175	. . .	2.95
12	1040	808	1090	1050	1050	. . .	2.64
13	920	719	950	920	925	. . .	2.34
14	800	640	830	800	800	.0830	2.03
15	720	570	720	720	700	720	1.83
16	640	508	650	630	610	650	1.63
17	560	452	580	540	525	580	1.42
18	480	403	490	470	450	490	1.22
19	400	359	420	410	400	400	1.02
20	360	320	350	350	350	350	.91
21	320	284	320	320	310	315	.81
22	280	253	280	280	280	295	.71
23	240	226	250	250	250	270	.61
24	220	201	220	230	225	250	.56
25	200	179	200	200	200	230	.51
26	180	159	180	180	180	205	.46
27	164	142	160	170	170	187	.42
28	148	126	140	160	160	165	.38
29	136	113	130	150	150	155	.34
30	124	100	120	140	140	137	.31
31	116	89	100	135	130	122	.29
32	108	79	90	130	120	112	.27
33	100	71	80	110	110	102	.25
34	92	63	70	100	100	95	.23
35	84	56	50	95	95	90	.21
36	76	50	40	90	90	75	.19
37	68	44	. . .	85	85	65	.17
38	60	39	. . .	80	80	57	.15
39	52	35	. . .	75	75	50	.13
40	48	31	. . .	70	70	45	.12
41	44	. . .	. . .	. . .	. . .	. . .	.11
42	40	. . .	. . .	. . .	. . .	. . .	.10
43	36	. . .	. . .	. . .	. . .	. . .	.09
44	32	. . .	. . .	. . .	. . .	. . .	.08
45	28	. . .	. . .	. . .	. . .	. . .	.07
46	24	. . .	. . .	. . .	. . .	. . .	.06
47	20	. . .	. . .	. . .	. . .	. . .	.05
48	16	. . .	. . .	. . .	. . .	. . .	.04
49	12	. . .	. . .	. . .	. . .	. . .	.03
50	10	. . .	. . .	. . .	. . .	. . .	.02

thus giving in a tabular form full information regarding the present method of wire measurement.

10. **The Circular Mil.** — A convenient trade convention for the measurement of wire has arisen in the use of the so-called "Circular Mil," the "Mil" being the name for the one-thousandth of an inch. The diameter, therefore, of a wire expressed in mils is its diameter in thousandths of an inch with the decimal point removed. If the diameter of any wire expressed in mils be squared, a number is obtained which is proportional to the actual area of the wire itself, and is termed the "circular millage" of the wire.

In Fig. 5 is the diagrammatic representation of a wire, each of the small circles symbolizing a unit wire, one mil, or one-thousandth of an inch, in diameter. It will be noticed that the diameter of the wire is ten small circles long, and therefore the wire is ten mils in diameter. The square of ten being 100, the circular millage of this wire would be 100 circular mils. As the area of a circle is the square of its diameter multiplied by .7854, in order to convert the circular millage of any wire into its actual area in square inches, the circular millage must be multiplied by .7854, and the requisite decimal places pointed off. Thus, in the previous example, a wire of 100 circular mils has an actual area of $.01 \times .01 \times .7854 = .00007$ square inches. Inasmuch as the circular millage is proportional to the actual area in square inches of the wire, it forms an exceedingly easy and convenient number for the purposes of calculation, and is widely used in this connection.

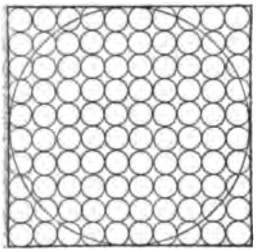


Fig. 5. — Diagram of the Circular Mil.

11. The use of the circular mil leads to other convenient conventions. Thus a wire of one mil area and one foot in length may be called a *unit wire;* and if the various properties of the unit wire be known, such as its electrical resistance, its tensile strength, weight, cost, etc., the similar properties of any other wire of any size or length can be at once determined by simple multiplication or division. Take copper wire for example; the following values for the unit wire (a mil foot) are commonly found:

Electrical resistance soft pure copper, 50.4 F.	......	10.00 ohms.
" " commercial wire	....	10.10 ohms.
" " hard commercial wire	....	10.30 ohms.

Unit Wire.

Weight.	.00000302	lbs.
Strength soft.	.0275	lbs.
Strength hard.	.0510	lbs.
Cost at 15 cents per lb.	.0000453	cents.

Tensile strength is affected by the process of drawing, and consequently the strength per unit of area is not constant, but will vary with the size of the wire. The constant here given is an average which will be too small for the finer sizes, and too large for the coarser ones.

Now, if any other wire has a circular millage of $C.\ M.$, a length of L feet and cost of p cents per lb., its various properties are found at once by the following formulæ:

$$\text{Resistance (soft)} = \frac{10L}{C.M.} \text{ ohms.}$$

$$\text{Resistance (hard)} = \frac{10.30L}{C.M.} \text{ ohms.}$$

$$\text{Weight} = .00000302 \times L \times C.M. \text{ pounds.}$$

$$\text{Tensile strength soft} = .0275 \times C.M. \text{ pounds.}$$

$$\text{Tensile strength hard} = .0510 \times C.M. \text{ pounds.}$$

$$\text{Cost} = .0000453 \times L \times C.M. \times \frac{p}{15} \text{ cents.}$$

12. Another convention used particularly in building telegraph and telephone lines is known as the "mile-ohm." The mile-ohm is the weight of a piece of wire one mile long which has a resistance of one ohm. Evidently the better the conductor the less the mile-ohm will weigh, because the wire will be smaller. Thus for copper the mile-ohm at 60° F. is 859 lbs., and for iron 4600 lbs.

13. Copper Wire. — It has been within recent years only that the development of the uses of electrical energy has been sufficiently important to draw careful attention to the materials to be employed in line construction. Previously to 1880, electric lines were almost exclusively confined to those used by the telegraph, which, with the exception of the submarine cables, were entirely constructed of iron wire. The electrical resistance of iron wire is some seven or more times greater than that of pure copper, yet it has only been within the last two decades that the state of copper metallurgy was sufficiently advanced to render possible the production of pure copper in commercial

quantities. Experiment also indicated that very small quantities of various impurities increased the electrical resistance in an enormous ratio. This increase in resistance, due to the admixture of other substances, is indicated in TABLE No. 3.

TABLE No. 3.

Variation in the resistance of copper due to varying purity.

Assuming pure copper to have a conductivity of 100 :—

The best refined copper would be	99.0
Alloy of copper and silver, equal parts	86.0
Copper containing 4 per cent of silicon	75.0
" " 12 " " " "	54.0
Silicon bronze wire	35.0
Copper with 10 per cent lead	30.0
Phosphor bronze	29.0
Bronze containing 35 per cent zinc	21.0
" " iron	16.0
Aluminum bronze	12.6
Siemens steel	12.0
Arsenical copper containing 10 per cent arsenic	9.0
Phosphor bronze with 10 per cent of tin	6.5
Phosphor bronze with 9 per cent phosphorus	4.9

From the preceding figures it will be very apparent that high electrical conductivity can only be obtained by the selection of the purest copper. It is not surprising, therefore, that the developments of electrical industries have been followed by a marked improvement in copper metallurgy. The cable extending between Calais and Dover, laid in 1851, had a conductivity of 42 per cent of that of pure copper; the Atlantic cable of 1856 had 50 per cent; the Red Sea cable in 1857, 75 per cent; the Atlantic cable of 1865, 96 per cent. These figures give approximately the rate of improvement in the manufacture of copper for electrical conductors, but it was necessary to await the advent of the modern dynamo in order to produce electrically pure copper at such prices as would permit of a wide commercial application. As long as electrical circuits were confined to telegraphic transmission in which the currents used were exceedingly small, the amount of line resistance was not a very important factor. As soon, however, as the problem was presented of transmitting

large quantities of electrical energy, it became imperative to seek some better material. At present the use of iron and steel wire is confined to circuits carrying but very small amounts of current, hard-drawn copper wire being universally adopted for lines having currents of any magnitude.

14. To properly design an electrical circuit, all of the mechanical and electrical properties of the material to be used must be thoroughly known. These properties for hard-drawn copper wire will be found in TABLES NOS. 5, 6, and 7. In addition to its superior conductivity, copper presents a great advantage in durability. Even in the open country, and with all possible protection, iron rusts rapidly; while in the smoky air of most cities, iron lines rarely last more than three years, and cases have been known wherein iron wires have been entirely corroded within a few months. With copper, on the contrary, it is found that the wire becomes rapidly coated with a thin layer of sulphide of copper, probably not over one-thousandth of an inch in thickness, which seems to entirely protect the wire from any subsequent action. At any rate, few copper lines have as yet been in existence long enough for any perceptible corrosion to have made itself manifest.

15. Composite Wire. — From time to time attempts have been made to use a composite wire, which should consist of a steel core, carrying an external sheath of copper; the idea being that the steel interior would add sufficient tensile strength to enable long spans to be used, while the external covering of copper would provide the necessary conductivity for the current. To a certain extent these experiments have been successful; but the use of composite wire has never extended beyond telegraphic or telephonic circuits, and now has fallen into disuse in the presence of the superior article of hard-drawn copper.

In telephonic circles the idea of composite wire has been revived, in the hope of improving the talking ability of long lines, by providing a medium of higher magnetic permeability. Theoretically, a telephone circuit using a wire with a copper core and an iron sheath ought to talk better than the simple copper wire. Mechanical difficulties of manufacture seem to enhance the cost of such wire to an extent which renders its use questionable from an economic standpoint except under special circumstances. Occasionally it is a valuable aid to the con-

structor, and so the properties of "Bimetallic Wire," as it is called, are given in Table 4.

TABLE No. 4.

Bimetallic Wire.

Numbers, B. & S. G.	Diameters in Mils.	Weights per Mile, Pounds.	Breaking Weight, Pounds.
0000	460	3200	10500
000	410	2537	8600
00	365	2022	7000
0	325	1620	5700
1	289	1264	4600
2	258	1003	3800
3	229	797	3200
4	204	629	2600
5	182	490	1790
6	162	398	1500
7	144	314	1210
8	128	246	1020
9	114	203	850
10	102	157	660
11	91	127	520
12	81	100	410
14	64	63	260
16	51	40	160
18	40	25	100

In the past bimetallic wire has proved specially useful where high conductivity and great strength must be combined; as for example the crossing of a wide river by a single span of open wire. Cases of this kind are in successful operation where the spans are over 2000 ft. long.

TABLE No. 5. — Properties of Copper Wire.

Number B. & S. Gauge.	Diameter "d" in Mils. 1 Mil = .001 Inch.	Area in: Circular Mils d^2.	Area in: Square inches $d^2 \times .7854$.	Weight per Unit of Length. Specific Gravity, 8.89. Pounds per 1000 Feet.	Pounds per Mile.	Feet per Pound.	Ohms per Pound, Annealed.	Resistance in International Ohms at 68° F. According to Matthiessen's Standard of Resistivity. Ohms per 1000 Ft. An-nealed.	Ohms per 1000 Ft. Hard-Drawn.	Ohms per Mile. An-nealed.	Ohms per Mile. Hard-Drawn.	Feet per Ohm An-nealed.	Tensile Sgth. in Pounds. An-nealed.	Tensile Sgth. in Pounds. Hard-Drawn.	Elong. % in 1 Foot. An-nealed.	Elong. % in 1 Foot. Hard-Drawn.	Safe Cur. Amperes. Bright Wire Paneled.	Safe Cur. Amperes. Black Wire in Free Air.	Turns per Lin. In. Cotton Covered.	Temperature Coefficients. Temp. in Deg. C.	Temperature Coefficients. Resistance per Mil-Foot; Ohms.
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
0000	460.000	211600.00	.166190	640.5	3381.4	1.561	.00007639	.04893	.050036	.25835	.26419	20440	5650	9975	45.0	5.00	146.0	346.0	1.6	0	1.00000
000	409.642	167806.43	.131794	508.0	2682.2	1.969	.0001215	.06170	.063094	.32577	.33314	16210	4475	7900	45.0	5.00	127.0	292.0	1.6	1	1.00387
00	364.796	133076.66	.104518	402.8	2126.8	2.482	.0001931	.07780	.079558	.41079	.42007	12850	3550	6250	45.0	4.00	110.0	247.0	1.8	2	1.00776
0	324.861	105534.50	.082887	319.5	1686.9	3.130	.0003071	.09811	.10033	.51802	.52973	10900	2800	4950	45.0	4.00	95.0	209.0	2.8	3	1.01166
1	289.296	83692.67	.065732	253.3	1337.2	3.947	.0004883	.1237	.12649	.65314	.66790	8083	2225	3925	42.0	4.00	83.0	177.0	3.2	4	1.01558
2	257.626	66371.31	.052128	200.9	1060.6	4.977	.0007765	.1560	.15953	.82368	.84239	6410	1750	3125	39.0	3.00	72.0	148.0	3.4	5	1.01950
3	229.422	52634.37	.041339	159.3	841.09	6.276	.001235	.1967	.20114	1.0386	1.0621	5084	1400	2450	36.0	3.00	63.0	127.0	3.9	6	1.02343
4	204.307	41741.32	.032784	126.4	667.39	7.914	.001936	.2480	.25361	1.3094	1.3392	4031	1100	1950	33.0	2.50	55.0	108.0	4.5	7	1.02738
5	181.941	33102.37	.025999	100.2	529.06	9.980	.003122	.3128	.31987	1.6516	1.6889	3197	875	1550	30.5	2.50	48.0	91.0	5.0	8	1.03134
6	162.022	26251.37	.020618	79.46	419.55	12.58	.004963	.3944	.40332	2.0825	2.1295	2535	700	1225	28.0	2.00	42.0	78.0	5.6	9	1.03531
7	144.285	20818.35	.016351	63.02	332.75	15.87	.007892	.4973	.50854	2.6258	2.6850	2011	550	975	26.0	2.00	37.0	66.0	6.2	10	1.03929
8	128.490	16509.64	.012967	49.98	263.89	20.01	.01255	.6271	.64127	3.3111	3.3859	1595	425	775	24.0	1.75	32.0	56.0	7.0	11	1.04328
9	114.434	13092.75	.010283	39.63	209.24	25.23	.01995	.7908	.80876	4.1753	4.2769	1265	325	600	21.5	1.75	28.2	48.0	7.5	12	1.04728
10	101.897	10383.02	.0081548	31.43	165.95	31.82	.03173	.9972	1.0199	5.2657	5.3848	1003	275	475	19.0	1.50	24.9	41.0	8.5	13	1.05129
11	90.743	8234.11	.0064656	24.93	131.63	40.12	.05045	1.257	1.2854	6.6369	6.7869	795.3	200	375	17.5	1.50	21.9	35.0	9.7	14	1.05532
12	80.808	6529.95	.0051287	19.77	104.39	50.59	.08022	1.586	1.6218	8.3741	8.5633	630.7	175	300	16.0	1.25	19.3	30.0	11.2	15	1.05935
13	71.962	5178.48	.0040672	15.68	82.791	63.79	.1276	1.999	2.0443	10.555	10.794	500.1	125	225	14.5	1.25	17.0	25.8	12.0	16	1.06339
14	64.084	4106.72	.0032254	14.43	76.191	80.44	.2028	2.521	2.5779	13.311	13.612	396.6	100	200	13.0	1.25	15.0	22.2	13.0	17	1.06745
15	57.069	3256.78	.0025579	9.858	52.050	101.4	.3225	3.179	3.2508	16.785	17.135	314.5	80	150	12.0	1.25	13.3	19.1	15.3	18	1.07152
16	50.821	2582.74	.0020285	7.818	41.277	127.9	.5128	4.009	4.0996	21.168	21.646	249.4	60	125	11.0	1.25	11.8	16.5	16.7	19	1.07559
17	45.257	2048.29	.0016087	6.200	32.736	161.3	.8153	5.055	5.1692	26.691	27.294	197.8	50	90	10.0	1.25	10.4	14.2	17.7	20	1.07968
18	40.303	1624.30	.0012757	4.917	25.960	203.4	1.296	6.374	6.5180	33.655	34.416	156.9	40	70	9.0	1.25	9.2	12.3	19.5	21	1.08378
19	35.890	1288.13	.0010117	3.899	20.595	256.5	2.061	8.038	8.2196	42.441	43.400	124.4	30	60	8.0	1.20	8.2	10.7	22.7	22	1.08788
20	31.961	1022.53	.00080231	3.092	16.324	323.4	3.278	10.14	10.372	53.539	54.749	98.66	25	45	7.0	1.20	7.2	9.2	27.0	23	1.09200
21	28.463	810.12	.00063626	2.452	12.946	407.8	5.212	12.78	...	67.479	...	78.24	23	39	6.0	1.00	...	...	31.0	24	1.09612
22	25.346	642.45	.00050457	1.945	10.268	514.2	8.287	16.32	...	85.114	...	62.05	21	32	5.0	1.00	...	...	34.4	25	1.10026
23	22.572	509.49	.00040015	1.542	8.142	648.4	13.18	20.32	...	107.29	...	49.21	18	26	5.0	1.00	...	...	38.2	26	1.10440
24	20.101	404.04	.00031733	1.223	6.457	817.6	20.95	25.63	...	135.33	...	39.02	16	20	5.0	1.00	...	...	42.3	27	1.10856
25	17.901	320.42	.00025166	.9699	5.121	1031	33.32	32.31	...	170.59	...	30.95	12	16	5.0	1.00	...	...	47.0	28	1.11272
26	15.940	254.10	.00019958	.7692	4.061	1300	52.97	40.75	...	215.16	...	24.54	...	...	...	...	...	...	52.0	29	1.11689
27	14.196	201.52	.00015827	.6100	3.221	1639	84.23	51.35	...	271.29	...	19.46	...	...	...	...	...	...	57.0	30	1.12107
28	12.642	159.81	.00012551	.4837	2.554	2067	133.9	64.70	...	242.09	...	15.43	...	...	...	...	...	...	63.4	40	1.16332
29	11.258	126.74	.000099536	.3836	2.025	2607	213.0	81.70	...	431.37	...	12.24	...	...	...	...	...	...	70.1	50	1.20625
30	10.025	100.51	.000078936	.3042	1.606	3287	338.6	103.0	...	543.84	...	9.707	...	...	...	...	...	...	77.1	60	1.24965
31	8.928	79.70	.000062599	.2413	1.274	4145	538.4	129.9	...	685.87	...	7.698	...	...	...	...	...	...	84.6	70	1.29327
32	7.950	63.20	.000049643	.1913	1.010	5227	856.2	163.8	...	864.87	...	6.105	...	...	...	...	...	...	92.7	80	1.33681
33	7.080	50.13	.000039368	.1517	.801	6591	1361	206.6	...	1090.8	...	4.841	...	...	...	...	...	...	101.6	90	1.37995
34	6.305	39.75	.000031221	.1203	.635	8311	2165	260.5	...	1375.5	...	3.839	...	...	...	...	...	...	112.1	100	1.42231
35	5.615	31.52	.000024759	.09543	.504	10482	3441	328.4	...	1734.0	...	3.045	...	...	...	...	...	...	119.7	...	...
36	5.000	25.00	.000019635	.07568	.400	13217	5473	414.2	...	2187.0	...	2.414	...	...	...	...	...	...	130.6	...	...
37	4.453	19.83	.000015574	.06001	.317	16666	8702	522.2	...	2757.3	...	1.915	...	...	...	...	...	...	140.6	...	...
38	3.965	15.72	.000012345	.04759	.251	21015	13870	658.5	...	3476.8	...	1.519	...	...	...	...	...	...	151.0	...	...
39	3.531	12.46	.0000097923	.03774	.199	26500	22000	830.4	...	4384.5	...	1.204	...	...	...	...	...	...	163.4	...	...
40	3.145	9.88	.0000077634	.02993	.158	33416	34980	1047.0	...	5528.2	...	.955	...	...	...	...	...	...	177.6	...	...

TABLE NO. 6.—SAFE CURRENTS FOR PANELED WIRE.

Applies to insulated copper wires of 98% conductivity, carrying continuous currents, encased in wooden paneling, so that the temperature elevation of any wire shall not, with the proposed current, exceed 18° F. or 10° C.

AMPERES.	NUMBER IN B. & S.	CIRCULAR MILS.	AMPERES.	NUMBER IN B. & S.	CIRCULAR MILS.	AMPERES.	NUMBER IN B. & S.	CIRCULAR MILS.
1,000	. . .	2160900	225	. . .	297025	15	11	8226
900	. . .	1876900	200	. . .	254016	12	12	6528
800	. . .	1612900	174	0000	211600	10.5	13	5184
700	. . .	1345600	147	000	167805	9.0	14	4110
600	. . .	1100401	124	00	133079	7.25	15	3260
550	. . .	976144	103	0	105592	6.00	16	2581
500	. . .	861184	87	1	83694	5.50	17	2044
475	. . .	804609	73	2	66373	4.00	18	1624
450	. . .	748225	61	3	52634	3.25	19	1253
425	. . .	692224	52	4	41742	2.75	20	1024
400	. . .	640000	43	5	33102	2.25	21	820
375	. . .	586756	36	6	26244	2.00	22	626
350	. . .	535824	30	7	20822	1.75	23	510
325	. . .	485809	25	8	16512	1.50	24	404
300	. . .	435600	22	9	13110	1.25	25	320
275	. . .	388129	18	10	10381	1.00	26	254
250	. . .	342225	. . .	. . .	. . .	. . .	. . .	. . .

TABLE NO. 7.

Fall of Potential in Copper Wire.

NUMBER B. & S. GAUGE.	CIRCULAR MILS.	FALL OF POTENTIAL IN VOLTS PER AMPERE PER 1000 FEET.	NUMBER B. & S. GAUGE.	CIRCULAR MILS.	FALL OF POTENTIAL IN VOLTS PER AMPERE PER 1000 FEET.	NUMBER B. & S. GAUGE.	CIRCULAR MILS.	FALL OF POTENTIAL IN VOLTS PER AMPERE PER 1000 FEET.
0000	211600.00	.0505318	5	33102.00	.3230183	13	5178.40	2.064841
000	167805.00	.0637158	6	26250.50	.4073238	14	4106.80	2.668524
00	133079.40	.0803503	7	20816.00	.5136713	15	3256.70	3.208450
0	105592.50	.1012593	8	16509.00	.6476743	16	2582.90	4.139673
1	83694.20	.1277612	9	13094.00	.8165943	17	2048.20	5.220349
2	66373.00	.1610920	10	10381.00	1.03	18	1624.30	6.582833
3	52634.00	.2031469	11	8234.00	1.298521	19	1252.40	8.537567
4	41742.00	.2561507	12	6529.90	1.637494	20	1021.50	10.46789

16. Aluminum Wire.—A few years ago improvements in the process of manufacturing aluminum reached such perfection that the metal became a commercial possibility, and is now available as a conductor material. While aluminum is an exceedingly light metal, its tensile strength and conductivity are low compared to hard drawn copper. Aluminum conductors, therefore, to have the same resist-

ance, must be much larger than copper, and so expose a greater area to the action of the wind, and upon which snow may collect. As now manufactured, aluminum wire shows an average resistance of about 17.6 ohms per mil-foot at 75° F. This corresponds to 423 lbs. per mile-ohm. The tensile strength varies from 20,000 lbs. to 50,000 lbs., averaging 33,000 lbs. per square inch or .002592 lbs. per circular mil. But different manufacturers produce several different grades of commercial metal, and the product of one firm is not always exactly like that of others. The Pittsburg Reduction Co. give the following constants for some of their output:

	Aluminum.	Copper.
Specific gravity	2.68	8.93
Relative specific gravity	1.00	3.33
Conductivity	54% to 63%	96% to 99%
Tensile strength per sq. in., aluminum	24,000 to 55,000 lbs.	
Tensile strength per sq. in., copper	30,000 to 65,000 lbs.	

The different grades and consequent different mechanical and electrical characteristics of both aluminum and copper make it impossible to give absolute ratios between the two metals for weight, tensile strength, price, etc., for cross-sections of equal conductivity.

On account of the lower conductivity of aluminum, the cross-sections of aluminum and copper of same resistance will be in the inverse ratio of their conductivities. Thus if an aluminum wire of 159,000 c.m. be assumed, the copper wire of equal conductivity is found by the following proportions (taking average value):

$$97 : 61 :: 159{,}000 : x;$$
$$x = 100{,}000.$$

This ratio is approximately the same as that between wires separated by one number in the Brown & Sharp gauge. Thus No. 1 aluminum wire is equivalent to No. 3 copper; No. 2 is equivalent to No. 4 copper, etc.

The comparative weights of equal volumes of aluminum and copper are as 1 for aluminum and 3.33 for copper. Therefore the relative weights of given lengths of same conductivity will be as 47.77 for aluminum to 100 for copper ($159\times1 : 100\times3.33 :: 47.77 : 100$),

or, the weight per mile or per thousand feet of aluminum wire is 47.77 per cent. of the weight of the same length of copper of same conductivity.

The question of the relative cost of aluminum or copper is a question not of price per pound, but price per mile or of any given length of equal conductivity. The weight per mile of aluminum is 47.77 per cent. of the weight of copper of same length and of same conductivity. Thus aluminum conductors will cost per mile the same as copper of equal conductivity if the price per pound of the copper is 47.77 per cent. of the price per pound for the aluminum. Take, for example, copper at 20 cents per pound and aluminum at 41.87 cents per pound. The cost per mile for wire of same conductivity in either metal will be the same. (100 lbs. copper at 20 cents = $20.00; 47.77 lbs. aluminum at 41.87 cents = $20.00.)

Reduced to the form of rules the calculations are as follows:

To obtain aluminum price corresponding to a known copper price, divide the cost of copper per pound by .4777 or multiply by 2.1.

To obtain copper price corresponding to a known aluminum price, multiply the cost of aluminum per pound by .4777 or divide by 2.1.

In case aluminum of conductivity other than 61% is used, the factors (.4777, 2.1, and 159) as obtained above would be correspondingly changed.

Table No. 8 gives the factors for the different conductivities of aluminum as compared with 97% conductivity copper.

The following specification shows the quality obtainable in a No. 2 wire of 59% conductivity:

Weight.—The weight shall not exceed 320 pounds per mile.

Conductivity and Resistance.—Conductivity not to be less than 59 in Matthiessen Standard Scale, so the wire must not have a resistance at 15.5° C. greater than .2443 ohms per 1000 feet.

Tensile Strength and Elastic Limit.—The tensile strength not to be less than 29,000 pounds per square inch and the elastic limit not less than 14,000 pounds per square inch.

Elongation.—The elongation of tension test pieces not to be less than 10 per cent. in 2 inches.

Ductility.—Wire shall be capable of being wrapped six turns around its own diameter, unwrapped, and again wrapped six turns

TABLE 8.

Factors for the Different Conductivities of Aluminum.

Conductivity of Aluminum, per cent.	63	62	61	60	59	58	57	56	55	54
Relative cross-section (Copper equal 100.)	154	156.5	159	161.7	164.4	167.3	170.2	173.2	176.3	179.7
Weight of aluminum (weight of copper of equal length and equal resistance equal 100)........	46.25	47	47.77	48.55	49.38	50.24	51.11	52.02	52.97	53.95
Tensile Strength — Factor by which to multiply tensile strength per sq. in. of aluminum to obtain tensile strength per sq. in. required in a copper wire of equal resistance in order to secure same breaking strength.	154	156.5	159	161.7	164.4	167.3	170.2	173.2	176.3	179.7
Price — Factor by which to multiply copper price per lb. to obtain equivalent price of aluminum; also factor by which to divide aluminum price per lb. to obtain equivalent price of copper	2.16	2.13	2.1	2.06	2.03	1.99	1.96	1.92	1.89	1.85
Price — Factor by which to divide copper price per lb. to obtain equivalent price of aluminum; also factor by which to multiply aluminum price to obtain equivalent price of copper...........	.4625	.47	.4777	.4855	.4938	.5024	.5111	.5202	.5297	.5395

around its own diameter in the same direction as the first wrapping without showing any cracks.

In case a No. 2 wire of 62 conductivity is desired, the specification above would be changed in the following points:

Conductivity and Resistance.—Conductivity shall not be less than 62 in Matthiessen Standard Scale, so the wire must not have a resistance at 15.5° C. greater than .2322 ohms per 1000 feet.

Tensile Strength and Elastic Limit.—The tensile strength shall not be less than 25,000 pounds per square inch and the elastic limit not less than 11,000 pounds per square inch.

Elongation.—The elongation of tension test pieces shall not be less than 9 per cent. in 2 inches.

On account of the small sizes of wire used for telephone and telegraph work, the breaking of the wire must be specially guarded against. So a wire of a slightly reduced conductivity but high tensile strength is advised, the low conductivity being unobjectionable. Table No. 9 gives the characteristics of telephone wire.

TABLE NO. 9.

Resistance and Tensile Strength of Aluminum Telephone Wire.

Grade.	Conductivity.	Comparative Section of Equal Conductivity Copper at 100.	Comparative Weight of Given Lengths of Equal Conductivity Copper at 100.
A0	62%	156.4	47
A75	58%	167	50.2
A2	54%	180	54

Number in B. & S. Gauge.	Diameter in Mills.	GRADE A0. Resistance per 1000 Ft. at 75° F.	GRADE A0. Tensile Strength per Sq. In.	GRADE A75. Resistance per 1000 Ft. at 75° F.	GRADE A75. Tensile Strength per Sq. In.	GRADE A2. Resistance per 1000 Ft. at 75° F.	GRADE A2. Tensile Strength per Sq. In.
		Ohms	Lbs.	Ohms.	Lbs	Ohms.	Lbs.
4	204.31	.4012	27000	.4288	33000	.4605	40000
5	181.94	.5058	27500	.5408	34000	.5818	42000
6	162.02	.6380	28000	.6820	35000	.7325	44000
7	144.28	.8044	29000	.8600	36000	.9235	46000
8	128.49	1.034	30000	1.105	37000	1.187	48000
9	114.43	1.278	32000	1.367	39000	1.468	50000
10	101.89	1.613	33000	1.724	40000	1.852	51000
11	90.74	2.033	35000	2.173	41000	2.335	53000
12	80.81	2.565	39000	2.741	42000	3.084	55000

The difficulty of connecting separate pieces of aluminum wire was an obstacle that delayed the general use of this conductor material.

Small wires can be joined by twisting the ends around each other, as is usually done in telephone and telegraph work.

For larger sizes the sleeve joint or compression joint shown in Fig. 6 is preferable. Joints of this kind are easily and quickly made; they possess mechanical strength and conductivity equal to that of the wire. The extended area of the contact surfaces, the

Fig. 6. Aluminum Cable Joints.

adhesion by which the surfaces are held together, and the protection from corrosion insure a permanent joint.

Cable joints are made by inserting the ends of the cable into a cast aluminum sleeve of proper size, the ends butting together in the center. The sleeve is then inserted between dies in a hydraulic jack and pressure applied which causes the metal of the sleeve and of the cable to flow together into a solid homogeneous mass; and the sleeve takes hold of the cable so strongly that when tested the cable will be pulled in two instead of pulling out.

Such joints are made in the field by means of a portable hydraulic jack and dies. The entire weight of these tools is less than 200 lbs., the largest piece weighing about 150 lbs.

There is a modified form of this joint in which the compression is all done in the factory, terminals being compressed upon the ends of the cable, the free ends of the terminals being threaded, one end having a right-hand and the other a left-hand thread.

When two ends are to be united, this is effected by screwing into the terminals a right- and left-hand threaded stud, and the use of this joint enables the ends of the cable to be connected in less than a minute, when the actual work of connecting comes to be done.

The general properties of pure aluminum wires of all sizes are shown in TABLES 10, 11, and 12.

17. Attempts have also been made to use the various alloys of copper with silicon and phosphorus, known under the names of phosphor-bronze and silicon-bronze. While these alloys have very high tensile strength, in some cases exceeding 80,000 pounds to the square inch, their conductivity is so low that they have had but very little commercial extension. In a few cases electric railways have used silicon-bronze for trolley wire, but the practice at the present time is almost exclusively confined to the adoption of hard-drawn copper. The properties of silicon-bronze are given in TABLES NOS. 13, 14, and 15 (see following pages).

18. **Iron Wire.**—Iron wire is never used for the transmission of large amounts of electrical energy because its resistance is so great as to make line losses prohibitory. Formerly all telegraph and telephone lines were built of iron, but of late years first-class telephone construction has used copper exclusively, and even in the telegraphic field the use of iron is declining.

TABLE No. 10.

Resistance of Aluminum Wire.

B. & S. Nos.	Resistance at 75° F., Ohms. Per 1000 Ft.	Per Mile.	Feet per Ohm.	Ohms per Pound.
0000	.08177	.43172	12229.8	.00042714
000	.10310	.54440	9699.0	.00067022
00	.13001	.68645	7692.0	.00108116
0	.16385	.86515	6245.4	.0016739
1	.20672	1.09150	4637.35	.0027272
2	.26077	1.37637	3836.22	.0043441
3	.32872	1.7357	3036.12	.0069057
4	.41448	2.1885	2412.60	.0109773
5	.52268	2.7597	1913.22	.017456
6	.65910	3.4802	1517.22	.027758
7	.83118	4.3885	1203.12	.044138
8	1.06802	5.5355	964.18	.070179
9	1.32135	6.9767	756.78	.111561
10	1.66667	8.8000	600.00	.17467
11	2.1012	11.0947	475.908	.28211
12	2.6497	13.9900	377.412	.44856
13	3.3412	17.642	299.298	.71478
14	4.3180	22.800	231.582	1.16225
15	5.1917	27.462	192.612	1.7600
16	6.6985	35.368	149.286	2.8667
17	8.4472	44.602	118.380	4.5588
18	10.6518	56.242	93.882	7.2490
19	13.8148	72.942	72.384	12.1916
20	16.938	89.430	59.0406	18.328
21	21.358	112.767	46.8222	29.142
22	26.920	142.138	37.1466	46.316
23	33.962	179.32	29.4522	73.686
24	42.825	226.12	23.3508	117.170
25	54.000	285.12	18.5184	186.28
26	68.113	359.65	14.6814	296.32
27	85.865	453.37	11.6460	485.56
28	108.277	571.70	9.2358	749.02
29	136.535	720.90	7.3242	1190.97
30	172.17	908.98	5.8087	1893.9
31	212.12	1119.98	4.7144	2941.5
32	273.97	1445.45	3.6528	4788.9
33	345.13	1822.3	2.8974	7610.7
34	435.38	2298.8	2.2969	12109.4
35	548.92	2898.2	1.8218	19251
36	692.07	3654.2	1.4449	30600
37	872.93	4609.2	1.1456	48661
38	1100.62	5811.2	.9086	76658
39	1387.47	7325.8	.7207	121881
40	1749.50	9236.8	.5716	193835

Pure iron wire shows a resistance of about six times that of copper, giving 57.06 ohms per mil-foot at 32° F. and 4218 lbs. per mile-ohm. The best commercial iron wire shows a resistance of 59.6 ohms per mil-foot and a weight per mile-ohm of from 4500 to 4600 lbs. The best Swedish iron wire has a tensile strength of 50,000 lbs. per square inch, an elongation of 16 per cent., and should show on a torsion test 18 to 20 twists in a length of 6 inches. The various grades of steel are much

TABLE No. 11.

Table of Weights of Weather-proof Aluminum Wire.

Number B. & S. Gauge.	Triple Braid.		Double Braid.	
	Weight per 1000 Feet.	Weight per Mile.	Weight per 1000 Feet.	Weight per Mile.
0000	284	1497	266	1402
000	234	1235	218	1151
00	195	1031	169	893
0	162	856	127	671
1	131	694	103	546
2	110	581	90	475
3	89	470	74	390
4	72	380	62	327
5	63	333	55	291
6	56	294	45	236
8	37	196	35	186
10	29	154	23	122
12	19	101	16	86
14	16	86	11	60
16	10	50	8	40
18	8	40	7	35

TABLE No. 12.

Stranded Aluminum Wire.

(Diameter and Properties.)

Number B. & S. Gauge.	Circular Mils.	Diameters.		Weight in Pounds.			Resistance in Ohms at 75° F. per 1000 Feet.
				Bare.		Triple Braid Insulated.	
		Decimal Parts of an Inch.	Nearest 32d of an Inch.	Per 1000 Feet.	Per Mile.	Per 1000 Feet.	
....	1000000	1.152	1 5/16	920	4860	1408	.01675
....	950000	1.125	1 1/8	874	4617	1340	.01763
....	900000	1.092	1 3/32	828	4374	1270	.01861
....	850000	1.062	1 1/16	782	4131	1202	.01969
....	800000	1.035	1 1/32	736	3888	1135	.02092
....	750000	.999	1	690	3645	1067	.02232
....	700000	.963	31/32	644	3402	1001	.02392
....	650000	.927	15/16	598	3159	938	.02575
....	600000	.891	29/32	552	2916	878	.02789
....	550000	.855	7/8	506	2673	806	.03044
....	500000	.819	13/16	460	2430	740	.03347
....	450000	.770	25/32	414	2187	665	.03720
....	400000	.728	3/4	368	1924	567	.04184
....	350000	.679	11/16	322	1701	502	.04782
....	300000	.630	5/8	276	1458	436	.05585
....	250000	.590	19/32	230	1215	375	.06698
0000	211600	.530	17/32	195	1028	280	.07912
000	167805	.470	15/32	155	816	232	.09958
00	133079	.420	7/16	123	647	192	.12563
0	105534	.375	3/8	97	513	155	.1584
1	83694	.330	11/32	77	407	132	.2004
2	66373	.291	5/16	61	323	108	.2515
3	52634	.261	9/32	48.5	256	88	.3182
4	41742	.231	1/4	38.5	203	72	.4012

used in wire-making. The softest makes of Bessemer and open-hearth show from 5000 to 6000 lbs. per mile-ohm and a tensile strength of from 60,000 to 65,000 lbs. per square inch. Cast steel can be made with a tensile strength of 150,000 to 200,000 lbs. per square inch, but its resistance is then very high, being from 10,000 to 15,000 lbs. per mile-ohm. As the soft steels are much less expensive than the pure irons, it is cheaper to use steel wire one or two gauge numbers larger than to purchase the wire of higher conductivity.

19. Galvanizing and Tinning. — As a protection against corrosion, it is customary to coat iron and steel wire with a thin film of zinc, which, being not readily oxidized, serves as a barrier against the destructive action of the elements. While for open country lines this expedient is of considerable value, for city work galvanizing has relatively but little importance; for the various sulphur compounds, so largely present in the smoky atmosphere of towns, act with great rapidity on zinc, cutting away the film, and leaving the iron unprotected. Furthermore, with the best possible care in galvanizing, the coating is never perfectly continuous; and subsequent mechanical operations frequently cut through the zinc, exposing the iron, which immediately commences to oxidize. It has even been asserted that, in view of the inevitable discontinuity of the protecting film, the zinc was a source of evil, forming with the iron a voltaic pair, thus aiding corrosion.

The operation of galvanizing is accomplished by immersing the coil of wire in a pickling bath of dilute sulphuric acid, which serves to remove the scale, rust, and grease, leaving a chemically pure surface for the reception of the zinc. The coil is then placed upon a reel from which it is slowly unrolled, being drawn through a bath of molten zinc, the surface of which is covered with a layer of salammoniac or similar flux. It is necessary that the wire should be immersed in the bath for a sufficient time to become fully heated, in order that the zinc coating may be firmly coherent. As the wire emerges from the bath, the superfluous zinc is wiped away by means of an asbestos roller or similar device. Galvanized wire should be very carefully inspected to see that the zinc coating is, on the one hand, thoroughly continuous; and that, upon the other, the superfluous zinc has been carefully removed, freeing the wire from bunches and lumps, and leaving it with a smooth and polished surface. It is also advisable to test galvanized samples by immers-

TABLE NO. 13.

Properties of the Aluminum Brass and Bronze Co.'s Silicon Bronze Wire.

Diameter in Inches.	Sectional Area in Square Inches.	Weight of one Mile in Pounds.	GRADE B. Silicon Bronze with a tensile strength of 80,000 lbs. per sq. inch and a ductility of 110 twists in 6 inches.		GRADE C. Silicon Bronze with a tensile strength of 90,000 lbs. per sq. inch and a ductility of 75 twists in 6 inches.		GRADE D. Silicon Bronze with a tensile strength of 100,000 lbs. per sq. inch and a ductility of 60 twists in 6 inches.	
			Tensile Strength in Pounds.	Resistance per Mile in Ohms.	Tensile Strength in Pounds.	Resistance per Mile in Ohms.	Tensile Strength in Pounds.	Resistance per Mile in Ohms.
0.005	.0000196	.4	1.6	5119	1.8	5119	2	5119
.010	.0000785	1.6	6.3	1280	7.1	1280	8	1280
.015	.0001767	3.6	14.1	569	16.0	569	18	569
.020	.0003142	6.4	25.1	320	28.3	320	31	320
.025	.0004909	10.	39.2	205	44.2	205	49	204.7
.030	.0007069	14.4	56.6	142	63.6	142	71	142.2
.035	.0009621	19.6	77.6	105	85.4	105	93	104.5
.040	.0012566	25.6	100	80	113.0	80	126	80.
.045	.0015904	32.4	127	63	143	63	159	63.22
.050	.0019635	40.	157	51	177	51	196	51.19
.055	.0023758	48.4	190	42	214	42	238	42.32
.060	.0028274	57.6	226	35	254	35	283	35.56
.065	.0033183	67.6	265	30	299	30	332	30.33
.070	.0038484	78.4	307	26	346	26	385	26.12
.075	.0044179	90.	353	23	400	23	445	22.76
.080	.0050265	102.4	402	20	452	20	503	20.00
.085	.0056745	115.6	454	17.7	511	17.7	567	17.71
.090	.0063617	129.6	509	15.8	573	15.8	636	15.80
.095	.0070882	144.4	567	14.2	638	14.2	709	14.18
.100	.007854	160.0	628	12.8	707	12.8	785	12.80
.105	.008659	176.4	693	11.6	779	11.6	866	11.63
.110	.009503	193.6	760	10.6	855	10.6	950	10.58
.115	.010387	211.6	831	9.7	935	9.	1039	9.68
.120	.011310	230.4	905	8.9	1018	8.9	1131	8.88
.125	.012272	250.	982	8.2	1105	8.2	1227	8.19
.130	.013273	270.4	1062	7.6	1194	7.6	1329	7.57
.135	.014314	291.6	1145	7.0	1288	7.0	1430	7.03
.140	.015394	313.6	1232	6.5	1386	6.5	1539	6.53
.145	.016513	336.4	1321	6.1	1486	6.1	1651	6.08
.150	.017672	360.	1414	5.7	1591	5.7	1767	5.69
.155	.018869	384.4	1510	5.3	1698	5.3	1887	5.35
.160	.020106	409.6	1608	5.0	1810	5.0	2010	5.00
.165	.021382	435.6	1711	4.7	1924	4.7	2138	4.70
.170	.022698	462.4	1816	4.4	2043	4.4	2270	4.43
.175	.024053	490.	1924	4.2	2165	4.2	2405	4.18
.180	.025447	518.4	2036	4.0	2290	4.0	2545	3.95
.185	.026880	547.6	2150	3.7	2419	3.7	2688	3.74
.190	.028353	577.6	2268	3.6	2552	3.6	2835	3.55
.195	.029865	608.4	2389	3.4	2688	3.4	2987	3.37
0.200	.031416	640.	2513	3.2	2826	3.2	3142	3.20

TABLE No. 14.

Properties of the Aluminum Brass and Bronze Co.'s Silicon Bronze Wire.

Diameter in Inches.	Sectional Area in Square Inches.	Weight of one Mile in Pounds.	GRADE E. Silicon Bronze with a tensile strength of 130,000 lbs. per sq. inch and a ductility of 45 twists in 6 inches.		GRADE F. Silicon Bronze with a tensile strength of 150,000 lbs. per sq. inch and a ductility of 30 twists in 6 inches.		GRADE A. Comp. Silicon Bronze with a tensile strength of 75,000 lbs. per sq. inch and a ductility of 75 twists in 6 inches.	
			Tensile Strength in Pounds.	Resistance per Mile in Ohms.	Tensile Strength in Pounds.	Resistance per Mile in Ohms.	Tensile Strength in Pounds.	Resistance per Mile in Ohms.
0.005	.0000196	.4	2.5	10238	2.9	10238	1.5	2560
.010	.0000785	1.6	11.2	2560	11.9	5620	6.	640
.015	.0001767	3.6	22.0	1138	26.5	1138	13.2	285
.020	.0003142	6.4	40.8	640	47.2	640	23.6	160
.025	.0004909	10.	63.8	409	73.6	409	36.8	102
.030	.0007069	14.4	91.9	284	106	284	53	71
.035	.0009621	19.6	120.3	209	139	209	70	52
.040	.0012566	25.6	163	160	188	160	94	40
.045	.0015904	32.4	207	126	239	126	120	32
.050	.0019635	40.	255	102	295	102	148	25
.055	.0023758	48.4	309	85	356	85	178	21
.060	.0028274	57.6	367	71	424	71	214	17.8
.065	.0033183	67.6	431	61	498	61	249	15.2
.070	.0038484	78.4	500	52	577	52	289	13.1
.075	.0044179	90.	578	45	667	45	334	11.4
.080	.0050265	102.4	653	40	*754	*40	377	10
.085	.0056745	115.6	738	35	856	35	428	8.8
.090	.0063617	129.6	827	31	954	31	477	7.9
.095	.0070882	144.4	921	28	1063	28	532	7.1
.100	.007854	160.0	1021	25	1183	26	592	6.4
.105	.008659	176.4	1127	23	1298	23	649	5.8
.110	.009503	193.6	1235	21	1425	21	713	5.3
.115	.010387	211.6	1350	19.3	1558	19.3	779	4.8
.120	.011310	230.4	1470	17.8	1696	17.8	848	4.4
.125	.012272	250.	1595	16.4	1841	16.4	921	4.1
.130	.013273	270.4	1726	15.1	1991	15.1	995	3.8
.135	.014314	291.6	1861	14.1	2147	14.1	1073	3.5
.140	.015394	313.6	2000	13.1	2309	13.1	1155	3.2
.145	.016513	336.4	2147	12.2	2478	12.2	1239	3.0
.150	.017672	360.	2297	11.4	2651	11.4	1326	2.8
.155	.018869	384.4	2453	10.7	2830	10.7	1415	2.6
.160	.020106	409.6	2614	10.0	3016	10.0	1508	2.5
.165	.021382	435.6	2780	9.4	3207	9.4	1603	2.4
.170	.022698	462.4	2951	8.9	3404	8.9	1702	2.2
.175	.024053	490.	3127	8.4	3608	8.4	1804	2.1
.180	.025447	518.4	2309	7.9	3817	7.9	1908	1.9
.185	.026880	547.6	3494	7.5	4032	7.5	2016	1.8
.190	.028353	577.6	3686	7.1	4253	7.1	2126	1.8
.195	.029865	608.4	3883	6.7	4480	6.7	2240	1.7
0.200	.031416	640.	4084	6.4	4712	6.4	2356	1.6

TABLE No. 15.

Table of Silicon Bronze Wire Weights and Electrical Resistances.

MANUFACTURED BY THE PHOSPHOR BRONZE COMPANY, LIMITED. LONDON.

Nearest B. W. G.	Diameter in Mils.	Diameter in Millimeters.	Sectional Area in Millimeters.	Weight per Kilometer in Kilograms.	Weight per Mile in Pounds.	Quality A for Telegraph Lines, etc.		Quality B for Railway Telegraphs, etc.		Quality C for Telephone Lines, etc.	
						Electrical Resistance at 0° C in Ohms per Kilo.	Electrical Resistance at 32° F in Ohms per Mile.	Electrical Resistance at 0° C in Ohms per Kilo.	Electrical Resistance at 32° F in Ohms per Mile.	Electrical Resistance at 0° C in Ohms per Kilo.	Electrical Resistance at 32° F in Ohms per Mile.
8	158	4.0	12.5664	112.00	400	1.32	2.12	1.54	2.47	...	...
9	148	3.75	11.0446	98.44	348	1.51	2.42	1.83	2.94	...	...
9	138	3.50	9.6211	86.75	304	1.73	2.77	2.09	3.35	...	...
10	128	3.25	8.2968	73.94	261	2.01	3.22	2.43	3.86	...	...
11	118	3.0	7.0685	63.00	223	2.36	3.78	2.85	4.57	...	...
11	114	2.9	6.6052	58.87	210	2.53	4.05	3.05	4.90	...	...
11	110	2.8	6.1575	54.88	195	2.71	4.33	3.28	5.25	...	...
12	106	2.7	5.7255	51.03	181	2.91	4.65	3.52	5.64	...	...
12	102	2.6	5.3093	47.32	168	3.14	5.02	3.80	6.09	...	...
12	99	2.5	4.9087	43.75	155	3.40	5.44	4.11	6.60	...	...
13	95	2.4	4.5238	40.32	143	3.69	5.91	4.29	6.90	...	...
13	91	2.3	4.1547	37.03	131	4.02	6.43	4.46	7.15	...	...
13	87	2.2	3.8013	33.88	120	4.39	7.02	5.33	8.55	...	...
14	83	2.1	3.4636	30.87	110	4.82	7.71	5.82	9.32	...	...
14	79	2.0	3.1415	28.00	100	5.31	8.50	6.42	10.30	12.24	19.60
14	75	1.9	2.8352	25.27	92	5.89	9.43	7.00	11.26	13.56	21.70
15	71	1.8	2.5446	22.68	82	6.56	10.50	7.93	12.70	15.11	24.18
15	67	1.7	2.2698	20.23	73	7.37	11.79	8.89	14.25	16.94	27.00
16	63	1.6	2.0106	17.92	64	8.31	13.29	10.04	16.09	19.13	31.35
17	59	1.5	1.7671	15.75	55½	9.45	15.12	11.42	18.30	21.77	35.00
17	55	1.4	1.5393	13.72	48	10.85	17.36	13.11	21.00	24.98	40.00
18	51	1.3	1.3273	11.83	42	12.59	20.14	15.20	24.40	28.98	46.00
18	48	1.25	1.2272	10.93	38½	13.64	21.82	16.35	26.19	30.65	49.00
18	47	1.2	1.1309	10.08	36	14.77	23.63	17.87	28.80	34.01	54.00
19	43	1.1	0.9502	8.47	30	17.58	28.12	21.24	34.00	40.47	66.00
19	40	1.0	0.7854	7.00	25	21.28	34.00	25.70	42.00	48.98	79.00
20	36	0.9	0.6362	5.67	20	...	...	...	...	60.46	98.00
21	31	0.8	0.5026	4.48	16	...	...	...	...	73.40	118.00

ing them for several minutes in a solution of sulphate of copper. Any discontinuity in the coating is thereby immediately made manifest by a red spot of precipitated copper. Copper wire which is to be insulated with any of the rubber compounds should be protected by a coating of tin applied in a similar manner, for in the absence of this protection the sulphur universally present in rubber insulations is likely to combine with the copper. Indeed, there are cases on record where wires of small diameter have been by this cause entirely corroded away, and the circuit thus destroyed. Protective

circuits, such as those used for fire and burglar alarm signals, should be specially guarded against this evil.

20. Insulated Wire. — Since the widespread introduction of currents of high pressure, it has become exceedingly important to protect all exposed wires with a sufficient coating of insulating material, so that such circuits may be rendered reasonably safe, and may not become sources of danger to human life. As a result, the wire manufacturers have adopted the custom of covering their product with insulating material of various kinds. This insulating material usually takes the form of a hard braid, composed of either cotton or hemp woven onto the wire, and saturated with some of the compounds of india rubber, or with one of the pitches, tars, or resins. It is obvious that layer after layer of this insulating material may be wound upon the wire, so as to make a covering of almost any desired thickness. When selecting an insulating covering, great attention should be paid to its power of resisting abrasion. The greatest enemy to overhead electric circuits is found in the various tree branches which constantly, by the wind, are brought into contact with the wire, and tend to abrade and destroy the insulation. The insulating covering, therefore, should be tested by rubbing it against a coarse surface similar to the bark of a tree, and noting the time that is required to cut through and expose the wire. Where trees are numerous along the route, no fibrous insulation will stand for a long period of time. A very successful expedient, however, has been found in the device of covering the wire in its passage through the trees with a coating of bamboo. This coating is obtained by sawing ordinary fishing-poles longitudinally, and with a gouge cutting out the knots which occur in the cane. After being prepared in this manner the bamboo may be lashed upon the wire, and it is found that the hard silicious surface of the cane will resist for almost an indefinite period the abrasive action of swaying branches. Other attempts have been made to secure a reliable "tree-wire" by covering the first coating of insulation upon the conductor itself, with an armor formed of a braid of steel wire, or a strip of iron or other metal, spirally wrapped around the insulation. Doubtless such expedients could be made successful; but in order to save expense and secure flexibility, the armor in the usual commercial forms is so light that it rarely survives, for a considerable period, the action of rust

and abrasion. So, while the bamboo expedient is clumsy, it is successful. As insulated wire is usually sold by weight, information as to the gross weight per mile of the more common commercial forms becomes of value to the designer. In TABLE No. 16 the present commercial forms are tabulated.

The insulating properties of a given covering are usually tested by coiling a length of the wire of from 100 to 1000 feet, depending on the quality of the insulation, in a tub of water, and measuring the leakage current with a sensitive galvanometer. If possible, the battery employed should have at least double the voltage of the current for which the wire is subsequently to be used. For telephone and telegraph cables, it is usual to use from 200 to 500 volts. For other circuits, the test is made with two or three times the voltage that is to be used on the line. After prolonged soaking in the testing tub, the wire battery and galvanometer are joined up in series, the remaining pole of the battery being connected to a plate of metal placed in the tub, and the leakage current measured. While 100 megohms per mile is a very common requirement, it is impossible, in view of the widely varying character of insulations and circuits, to give a definite standard.

21. Flexible Cable. — In this country it is not uncommon to use wire in a single rod up to $\frac{1}{2}$ inch in diameter. Large sizes are, however, exceedingly stiff and difficult to handle, so that for greater diameters, and in many cases for $\frac{1}{2}$ inch or less, it becomes essential to use a stranded conductor in order to obtain the requisite flexibility. Twisted cables are from 10 to 15 per cent more expensive than solid conductors; for, owing to the spiral arrangement of the strands, there is from 1 to 3 per cent more metal per unit of cross-section and length than in a solid conductor of equal resistance. The process of manufacture, also involving two or three additional handlings, adds to the cost; and as the stranded conductor is more bulky, a considerably larger quantity of insulation is required. The ease of erection, however, will in many cases, largely if not entirely, compensate for the increased expenditure in raw material in the stranded conductor. The properties of common commercial flexible cable may be found in TABLE No. 17.

The use of flexible cable may obviously be avoided by stringing a sufficient number of separate wires, and joining them in multiple

TABLE No. 16.

Giving Approximate Weights per Mile of Various Insulated Wires. All Weights are in Lbs. per Mile.

Number B. & S. Wire Gauge.	Area in Circular Mils.	KINDS OF INSULATION AND MAKER.														B. & P. Wire.	Aluminium, Brass, and Bronze Co.	Underwriter's Wire.
		J. A. Roebling's Sons.			Edison Co.				Simplex Co.			Ansonia Brass and Copper Co.						
		Bare.	Double Braid.	Triple Braid.	Grade 1.		Tree Wire.		T. Z. R.	Braided.	Plain Rubber.	Weather Proof.		Weather and Fire Proof.				
					Solid.	Stranded.	Solid.	Stranded.				Double Braid.	Triple Braid.	Double Braid.	Triple Braid.			
...	500000	8448	...	9858	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	450000	7603	...	8870	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	400000	6758	...	7883	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	350000	Stranded 5914	...	6901	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	300000	5089	...	5914	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	250000	4224	...	4926	...	...	...	...	...	...	...	...	...	...	...	...	...	...
0000	211600	3386	3712	3902	3828	4065	4013	4530	3940	4890	4125	3696	3960	3538	3977	...	3727	3960
000	167805	2685	2936	3157	3062	3210	3221	3609	3200	4270	3340	3036	3168	2930	3136	...	3181	2957
00	133079	2129	2397	2561	2534	2465	2693	2914	2520	3550	2700	2429	2640	2323	2640	2425	2347	2600
0	105592	1688	1932	2086	1980	2028	2112	2402	2020	3010	2205	1917	2006	1880	1930	1895	1920	1990
1	83694	1339	1521	1653	1621	1748	1763	1921	1640	2175	1745	1531	1637	1500	1605	1491	1509	1600
2	66373	1062	1225	1326	1294	1420	1409	1573	1255	1685	1400	1257	1320	1240	1320	1180	1133	1270
3	52634	842	987	1082	1030	1003	1129	1161	1015	1360	1165	1019	1056	1003	1108	940	1056	1025
4	41742	668	803	887	776	845	871	924	810	1010	905	764	845	834	892	758	834	1880
5	33102	530	649	734	639	745	713	797	665	860	730	607	692	670	712	620	681	720
6	26244	420	528	597	523	618	634	687	550	715	620	501	581	538	580	505	571	575
7	20817	333	407	465	422	496	491	565	470	540	450	411	465	449	491	415	...	450
8	16512	264	333	391	349	396	412	423	400	460	388	335	380	364	422	344	364	390
9	13110	210	275	322	290	343	343	364	340	370	310	...	...	...	...	...	...	300
10	10381	166	227	269	225	269	264	333	280	340	265	225	257	252	290	241	264	285
11	8226	132	190	227	196	...	237	...	...	...	...	...	...	...	...	...	...	190
12	6528	105	153	195	164	...	195	...	195	220	165	142	180	163	200	146	176	158
13	5184	...	...	...	137	...	174	...	...	...	...	...	...	...	...	...	...	132
14	4110	...	...	...	121	...	148	...	128	155	110	95	116	105	137	100	117	111
15	3260	...	...	...	106	...	126	...	...	...	...	...	...	...	...	...	...	85
16	2581	...	...	...	79	...	100	...	95	102	73	74	85	79	106	65	81	74
17	2044	...	...	...	68	...	85	...	...	...	...	...	...	...	...	...	...	68
18	1624	...	...	...	60	...	74	...	80	75	53	...	...	...	...	43	59	64
19	1253	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	58
20	1024	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	52

TABLE NO. 16. — *Continued.*

Number B. & S. Wire Gauge.	Area in Circular Mils.	KIND OF INSULATION AND MAKER.																	
		SHIELD BRAND.				Bishop Rubber Wire.	AMERICAN CIRCULAR LOOM CO.				OKONITE CO.								
											Plain Okonite.			Braided Insulation.			Candee Wire.		
		Stranded.	Double Braid.	Triple Braid.	Quadruple Braid.		No. 1.	No. 2.	No. 3.	No. 4.	Solid.	Stranded.	Switch Cords.	Solid.	Stranded.	Switch Cords.	Copper.	Iron.	Armor Cable.
...	500000	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	450000	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	400000	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	350000	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	300000	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	250000	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
0000	211600	3975	3690	3911	4106	...	3787	4023	4273	4533	...	5723	...	...	5988	...	4100	...	25476
000	167805	3099	3070	3250	3412	...	2983	3255	3385	3663	...	4176	...	...	4408	...	2960	...	22040
00	133079	2455	2390	2523	2657	...	2369	2640	2721	3000	3514	3792	3384	3684	3978	3534	2403	...	20883
0	105592	1927	1865	1955	2046	...	1917	2160	2413	3686	2830	3036	3220	2970	3206	3390	1942	...	18930
1	83694	808	1477	1548	1619	...	1528	1745	1942	2169	2150	2390	2364	2270	2522	2491	1536	...	15365
2	66373	607	1169	1224	1279	...	1231	1426	...	...	1633	2090	2005	1738	2222	2128	1249	...	14668
3	52634	623	930	975	1020	...	993	1168	...	...	1364	1558	1385	1450	1666	1471	1004	1065	13365
4	41742	634	770	830	840	1050	803	961	...	...	1063	1280	969	1144	1366	1050	800	894	12544
5	33102	391	630	690	700	947	650	792	...	...	853	940	790	930	1024	1509	650	...	11884
6	26244	...	515	565	581	710	528	660	...	...	687	845	690	755	926	861	524	674	11126
7	20817	...	420	449	488	633	429	546	...	...	531	635	528	587	703	588	431	...	7967
8	16512	...	350	395	415	492	394	476	...	...	426	561	625	461	629	699	352	485	7892
9	13110	...	255	...	...	447	285	380	...	...	332	...	...	361	...	...	285	418	...
10	10381	...	170	...	...	344	233	321	...	...	295	...	...	324	...	...	230	348	...
11	8226	...	110	...	...	352	192	269	...	...	...	...	...	...	...	...	...	...	...
12	6528	...	70	...	...	250	159	232	...	...	176	...	...	194	...	...	169	241	...
13	5184	...	60	...	...	174	130	...	...	...	...	...	...	...	...	...	...	...	...
14	4110	...	...	...	...	132	108	...	...	...	126	...	...	143	...	...	110	154	...
15	3260	...	...	...	...	120	...	...	...	...	...	...	...	...	...	...	...	...	...
16	2581	...	...	...	...	...	...	...	...	...	82	...	...	98	...	...	69	101	...
17	2044	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
18	1624	...	...	...	...	...	...	...	...	...	58	...	...	73	...	...	51	63	...
19	1253	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
20	1024	...	...	...	...	...	...	...	...	...	50	...	...	64	...	...	...	44	...

TABLE No. 16. — *Continued.*

Number B. & S. Wire Gauge.	Area in Circular Mils.	KINDS OF INSULATION AND MAKER.									
		AMERICAN ELECTRICAL WORKS.		HOLMES, BOOTH, & HAYDEN.	A. F. MOORE.		N. Y. INSULATED WIRE CO.		OKONITE WIRE BY BIRMINGHAM GAUGE.		
									B.W.G. No.	Telephone and Telegraph.	
		Underwriters' Braided Wire.	Weather-proof Wire.	K. K. Triple Braid.	Underwriters' Wire.	Fire and Weather-proof Wire.	Line Wire.	Insulated Wire.		Plain Insulation.	Braided Insulation.
...	500000	...	...	...	...	...	...	...	10	369 to 387	394 to 413
...	450000	...	...	...	...	...	...	...	12	275 to 290	295 to 313
...	400000	...	...	...	...	...	...	...	14	176 to 321	194 to 351
...	350000	...	...	...	...	...	...	...	16	126 to 288	143 to 318
...	300000	...	...	...	...	...	...	...	18	81 to 299	96 to 329
...	250000	...	...	...	...	...	...	...	20	56 to 65	71 to 80
0000	211600	...	...	4012	3960	4694	...	...	22	45 to	60 to
000	167805	...	...	3168	3511	3458	...	...	24	29 to 334	43 to 48
00	133079	2376	2444	2640	2323	2725	...	...	...	...	...
0	105592	1848	1742	2112	1848	2202	2112	...	...	...	...
1	83694	1531	1425	1669	1499	1763	1758	...	...	...	...
2	66373	1267	1077	1473	1240	1463	...	...	...	...	...
3	52634	1030	934	1267	1003	1177	1642	...	...	...	...
4	41742	818	739	834	818	961	1019	...	...	...	...
5	33102	660	581	681	660	919	755	...	...	...	...
6	26244	555	501	575	538	634	586	...	...	...	...
7	20817	424	386	424	428	554	...	...	...	...	...
8	16512	371	343	375	364	447	387	...	...	...	...
9	13110	290	259	290	285	359	...	...	...	...	...
10	10381	264	238	274	248	317	...	...	...	...	...
11	8226	211	185	208	...	...	...	...	...	...	...
12	6528	153	137	179	150	185	...	...	...	...	...
13	5184	...	...	...	...	...	...	...	...	...	...
14	4110	...	...	...	...	...	...	...	...	...	...
15	3260	...	...	...	...	...	...	...	...	...	...
16	2581	...	...	...	...	...	...	...	...	...	...
17	2044	...	...	...	...	...	...	...	...	...	...
18	1624	...	...	...	...	...	...	...	...	...	...
19	1253	...	...	...	...	...	...	...	...	...	...
20	1024	...	...	...	...	...	...	...	...	...	...

TABLE No. 16. — *Continued.*

Number B. & S. Wire Gauge.	Area in Circular Mils.	KIND OF INSULATION AND MAKER.																		
		SHIELD BRAND.				Bishop Rubber Wire.	AMERICAN CIRCULAR LOOM CO.				OKONITE CO.									
											Plain Okonite.			Braided Insulation.			Candee Wire.			
		Stranded.	Double Braid.	Triple Braid.	Quadruple Braid.		No. 1.	No. 2.	No. 3.	No. 4.	Solid.	Stranded.	Switch Cords.	Solid.	Stranded.	Switch Cords.	Copper.	Iron.	Armor Cable.
...	500000	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	450000	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	400000	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	350000	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	300000	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
...	250000	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
0000	211600	3975	3690	3911	4106	...	3787	4023	4273	4533	...	5723	...	...	5988	...	4100	...	25476
000	167805	3099	3070	3250	3412	...	2983	3255	3385	3663	...	4176	...	...	4408	...	2960	...	22040
00	133079	2455	2390	2523	2657	...	2369	2640	2721	3000	3514	3792	3384	3684	3978	3534	2403	...	20883
0	105592	1927	1865	1955	2046	...	1917	2160	2413	3686	2830	3036	3220	2970	3206	3390	1942	...	18930
1	83694	808	1477	1548	1619	...	1528	1745	1942	2169	2150	2390	2364	2270	2522	2491	1536	...	15365
2	66373	607	1169	1224	1279	...	1231	1426	...	...	1633	2090	2005	1738	2222	2128	1249	...	14668
3	52634	623	930	975	1020	...	993	1168	...	...	1364	1558	1385	1450	1666	1471	1004	1065	13365
4	41742	634	770	830	840	1050	803	961	...	...	1063	1280	969	1144	1366	1050	800	894	12544
5	33102	391	630	690	700	947	650	792	...	...	853	940	790	930	1024	1509	650	...	11884
6	26244	...	515	565	581	710	528	660	...	...	687	845	690	755	926	861	524	674	11126
7	20817	...	420	449	488	633	429	546	...	...	531	635	528	587	703	588	431	...	7967
8	16512	...	350	395	415	492	394	476	...	...	426	561	625	461	629	699	352	485	7892
9	13110	...	255	...	...	447	285	380	...	...	332	...	...	361	...	...	285	418	...
10	10381	...	170	...	...	344	233	321	...	...	295	...	...	324	...	...	230	348	...
11	8226	...	110	...	...	352	192	269	...	...	...	...	...	...	...	...	...	...	...
12	6528	...	70	...	...	250	159	232	...	...	176	...	...	194	...	...	169	241	...
13	5184	...	60	...	...	174	130	...	...	...	...	...	...	...	...	...	...	...	...
14	4110	...	...	...	...	132	108	...	...	...	126	...	...	143	...	...	110	154	...
15	3260	...	...	...	...	120	...	...	...	...	...	...	...	...	...	...	...	...	...
16	2581	...	...	...	...	...	...	...	...	...	82	...	...	98	...	...	69	101	...
17	2044	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
18	1624	...	...	...	...	...	...	...	...	...	58	...	...	73	...	...	51	63	...
19	1253	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
20	1024	...	...	...	...	...	...	...	...	...	50	...	...	64	...	...	...	44	...

Table No. 16. — *Continued.*

Number B. & S. Wire Gauge.	Area in Circular Mils.	KINDS OF INSULATION AND MAKER. American Electrical Works. Underwriters' Braided Wire.	American Electrical Works. Weather-proof Wire.	Holmes, Booth, & Hayden. K. K. Triple Braid.	A. F. Moore. Underwriters' Wire.	A. F. Moore. Fire and Weather-proof Wire.	N. Y. Insulated Wire Co. Line Wire.	N. Y. Insulated Wire Co. Insulated Wire.	Okonite Wire by Birmingham Gauge. B.W.G. No.	Okonite. Telephone and Telegraph. Plain Insulation.	Okonite. Telephone and Telegraph. Braided Insulation.
...	500000	...	...	...	...	...	...	...	10	369 to 387	394 to 413
...	450000	...	...	...	...	...	...	...	12	275 to 200	295 to 313
...	400000	...	...	...	...	...	...	...	14	176 to 321	194 to 351
...	350000	...	...	...	...	...	...	...	16	126 to 288	143 to 318
...	300000	...	...	...	...	...	...	...	18	81 to 299	96 to 329
...	250000	...	...	...	...	...	...	...	20	56 to 65	71 to 80
0000	211600	...	...	4012	3960	4694	...	...	22	45 to	60 to
000	167805	...	...	3168	3511	3458	...	...	24	29 to 334	43 to 48
00	133079	2376	2444	2640	2323	2725	...	...	...	...	...
0	105592	1848	1742	2112	1848	2202	2112	...	...	...	...
1	83694	1531	1425	1669	1499	1763	1758	...	...	...	...
2	66373	1267	1077	1473	1240	1463	...	...	...	...	...
3	52634	1030	934	1267	1003	1177	1642	...	...	...	...
4	41742	818	739	834	818	961	1019	...	...	...	...
5	33102	660	581	681	660	919	755	...	...	...	...
6	26244	555	501	575	538	634	586	...	...	...	...
7	20817	424	386	424	428	554	...	...	...	...	...
8	16512	371	343	375	364	447	387	...	...	...	...
9	13110	290	259	290	285	359	...	...	...	...	...
10	10381	264	238	274	248	317	...	...	...	...	...
11	8226	211	185	208	...	...	...	...	...	...	...
12	6528	153	137	179	150	185	...	...	...	...	...
13	5184	...	...	...	...	...	...	...	...	...	...
14	4110	...	...	...	...	...	...	...	...	...	...
15	3260	...	...	...	...	...	...	...	...	...	...
16	2581	...	...	...	...	...	...	...	...	...	...
17	2044	...	...	...	...	...	...	...	...	...	...
18	1624	...	...	...	...	...	...	...	...	...	...
19	1253	...	...	...	...	...	...	...	...	...	...
20	1024	...	...	...	...	...	...	...	...	...	...

arc, in order to make up the necessary copper section. In fact, this is the plan most usually adopted; for all of the ordinary sizes of wire can readily be obtained in stock, while flexible cable is only made to order, at least in the larger sizes. The use of separate wires leads to greater expense in insulators, pole fixtures, greater weight of insulating material on the wire, and increased cost in stringing. Separate lines also entail a larger annual maintenance, so for all reasons cable is to be preferred when it is possible to obtain it. To facilitate

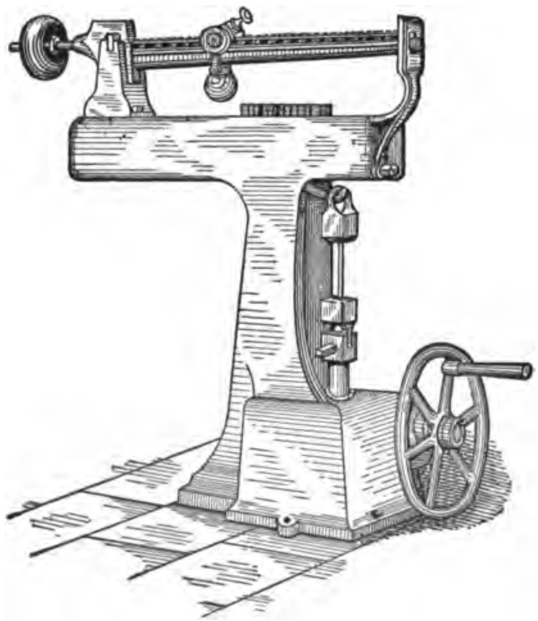


Fig. 7. Tension Testing-Machine.

calculation, TABLE No. 18 gives the circular millage of the various combinations of Nos. 000, 00, 0, 1 and 2 wire, from which a line of any copper cross-section may be calculated. After the total circular millage is found for the given line, it should be divided by 211,000 (the area of a 0000 wire), the quotient will be the *least* number of 0000 wires required; if there is a remainder, find the nearest number corresponding to it in the column headed "Circular millage" of TABLE No. 18, and in the column opposite will be found the least number and size of wires that will make up the required amount.

General Table of Reference for Stranded Cables. TABLE NO. 17. USED BY THE INDIA RUBBER, GUTTA PERCHA, & TELEGRAPH WORKS, L't'd.

No. of Wires in Strand.	Legal Standard Gauge of Each Wire.	Diameter.				Equivalent to Solid Wires.				Weight of Conductor.		Resistance at 60° F.	
		Of Each Single Wire.		Of the Strand.		Diameter.		Area.		Per Statute Mile.	Per Kilometer.	Per Statute Mile.	Per Kilometer.
		Inches.	Mm.	Inches.	Mm.	Inches.	Mm.	Sq. In.	Sq. Mm.	Lb.	Kilog.	Ohms.	Ohms.
3	25	.020	.508	.042	1.07	.034	.863	.0009	0.585	19	6	46.79	29.07
3	23	.024	.600	.051	1.29	.042	1.06	.0014	0.893	28	8	32.50	20.19
3	22	.028	.711	.059	1.50	.049	1.24	.0019	1.216	38	11	23.87	14.83
7	25	.020	.508	.060	1.54	.053	1.35	.0022	1.423	45	13	20.01	12.43
7	23	.024	.609	.072	1.83	.064	1.62	.0032	2.075	65	19	13.89	8.630
7	22	.028	.711	.084	2.13	.075	1.90	.0044	2.849	89	25	10.20	6.337
7	21½	.030	.762	.090	2.28	.080	2.03	.0050	3.242	102	29	8.893	5.525
7	21	. . .	. . .	. . .	. . .	.084	. . .	.0056	. . .	. . .	. . .	. . .	. . .
7	20½	.033	.838	.099	2.51	.088	2.23	.0061	3.923	124	35	7.342	4.561
7	20	.036	.914	.108	2.74	.096	2.43	.0072	4.65	147	42	6.175	3.835
7	19	.040	1.02	.120	3.04	.107	2.71	.0089	5.77	182	52	5.002	3.108
7	18	.048	1.22	.144	3.66	.128	3.25	.0128	8.30	262	74	3.473	2.158
7	17	.056	1.42	.168	4.27	.149	3.78	.0174	11.28	356	100	2.552	1.585
7	16	.064	1.63	.192	4.88	.171	4.34	.0229	14.73	465	132	1.953	1.213
7	15	.072	1.83	.216	5.49	.192	4.87	.0289	18.66	589	166	1.543	.9589
7	14	.080	2.03	.240	6.10	.213	5.41	.0356	22.98	727	205	1.253	.7785
7	13	. . .	. . .	. . .	. . .	.243	. . .	.0465	. . .	. . .	. . .	. . .	. . .
19	21	. . .	. . .	. . .	. . .	.395	. . .	.0153	. . .	. . .	. . .	. . .	. . .
19	20	.036	.914	.180	4.57	.159	4.03	.0198	12.74	402	113	2.261	1.404
19	19	.040	1.02	.200	5.08	.176	4.47	.0243	15.72	496	140	1.831	1.137
19	18	.048	1.22	.240	6.10	.211	5.35	.0349	22.66	715	201	1.271	.7897
19	17	.056	1.42	.280	7.10	.247	6.27	.0479	30.91	973	274	.9300	.5790
19	16	.064	1.63	.320	8.12	.282	7.16	.0624	40.25	1270	358	.7154	.4445
19	15	.072	1.83	.360	9.14	.317	8.05	.0789	50.96	1608	453	.5652	.3512
19	14	.080	2.03	.400	10.1	.352	8.94	.0973	62.77	1985	559	.4579	.2845
19	13	.092	2.34	.460	11.6	.404	10.7	.1282	83.20	2625	740	.3462	.2151
19	12	.104	2.64	.520	13.2	.458	11.6	.1647	106.3	3354	945	.2709	.1683
37	16	.064	1.63	.448	11.3	.394	10.0	.1219	78.6	2482	699	.3661	.2274
37	15	.072	1.83	.504	12.8	.443	11.2	.1541	99.58	3142	885	.2892	.1797
37	14	.080	2.03	.560	14.2	.493	12.5	.1909	122.9	3879	1093	.2343	.1456
37	13	.092	2.34	.644	16.3	.566	14.3	.2516	162.6	5130	1445	.1772	.1101
37	12	.104	2.64	.728	18.4	.640	16.2	.3217	207.7	6555	1847	.1386	.0861
61	13	.092	2.34	.828	21.0	.728	18.5	.4162	268.7	8477	2389	.1072	.0666
61	12	.104	2.64	.936	23.7	.823	20.9	.5319	343.4	10832	3052	.0839	.0521

TABLE NO. 18.

Giving the Circular Millage of the Various Combinations of Nos. $\frac{3}{0}$, $\frac{2}{0}$, $\frac{1}{0}$, 1, 2, and 3 Wire.

CIRCULAR MILLAGE.	WIRE COMBINATION.	CIRCULAR MILLAGE.	WIRE COMBINATION.
609119	$\frac{3}{0}+\frac{2}{0}+\frac{1}{0}+1+2+3$	273339	$\frac{3}{0}+\frac{1}{0}$
556485	$\frac{3}{0}+\frac{2}{0}+\frac{1}{0}+1+2$	255601	$\frac{1}{0}+1+2$
490112	$\frac{3}{0}+\frac{2}{0}+\frac{1}{0}+1$	252086	$\frac{2}{0}+2+3$
476040	$\frac{3}{0}+\frac{1}{0}+1+2+3$	251499	$\frac{3}{0}+1$
441314	$\frac{2}{0}+\frac{1}{0}+1+2+3$	238613	$\frac{2}{0}+\frac{1}{0}$
423406	$\frac{3}{0}+\frac{1}{0}+1+2$	234178	$\frac{3}{0}+2$
406418	$\frac{3}{0}+\frac{2}{0}+\frac{1}{0}$	224541	$\frac{1}{0}+2+3$
388680	$\frac{2}{0}+\frac{1}{0}+1+2$	220439	$\frac{3}{0}+3$
370506	$\frac{3}{0}+1+2+3$	216773	$\frac{2}{0}+1$
357033	$\frac{3}{0}+\frac{1}{0}+1$	202701	$1+2+3$
335780	$\frac{2}{0}+1+2+3$	199452	$\frac{2}{0}+2$
322307	$\frac{2}{0}+\frac{1}{0}+1$	189228	$\frac{1}{0}+1$
317872	$\frac{3}{0}+1+2$	185713	$\frac{2}{0}+3$
308235	$\frac{1}{0}+1+2+3$	171907	$\frac{1}{0}+2$
300884	$\frac{3}{0}+\frac{2}{0}$	150067	$1+2$
286812	$\frac{3}{0}+2+3$	136328	$1+3$
283146	$\frac{2}{0}+1+2$	119007	$2+3$

22. Testing and Inspection. — In all large contracts for wire, it is customary to locate an inspector at the manufactory, whose business it is to examine and see that the product complies fully with the specification requirements. The inspector must be provided with a machine for making tension tests, one for tortional tests, a wire gauge, a slide wire bridge, and an accurate scale for determining the weight of coils. One form of the tension-machine is shown in Fig. 7.

The apparatus consists of two clamps, by means of which the ends of each wire sample can be secured, a straining mechanism for applying a breaking stress to the sample, and a scale apparatus for measuring the force required to produce rupture. Though there are many other designs of this apparatus in commercial use, all embrace the same features. By the side of the specimen an apparatus is placed, consisting of two sliding scales, one of which is secured to the top of the sample, and the other to the bottom. As stress is applied to the wire it elongates, the amount being measured by the mutual displacement of the scales. The torsion testing-machine is indicated in Fig. 8.

A set of iron ways carries two clamps, to which the sample to be

examined can be secured. One clamp being movable longitudinally permits the length of test piece to be varied at pleasure. The fixed clamp carries a spindle that, by means of a crank, can be rotated, thus

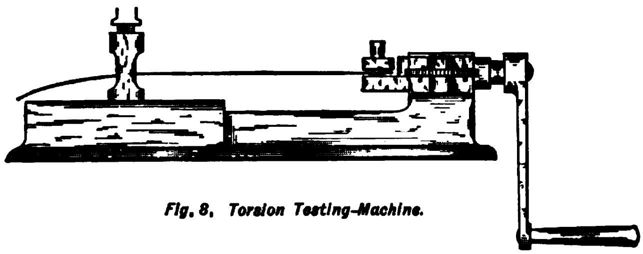


Fig. 8. Torsion Testing-Machine.

twisting the specimen. To the spindle is attached a counting-wheel to register the number of twists required to produce rupture. In making all physical examinations great care should be taken to see that the test pieces are carefully set in the axis of the testing-machine, that the stress is applied steadily and uniformly, and that the jaws do not injure the wire, thus giving rise to erroneous results.

Fig. 9 illustrates the best and most useful form of wire gauge, in the shape of a micrometer. In skillful hands this instrument will give accurate results to a ten-thousandth of an inch; and thus the actual size of the sample under examination may at various points be determined, and compared with the tabulated diameter of the desired gauge number. Determination may also be made of the roundness of the wire. Care should be constantly exercised to always exert a constant, though light, pressure on the micrometer screw, that there may be no springing of the apparatus, and that uniform readings may be obtained. The slide wire bridge, for determining the electrical properties of the samples under examination, will be found illustrated and described in the chapter on testing. For determining the weight of coils and the weight per mile, there is no better instrument than a good platform scale, carefully tested, and adjusted to be sensitive to a fraction of a pound.

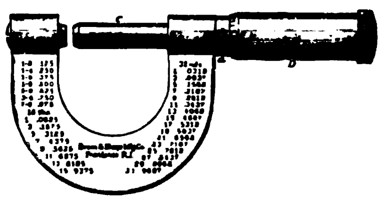


Fig. 9. Micrometer Wire Gauge.

23. Specifications for Line Wire. — As a guide to the selection of line wire, the following specifications are the latest issue by the British Postal Telegraph authorities.

I. SPECIFICATIONS FOR COPPER WIRE.

In this specification, the term "piece" shall be understood to mean a single length of wire without joint or splice of any description before being drawn, or in a finished wire; a "coil" shall be held to be a piece of wire in the form of a coil; and a "parcel" shall be any quantity of manufactured wire presented for examination and testing at any one time.

1. The wire shall be drawn in continuous pieces of the respective weights and diameters given in the Table hereunto annexed (*see* TABLE NO. 19), and every piece must be gauged for a diameter in one or more places.

TABLE NO. 19.

Data accompanying British P. O. Wire Specifications.

Weight per Statute Mile.		Approximate Equivalent Diameter.		Minimum Breaking Weight.	Minimum Number of Twists.	Maximum Resistance per Mile of Wire when Hard at 60° F.	Minimum Weight of Each Piece (or Coil) of Wire.
Standard.	Range Allowed.	Standard.	Range Allowed.				
Lbs.	Lbs.	Mils.	Mils.	Lbs.		Ohms.	Lbs.
100	97½	79	78	330	30 (In 3 inches.)	9.10	50
	102½		80				
150	146¼	97	95½	490	25 (In 3 inches.)	6.05	50
	153¾		98				
200	195	112	110½	650	20 (In 3 inches.)	4.53	50
	205		113¼				
400	390	158	155½	1250	25 (In 6 inches.)	2.27	80
	410		160¼				
600	585	194	191	1800	20 (In 6 inches.)	1.484	80
	615		196				
800	780	224	220½	2400	15 (In 6 inches.)	1.113	80
	820		226				

A maximum weight of 112 pounds for each coil is fixed for all sizes.

2. The wire shall be perfectly symmetrical, uniform in quality, pliable, free from scale, inequalities, flaws, splits, and other defects, and shall be subject to the tests hereinafter provided for.

3. Every piece may be tested for ductility and tensile strength, and 5% of the entire number of pieces may be cut and tested in any part. Pieces cut for this purpose shall not be brazed or otherwise joined together, but each length shall be bound up into a separate coil.

4. The wire shall be capable of being wrapped in six turns round a wire of its own diameter; unwrapped, and again wrapped in six turns round a wire of its own diameter in the same direction as the first wrapping, without breaking; and shall be also capable of bearing the number of twists set down in detail, without breaking. The twist-test is made as follows: The wire will be gripped by two vises, one of which will be made to revolve at a speed not exceeding one revolution per second. The twist thus given to the wire will be reckoned by means of an ink-mark, which forms a spiral on the wire during torsion, the full number of twists to be visible between the vises.

5. Test for tensile strength may be made with a lever or other machine which has the approval of the officer appointed on behalf of the Postmaster-General to inspect the wire, and hereinafter called the Inspecting Officer, who will be afforded all requisite facilities for proving the correctness of the machine.

6. The electrical resistance of each test-piece shall be reduced according to its diameter, and shall be calculated for a temperature of 60° F. Such test-piece shall measure not less than one-thirtieth part of an English statute mile.

7. If, after the examination of any parcel of wire, 5% of such parcel fail to meet all or any of the requirements of the specification and of the table, the whole of such parcel shall be rejected; and on no account shall such parcel, or any part thereof, be again presented for examination and testing; and this stipulation shall be deemed to be, and shall be treated as, an essential condition of the contract.

8. Each piece, when approved by the Inspecting Officer, shall be made into a coil, and be separately bound; and in no case shall two or more pieces be linked or otherwise jointed together. The eye of the coil shall be not less than 18 inches, nor more than 20 inches, in diameter.

9. Each coil of approved wire shall be weighed separately, and its weight (in English pounds avoirdupois) stamped on a soft copper label, which shall be provided by the contractors, the label being firmly affixed to the inner part of the coil. The contractors shall also provide the assistance necessary for properly affixing to each coil of approved wire, under the direction of the Inspecting Officer, a metallic seal which shall be provided by the Postmaster-General, the weight of this seal being deducted from the invoiced weight of the wire when each delivery is made, or on completion of the order, as may be arranged.

10. The approved wire shall be wrapped in canvas, and be delivered as required, securely packed in casks or cases.

II. SPECIFICATIONS FOR IRON WIRE.

For iron wire the same general characteristics are required in so far as quality of wire and amount of testing and inspection are concerned. The physical requirements, however, will be found in TABLE No. 20.

TABLE No. 20

The Specifications issued by the British Postal Telegraph Authorities for the Supply of Galvanized Iron Wire.

Weight per Mile. Required Standard.	Weight per Mile. Allowed. Minimum.	Weight per Mile. Allowed. Maximum.	Diameter. Required Standard.	Diameter. Allowed. Minimum.	Diameter. Allowed. Maximum.	Tests for Strength and Ductility. Breaking Weight. Minimum.	No. of Twists in 6 in. Minimum.	For Breaking Weight not less than	No. of Twists in 6 in. Minimum.	For Breaking Weight not less than	No. of Twists in 6 in. Minimum.	Resistance per Mile of the Standard Size at 60° Fahr. Maximum.	Constant, being Standard Weight by Resistance.	Weight of Each Piece (or Coil). Minimum.	Weight of Each Piece (or Coil). Maximum.
Lbs.	Lbs.	Lbs.	Mils.	Mils.	Mils.	Lbs.		Lbs.		Lbs.		Ohms.		Lbs.	Lbs.
800	767	833	242	237	247	2480	15	2550	14	2620	13	6.75	5400	90	120
600	571	629	209	204	214	1860	17	1910	16	1960	15	9.00	5400	90	120
450	424	477	181	176	186	1390	19	1425	18	1460	17	12.00	5400	90	120
400	377	424	171	166	176	1240	21	1270	20	1300	19	13.50	5400	90	120
200	190	213	121	118	125	620	30	638	28	655	26	27.00	5400	40	65

The most recent practice in American aerial line construction requires manufacturers to furnish line wire under the following requirements : —

COPPER WIRE.

1. **Finish.** — Each coil shall be drawn in one length and be exempt from all joints or splices. All wire shall be truly cylindrical and fully up to gauge specified for each size, and must not contain any scale, inequalities, flaws, cold shuts, seams, or other imperfections.

2. **Inspection.** — The purchaser will appoint an inspector, who shall be supplied by the manufacturer with all facilities which may be required for examining the finished product, or any of the processes of manufacture. The inspector shall have the privilege of overseeing the packing and shipping of the samples. The inspector will reject any and all wire which does not fully come up to all the specification requirements. The purchaser further reserves the right to reject on reception, any or all lots

of wire which do not fulfil the specifications, even though they shall previously have been passed or accepted by the inspector.

3. **Apparatus.** — The manufacturer must supply, at the mill, the necessary apparatus for making the examination called for. This apparatus shall consist of a tension testing-machine, a torsion testing-machine, an elongation gauge, an accurate platform scale, and an accurate bridge and battery. Each of these pieces of apparatus may be examined by, and shall be satisfactory to, the inspector.

4. **Packing for Shipment.** — When ready for shipment, each coil must be securely tied with not less than four separate pieces of strong twine, and shall be protected by a sufficient wrapping of burlap so the wire may not be injured during transportation. The wrappings shall be placed upon the wire bundles, after they have been coiled and secured by the twine. The diameter of the eye of each coil shall be prescribed by the inspector, and all coils shipped shall not vary more than two inches in the diameter of the eye.

5. **Weight.** — Each coil shall have its length and weight plainly and indelibly marked upon two brass tags, which shall be secured to the coil, one inside of the wrapping, and the other outside.

6. **Mechanical Properties.** — All wire shall be fully and truly up to gauge standard, as per B. and S. wire gauge. The wire shall be cylindrical in every respect. The inspector shall test the size and roundness of the wire by measuring each end of each coil, and also by measuring at least four places in the length of each coil. A variation of not more than one and one-half mils on either side of the specified wire gauge number will be allowed, and the wire must be truly round within one mil upon opposite diameters at the same point of measurement. The strength of the wire shall be determined by taking a sample from one end of each coil, 30″ in length. Of this piece, 18″ shall be tested for tension and elongation, by breaking the same in the tension testing-machine. The samples should show a strength in accordance with TABLE No. 21:—

TABLE No. 21.

Size of Wire, B. & S. Gauge.	Breaking Weight of Hard-drawn. Lbs.	Breaking Weight of Annealed. Lbs.	Size of Wire, B. & S. Gauge.	Breaking Weight of Hard-drawn. Lbs.	Breaking Weight of Annealed. Lbs.
0000	9,971	5,650	9	617	349
000	7,907	4,480	10	489	277
00	6,271	3,553	11	388	219
0	4,973	2,818	12	307	174
1	3,943	2,234	13	244	138
2	3,127	1,772	14	193	109
3	2,480	1,405	15	153	87
4	1,967	1,114	16	133	69
5	1,559	883	17	97	55
6	1,237	700	18	77	43
7	980	555	19	61	34
8	778	440	20	48	27

A variation of 1½ per cent on either side of the tabular limits will be accepted by the inspector. The elongation of the wire must be at least 3 per cent for all sizes larger than No. 1; 1½ per cent from No. 1 to No. 10, and 1 per cent for sizes less than No. 10, for hard-drawn copper wire. The remainder of the sample selected will be tested for torsion. The torsion sample will be twisted in the torsion testing-machine to destruction, one foot in length being placed between the jaws of the machine. Under these circumstances, hard-drawn copper wire shall show not less than 20 twists for sizes over No. 1; from 40 to 90 twists in sizes from No. 1 to No. 10; and not less than 100 twists in sizes less than No. 10. Should the sample selected from one end of each coil show failure to come up to the specifications, the inspector may take a second sample from the other end of the coil. If the average of the results from both samples shall be within the specifications, the coil shall be accepted; if not within the specifications, the coil shall be rejected. The weight per mile shall be determined by carefully weighing 2 per cent of the number of coils called for in the contract; and the weight thus obtained shall correspond, within 2 per cent, on either side of the result given in the following formulæ: —

$$\text{Weight per mile} = \frac{CM}{62.567};$$

$$\text{Weight per 1000 ft.} = \frac{CM}{330.353}.$$

7. **Electrical Properties.** — The electrical properties of the wire shall be determined by the inspector, selecting 3 per cent of the coils, and from them taking lengths of 100 ft., 500 ft., or 1000 ft., at his discretion, and measuring the conductivity of the same with a standard bridge. For soft-drawn copper wire, the following resistance per mil ft. will be assumed: —

TABLE NO. 22.

Temperature in Degrees F.	Resistance Legal Ohms.	Temperature in Degrees F.	Resistance Legal Ohms.
0	8.96707	60	10.20253
10	9.16413	70	10.42083
20	9.36473	80	10.64268
30	9.56887	90	10.86806
40	9.77655	100	11.09698
50	9.98777		

For hard-drawn wire, the resistance per mil ft. shall be 1.0226 times the foregoing figures. All wire shall be within 98 per cent of the above figures.

8. **Requirements for Iron Wire.** — All iron wire shall be subjected to the same general requirements as above specified for copper wire, and shall be inspected and tested in the same general manner. The wire shall be carefully annealed, without burning or undue rusting from the heat of the

furnace. It shall be soft, pliable, and capable of elongating not less than 15 per cent in lengths of one foot between the jaws of the testing-machine.

9. **Mechanical Properties.** — Weights and strengths of the various sizes shall be as follows: —

TABLE No. 23.

NUMBER B. W. G.	WEIGHT PER MILE.	BREAKING WEIGHT.	NUMBER B. W. G.	WEIGHT PER MILE.	BREAKING WEIGHT.
4	730	1898	10	260	676
6	540	1404	11	214	556
8	380	988	12	165	429
9	330	858	14	96	250

In general the weight per mile shall be $CM/72$. High tensile strength is not required; but, in general, wire must not break under less strain than two and one-half times its weight per mile in pounds.

10. **Torsion Tests.** — Torsion tests will be made as prescribed for copper wire, and the specimens must not fail under less than 80 twists in a length of one foot. In the mechanical requirements for iron wire, a variation of 3 per cent on either side of the specification limits will be accepted by the inspector.

11. The electrical resistance of iron wire will be determined in the same manner as specified for copper wire, excepting that the resistance of the iron wire shall be 6.081 times the resistance of the copper wire per mil ft. at 32° F., with an allowance of .278 per cent for every degree of increase of temperature.

12. **Galvanizing.** — When galvanized wire is called for, the galvanizing must be true and smooth over the entire length of all the coils, showing that the zinc has been carefully and evenly wiped off. The wire must show no black spots, scales, or inequalities. The galvanizing will be tested by plunging a sample of wire from 5 per cent of all the coils into a saturated solution of sulphate of copper, and permitting same to remain for 70 seconds. The wire will then be withdrawn and wiped clean. This operation will be repeated four times, at the end of which time, if the wire appears black, the galvanizing will be considered satisfactory, and the sample accepted. If, on the contrary, any precipitated copper is shown, the galvanizing will not be considered sufficiently well done, and the samples may be rejected.

24. The Tension of Aerial Lines. — When a wire is stretched between the points A and B, Fig. 10, situated at the same level, it describes a curve ADB, known under the name of the catenary.

The horizontal distance AB is termed the span, while the vertical distance CD between the lowest point of the curve and the

horizontal line AB is the deflection. The catenary may be referred to two rectangular co-ordinates, of which the vertical axis Oy passes through the lowest point of the curve, while the horizontal

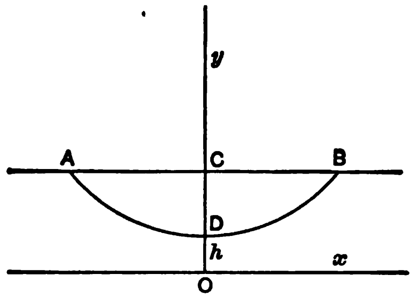


Fig. 10. Diagram of the Catenary.

axis is situated at such a distance "*h*," *below* the point D, or the lowest part of the curve, that if the tension of the wire at this point be represented by T, and the weight per unit of length by P, then

$$h = \frac{T}{P}. \tag{1}$$

Under these conditions the equation of the catenary may be represented by

$$y = \frac{h}{2}\left(e^{\frac{x}{h}} + e^{-\frac{x}{h}}\right), \tag{2}$$

in which e is the base of the Napierian system of logarithms. Developing the second number of this equation by McLaurin's formula,

$$y = h\left(1 + \frac{x^2}{1.2.h^2} + \frac{x^4}{1.2.3.4.h^4} + \text{etc.}\right). \tag{3}$$

Usually the tension T is so large in respect to the weight P that for all ordinary spans it is sufficient to represent the curve by the following equation, which is readily recognized as that of a parabola,

$$y = h + \frac{x^2}{2\,h}. \tag{4}$$

Calling the span a, the deflection f, and making $x = \frac{a}{2}$, from equation (4),

$$y = h + \frac{a^2}{8\,h}\ (5), \text{ and } f = \frac{a^2}{8\,h} = \frac{a^2 P}{8\,T}. \tag{6}$$

This last equation gives the deflection at the center, if the weight per unit of length is known, as well as the span and the tension at the lowest point. If the span, the deflection, and weight are known, it is easy to calculate the tension. The tension T_h at the highest point is given by the equation

$$T_h = T + Pf. \tag{7}$$

Except in cases where the deflection is very great, the tension T_h does not sensibly differ from the tension T at the lowest point; and without serious error it may be assumed that the tension calculated by formula (6) represents the tension throughout all points of the span. The length of the wire may be obtained from the equation

$$dl = \sqrt{dx^2 + dy^2} = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}. \tag{8}$$

Substituting for dy/dx its value from equation (4)

$$dl = dx\left(1 + \frac{x^2}{h^2}\right)^{\frac{1}{2}} = dx\left(1 + \frac{x^2}{2\,h^2} - \frac{x^4}{8h^4} + \text{etc.}\right), \tag{9}$$

an expression that for common spans reduces to

$$dl = dx\left(1 + \frac{x^2}{2\,h^2}\right) \text{ integrating,} \tag{10},$$

$$l = \int_{-\frac{a}{2}}^{+\frac{a}{2}} dx\left(1 + \frac{x^2}{2\,h^2}\right) = a + \frac{a^3}{24\,h^2}, \tag{11}$$

or

$$l = a + \frac{a^3P^2}{24\,T^2}. \tag{12}$$

If a equals the horizontal span, l the actual length of the wire between the insulators, f the deflection of the wire at the center, P the weight of one unit of length of the wire, and T the tension of the wire at its lowest point, the three following equations are approximate: —

1	2	3
$f = \frac{a^2P}{8\,T}$ (13)	$T = \frac{a^2P}{8f}$ (14)	$l = a + \frac{8f^2}{3a}$ (15)

25. Influence of the Variations of Temperature. — The tension in the wire deduced from these formulæ is obviously the tension at the time when the line is erected. If the temperature falls, the

wire tends to contract in proportion to the diminution of temperature. The elasticity of the wire, however, allows it to elongate somewhat under any increase in the tension that results from the contraction. The accumulation of sleet or snow upon the wire adds a very considerable amount to its weight, and consequently to the

Table No. 24.

Sags and Tensions to be Observed in Erecting Wires at Various Temperatures.

400 lbs. Iron Wire (No. 7½).

Span in Feet.	22° F. Low Winter Temperature.			40° F. Ordinary Winter Temperature.			58° F. Average Summer Temperature.			76° F. High Summer Temperature.		
	Sag.		Tension.	Sag.		Tension.	Sag.		Tension.	Sag.		Tension.
	Ft.	In.	Lbs.	Ft.	In.	Lbs.	Ft.	In.	Lbs.	Ft.	In.	Lbs.
300	3	1¾	270	3	9	227	4	3½	200	4	8⅞	180
270	2	6⅝	270	3	1¾	219	3	2¾	190	4	0⅞	169
240	2	0¼	270	2	7⅛	210	3	0⅜	178	3	5⅝	157
210	1	6½	270	2	1½	198	2	6½	164	2	10⅞	143
180	1	1⅝	270	1	8	184	2	0⅜	148	2	4¾	128
150	0	9½	270	1	3½	165	1	7¾	130	1	11¼	110

150 Lbs. Hard-Drawn Copper Wire (No. 12½).

Span in Feet.	22° F. Low Winter Temperature.			40° F. Ordinary Winter Temperature.			58° F. Average Summer Temperature.			76° F. High Summer Temperature.		
	Sag.		Tension.	Sag.		Tension.	Sag.		Tension.	Sag.		Tension.
	Ft.	In.	Lbs.	Ft.	In.	Lbs.	Ft.	In.	Lbs.	Ft.	In.	Lbs.
300	2	8	120	3	7	89	4	3⅞	74	4	11½	64
270	2	2	120	3	1	84	3	9½	69	4	4⅛	60
240	1	8⅝	120	2	6⅞	80	3	2½	64	3	8⅞	54½
210	1	3⅝	120	2	1⅞	73	2	8⅝	57½	3	2½	49
180	0	11⅝	120	1	9	66	2	3⅛	51	2	8¼	43
150	0	8	120	1	4⅝	58	1	10	44	2	2⅜	36½

100 Lbs. Hard-Drawn Copper Wire (No. 14).

Span in Feet.	22° F. Low Winter Temperature.			40° F. Ordinary Winter Temperature.			58° F. Average Summer Temperature.			76° F. High Summer Temperature.		
	Sag.		Tension.	Sag.		Tension.	Sag.		Tension.	Sag.		Tension.
	Ft.	In.	Lbs.	Ft.	In.	Lbs.	Ft.	In.	Lbs.	Ft.	In.	Lbs.
300	2	8	80	3	7	59	4	3⅞	49	4	11½	43
270	2	2	80	3	1	56	3	9½	46	4	4⅛	40
240	1	8⅝	80	2	6⅞	53	3	2½	42½	3	8⅞	36
210	1	3⅝	80	2	1¾	49	2	8⅝	38	3	2½	33
180	0	11⅝	80	1	9	44	2	3⅛	34	2	8¼	29
150	0	8	80	1	4⅝	39	1	10	29	2	2⅜	24

tension to which it is subjected. In order to ensure against accidents, it is therefore necessary to so adjust the tension of the wire when the line is erected, that under all ordinary circumstances no future stresses will be sufficient to exceed the elastic limit of the material. The limiting tension permissible is often assumed to be from one-fifth to one-quarter of the breaking-strain, the limit of elasticity usually being about one-third of the ultimate strength. Lines which are erected in the summer time should be allowed a much greater deflection than those placed in the winter, in order that the contraction due to cold weather may not cause the tension to exceed the safety limit. Long spans must have more slack than short ones, and straight lines must be given more deflection than those in which curves are of frequent occurrence; for the curve allows the line to give somewhat under the additional stresses introduced by contraction. While slack lines are safer so far as the tension on the wires is concerned, they are liable to give extreme trouble from swinging crosses, and from entanglement with neighboring wires. In TABLE No. 24 will be found data indicating the appropriate deflections and tensions to be observed in stringing the more common sizes of iron and copper wire at the various temperatures to be met with in practice. If, in equations (13), (14) and (15), a and P be made unity and if f be calculated as a percent of a, a table of coefficients can be formed from which the tension in any span of any weight of wire or its length can be at once calculated by simple multiplication. TABLE No. 25 is such a table of coefficients.

TABLE No. 25.

Table of Coefficients of Tension and Length for Wire Spans.

Deflection in Per Cent of Span.	Tension = Weight per Foot of Wire Multiplied by,	Total Length = Span Multiplied by,	Deflection in Per Cent of Span.	Tension = Weight per Foot of Wire Multiplied by,	Total Length = Span Multiplied by,
.15	83.33	1.0000058	.75	17.00	1.00015
.16	78.15	1.0000068	.80	15.63	1.00017
.17	73.65	1.0000077	.85	14.50	1.00020
.18	69.55	1.0000086	.90	13.75	1.00022
.19	65.62	1.0000096	.95	13.13	1.00024
.20	62.50	1.0000107	1.00	12.51	1.00027
.25	50.00	1.0000167	1.25	10.01	1.00042
.30	41.62	1.0000256	1.50	8.12	1.00064
.35	34.88	1.0000346	1.70	7.51	1.00074
.40	31.25	1.0000430	1.80	6.87	1.00088
.45	27.00	1.0000500	2.00	6.27	1.00106
.50	25.00	1.0000600	2.50	5.02	1.00167
.55	22.80	1.0000798	3.00	3.45	1.00254
.60	20.00	1.0001	4.00	3.16	1.00427
.65	19.00	1.00012	5.00	2.55	1.00667
.70	18.13	1.00013			

CHAPTER III.

CONSTRUCTION OF AERIAL CIRCUITS.

GENERAL LINE WORK.

Art. 26. Classification. — Assuming the location of the central station as selected, and the general design of the plant determined, the construction of the necessary circuit becomes a matter for consideration. Electrical circuits may be primarily divided into Aerial and Underground, depending upon whether the line is placed upon poles distributed over the surface, or carried through a conduit or other structure buried in the earth. Either form of circuit may be further classified as Metallic or Grounded. In the metallic circuit, the entire line is composed of wire extending from one pole of the generating-station back to the other pole. In the grounded circuit, a portion of the line is composed of wire, while for the remainder the earth as a return is used. Depending on the design of the plant, electrical circuits may still further be catalogued as Telegraph circuits, Telephone circuits, and Power circuits ; and the latter group may still further be subdivided into Lighting circuits, Motor circuits, and Railway circuits.

27. Aerial Lines. — Recently all electrical circuits were of the aerial type; as it has only been within recent years that the increasing multiplicity of overhead wires has caused the various forms of conduit to spring into existence, that the streets of the larger cities might be freed from the inconvenience of pole circuits.

Aerial lines are built by setting upright into the ground poles of sufficient strength to carry the weight of wire necessary for the circuits, and the various lines are supported on insulators placed on cross-arms attached to the tops of the poles. For open country lines wooden poles are used, white Canadian cedar, Northern pine, or chestnut being considered the best material. In the South, cypress or Southern pine is common ; but it does not weather as well as the Northern woods. The poles should be sound, live wood,

straight and true, and free from bad knots, shakes, large cracks, dry rot, and other defects. The poles should not be less than six inches in diameter at the top for lines through the open country, or from seven to eight inches for city work. The bark should be carefully stripped from the poles, which should then be shaved and trimmed, and the gains for the cross-arms cut. It is also customary to bolt the cross-arms to the poles previous to setting, as it is considerably more economical for this work to be performed on the ground level. The distance apart at which poles are set varies greatly with the nature of the line, and territory through which it is constructed. For light lines in the open country, from thirty-five to forty-two poles per mile is a common rule. For heavier city lines, forty to fifty poles per mile is usual; while for electric railway work, from forty-five to fifty-five poles to the mile are necessary.

28. Methods of Preservation. — In this country it is not customary to treat the poles in any way in reference to their preservation, excepting to give them one coat of paint previous to, and one after, erection. In Europe the custom of creosoting, or of treating them with various preserving solutions, is quite common. Besides the operation of creosoting, several methods of preservation have been suggested, which consist in soaking the poles in large tanks filled with solutions of. chloride of zinc, chloride of mercury, sulphate of iron, or sulphate of copper. It is the attempt of all of these processes to make the preserving solution saturate the wood, and form, with the various vegetable albumens, insoluble compounds, thus lengthening the life of the timber. With the exception of creosoting, all these methods involve considerable expense, and as yet have not given results that seem to justify the necessary outlay. Recently the process of "Vulcanizing," which simply consists in heating the poles for some hours in a closed cylinder to about 500° F., has given great satisfaction. The high temperature seasons the wood by coagulating the albumen, adding greatly to the life of the pole. The simplicity and cheapness of this process are greatly in its favor.

The treatment by the creosoting process consists in placing the poles in a strong iron vat from which the air can be exhausted. In the vacuum thus produced, all the sap and other juices of the wood flow outwards, and are carried away by a suitable system of piping. Live steam at a pressure of about 100 pounds to the square inch

is then admitted into the cylinder, and the poles thoroughly cooked and steamed for several hours. Subsequent to the steaming, crude petroleum oil is admitted into the cylinder under a hydrostatic pressure of 200 to 300 pounds. By means of this operation it is expected that all the fluids contained in the woods are extracted, and are replaced by the crude petroleum, contributing very materially to the life of the poles. Experiments have shown that lines constructed of poles treated in this manner are in perfectly good condition after twenty years of life.

29. The Height of Poles.—The height of the poles which it is necessary to use will depend very largely upon the magnitude or number of circuits which they are to sustain. For ordinary city work, for telegraph or telephone construction, a height of 40 to 60 feet is usually employed. In some cases, however, very notably in some of the large metropolitan lines, poles of 100 or even 125 feet in height have been erected, carrying a very large number of wires. Usually it is preferable to use a high pole rather than a low one, in order that the line may be thoroughly clear of the street, and may not become too much of an obstruction to neighboring buildings.

30. Cross-Arms.—The cross-arms, carrying insulators, should preferably be of yellow pine; they should be carefully sawed, true and square, and should be of sound hard wood, and thoroughly coated with mineral paint having good insulating properties. The cross-arms are usually $4\frac{1}{4}'' \times 3\frac{1}{4}''$, and vary from 3 to 10 feet in length, depending upon the number of insulators to be supported. Two wires are carried on a three-foot arm, and 10 on a ten-foot arm. The top of the arm is rounded with a circular chamfer, as shown at C, Fig. 11, to prevent the accumulation of snow and water. Frequently the arms, including the pins, are assembled at the factory and shipped complete. The cross-arms are usually set 20″ apart vertically along the gains, strongly bolted and braced. Two braces are allotted to each arm. They are of galvanized iron, $1\frac{1}{4}''$ wide, $\frac{1}{4}''$ thick, and 28″ long. One end of each brace of every pair is attached to the pole by some form of bolt, the other end being attached to the next arm above, between the second and third pin on the right and left hand of the pole, thus forming a bracket to steady the arm. It has been customary to secure the arm to the pole by means of two lag-bolts passed through the arms, and screwed into the pole. As the thread

of the lag-bolt destroys the fiber of the pole to such an extent that it is usually impossible to replace cross-arms after the line has been some years in service, it is at present considered better to fasten the arm with a single machine-bolt, passed entirely through both arm and pole, the bolt hole being cleanly bored with a sharp bit, accurately to fit the bolt. The bearings of the bolt-head and nut should be prevented from crushing the wood by ample washers. In lines constructed of machined poles, as in street railway work, when the pole-tops are all uniform in size, the cross-arms may best be fastened by means of a U-bolt that extends entirely around the pole. This method obviates any injury or weakening of the pole itself.

31. Pins. — The insulator pins should be of locust or oak. The pins have a turned shank 1¼″ in diameter to fit the hole in the cross-

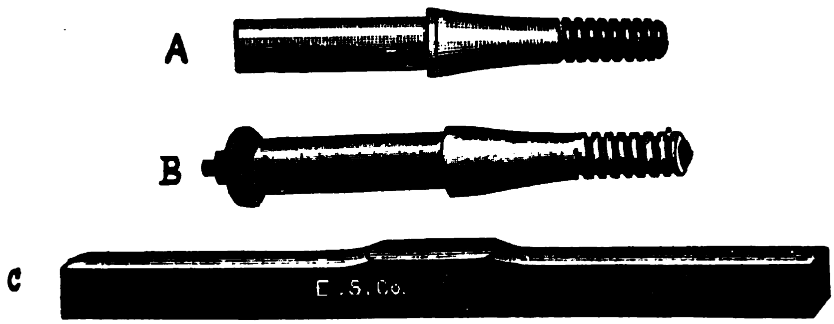


Fig. 11. Cross-Arms and Pins.

arm, and when in place are secured by a single wire nail driven through the arm. The upper part of the pin is threaded to match the particular insulator for which the line is designed. Where corners in the line occur, the sustaining power of the pin is re-enforced by securing it to the arm by a nut and washer. A corner pin is indicated at B, Fig. 11, and a common pin at A. The pins are placed 10″ to 12″ apart on the arm, with an allowance of 15″ between the two middle pins, in order that the wires supported by them may adequately clear the pole. The general aspect of a properly equipped wooden pole-top, for telephone or telegraph lines, is shown in Fig. 12 (p. 50).

32. The Facing of Arms. — In setting the poles, it is customary to place the cross-arms in such a manner that those on the

adjacent poles shall face each other, while on the next two poles they shall be turned back to back. The object of this disposition of the cross-arms is to prevent accident in case of the breaking of any of the wires. If the cross-arms were all set in one direction, an excessive stress, or some accidental cause, such as a broken pole, might wrench off one of the cross-arms, and the stress being transferred to the next one, like a row of blocks the whole line would go down. On the contrary, if the arms are set alternately facing and back to back, it is practically impossible to pull off more

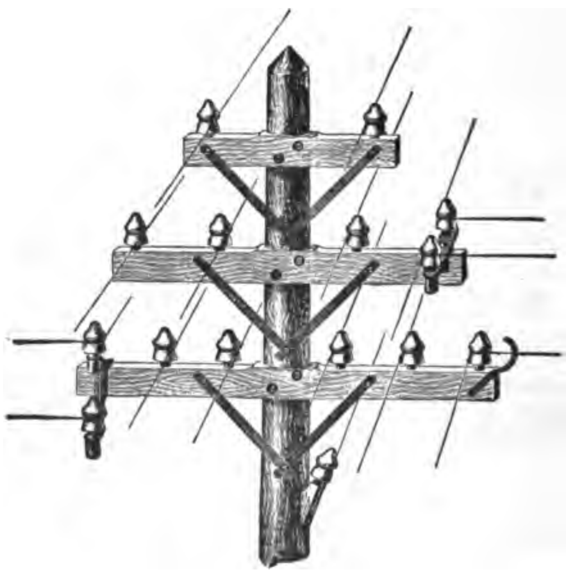


Fig. 12. Pole-Top.

than two sets of arms, and thus, frequently, a broken pole is kept from falling, and the line saved. To obtain good appearance, the poles should be carefully set plumb and true, heaving their tops essentially in a straight line. In city work this condition can usually be attained by using poles of the same height. In the open country, owing to the differences in level, it is essential to use poles of correspondingly varying heights in order to preserve uniformity.

33. Stresses. — The stresses to which a pole-line is subjected are primarily due to the weight of the wire and cable which the poles have to support, and to the longitudinal stresses due to the

tension upon them. Concerning the stresses to which the poles are subjected, the forces may be divided into three classes.

34. FIRST. — In straight line work the cross-arms and insulators transmit to the poles the weight of the wire and cables supported upon them. This stress acts vertically, being a direct load tending to break the arm at the center where it is secured to the pole, and to crush the pole as a column. In the winter, the weight thus thrown on poles may be very largely augmented by snow or sleet, with which the wires may become incrusted.

35. SECOND. — The action of the wind against the wires develops considerable lateral stress, which causes the poles to vibrate in a dangerous manner, and may break them at or near the surface of the ground.

36. THIRD. — Whenever a change of direction occurs in a pole-line, or wherever one or more wires or cables are terminated, the poles are subjected to a bending-stress, equivalent in the first case to the resultant of the tension in all of the circuits, pulling the poles sidewise; and in the second case, to a bending-moment derived from the sum of all the tensions in the wires, or other circuits which are ended, tending to pull the pole over longitudinally in the direction of the line.

Great care must be exercised in the design of all lines of magnitude, to see that, at the points of change of direction, or at the termination of any or all circuits, the poles are sufficiently strong or carefully braced to withstand these stresses.

In the case of a change of direction in a pole-line, the resultant of the line stress may be readily determined by the application of the well-known principle of the parallelogram of forces. By examining the tensions in the line on either side of the angle, and determining the resultant, the magnitude of the force tending to deflect the pole, and its direction, are readily ascertained. To counterbalance this tendency, the pole must be either stiff enough to stand the bending-moment, or else the top of the pole must be anchored in a direction opposed to the resultant stress, by means of a guy wire or rod.

37. Calculations for Pole Strength. — A pole subjected to a horizontal force is bent in the direction of the force; the fibers of the wood lying on the side of the pole toward the force being ex-

tended, and those on the opposite side being compressed. If, as is usually the case, the pole be a truncated cone, rupture occurs at the surface of the ground when the diameter of the pole at the point of application of the force is equal to, or greater than, $\frac{2}{3}$ of the diameter at the surface of the ground. When the diameter of the section at the point of application of the force is less, the rupture takes place above the surface of the ground, at that point where the diameter *is* $\frac{3}{2}$ of the diameter at the point of application of the force.

The horizontal force which a pole will support at the instant of rupture, when the section of rupture coincides with the section at the ground level, is shown by the formula —

$$F = \frac{\pi R^3 T_1}{4L}, \tag{16}$$

where R is the radius of the section at the surface of the ground, L is the height above the soil of the point of application of the force, and T_1 is the resistance to rupture per unit section of the substance which forms the pole.

Transposing and assuming 10 as a factor of safety, the formula becomes

$$R = \sqrt[3]{\frac{40\,FL}{\pi T_1}}. \tag{17}$$

Where the diameter of the section at the point of application of the force is less than $\frac{2}{3}$ of the diameter at the surface of the ground, and the section of rupture is above the ground level, the force F which will rupture the pole is given by the more complicated formula —

$$F = \frac{27\,\pi R_1^2 (R - R_1)\,T_1}{16\,L}, \tag{18}$$

in which R is the radius of the section at the surface of the ground, and R_1 the radius of the section at which the force is applied. F, in all cases, is the resultant of the horizontal forces acting on the pole, and a factor of safety of from 6 to 10 should be used.

The value of T_1 may be found in TABLES NOS. 26 and 27, compiled from standard authorities, which give the ultimate or breaking load of various kinds of wood, either in tension or compression.

TABLE No. 26.

Tensile Strength of Timber.

VARIETY.	BREAKING WEIGHT PER SQUARE INCH IN LBS.		
Ash, white	10,000	to	17,000
Ash, American	5,500	"	17,000
Ash, English	4,000	"	16,000
Beech	5,000	"	12,000
Birch	7,000	"	15,000
Cedar, Lebanon	10,000	"	12,000
Cedar, West Indian	4,000	"	7,500
Cedar, American			11,400
Chestnut	7,000	"	13,000
Cypress			6,000
Deal, Christiania			12,900
Elm	6,000	"	10,000
Oak			10,000
Pine, pitch			7,600
Pine, Riga			14,300
Pine, yellow	5,000	"	12,000
Pine, red			3,000
Poplar	5,500	"	7,000
Redwood, California			10,800
Spruce	5,000	"	10,000
Sycamore	9,000	"	13,000

TABLE No. 27.

Crushing Strength of Timber.

VARIETY.	CRUSHING WEIGHT IN LBS. PER SQUARE INCH.		
Ash	5,000	to	8,000
Beech			7,700
Birch	4,500	"	6,000
Cedar, red	4,500	"	5,900
Chestnut			5,350
Elm	6,000	"	10,000
Oak, American, white	4,000	"	9,000
Oak, English	6,500	"	9,500
Oak, Dantzic			7,700
Pine, pitch			6,800
Pine, yellow	5,300	"	6,500
Pine, red	6,000	"	7,500
Pine, white	5,000	"	6,000
Spruce, white	4,500	"	6,000

38. The preceding formulæ are generally applicable where the forces acting on a pole may be resolved into one horizontal component. Often the horizontal force is exceedingly large, as in a corner or terminal pole. In such cases, an unsupported pole of sufficient strength would be impracticable. By staying the pole with a guy-rod, a new set of conditions arise. Where a pole stands without guying, one-half of its fibers are in tension and the other half in compression. When the guy-rod is added, all the fibers of the pole are under compression, while the guy-rod is under tension.

Let F equal the known horizontal resultant of the forces which act on the pole; β, the angle between the guy-rod and the ground level (horizontal); α, the angle between the guy-rod and the pole (perpendicular); T, the tension on the guy-rod; and S, the crushing-force acting on the pole.

Then, $$T = \frac{F}{\cos\beta} \quad (19), \qquad \text{and } S = T\cos\alpha. \quad (20)$$

These formulæ are true, assuming that the guy-rod is attached to the pole at the point of application of the horizontal resultant force F.

Represent by P the total weight of pole cross-arms, guy-rod, fixtures, wires, and cables. The total crushing-force W acting on the pole is then $S + P$.

Considering the pole now as a long column fixed at one end, in which l is the distance in feet from ground to section just below cross-arms; d is the diameter in inches of the smallest section of the pole below the cross-arms; Hodgkinson's experiments indicate that the ultimate supporting power is given for pine columns by the expression, —

$$W \text{ (in short tons)} = 4\frac{d^4}{l^2}. \quad (21)$$

For any other kind of wood, the resistance to rupture will be in proportion to the respective ultimate crushing-strength, as given in Table No. 27.

39. In the case of the ordinary pole-line, provision must be made for two things — wind pressure, and the crushing-weight due to snow and ice on the wires. Assuming a wind pressure of 30 lbs. per sq. ft., the pressure exerted by the wind on an ordinary 40 ft. pole, measuring 7 in. in diameter at the top, and 14 ins. at the

ground level, will be approximately equivalent to a horizontal force of about 500 lbs. applied at the top of the pole; and a difference of 5 ft. in the length of the pole will make a difference of about 100 lbs. in the resultant pressure.

The pressure per cross-arm carrying 10 wires will be, approximately, 500 lbs. From such data, the total horizontal force acting on the top of the pole can be estimated, and applied in the formulæ. It must be understood that it is unnecessary to use all the preceding formulæ in calculating the size of every pole, as, to a great extent, the judgment of the designer must be exercised. A pole-line with only two or three wires, or a single cross-arm, would need no special precaution against crushing. In a large corner, or terminal pole, wind pressure is a comparatively small proportion of the total bending-force, and is amply provided for by the factor of safety. In each case the controlling destructive force must be provided for, allowing the factor of safety to provide for the others.

The sleet storm, or fall of damp snow, succeeded by a high wind, is the worst enemy of the pole-line. There are cases on record of ice incrustations on a No. 10 wire accumulating to such an extent as to make a continuous cylinder two inches in diameter. The best practice indicates the advisability of making the poles strong enough to withstand all the ordinary attacks of the elements; if then, under an excessive snow load, some of the circuits are ruptured, the repair job is an easy one. A broken pole is much more difficult to replace, and in the act of falling is apt to drag with it a long section of line, thus extending the damage over a large territory.

40. Guying. — In the open country, there is little or no objection to the practice of re-enforcing by means of guy-rods; but in city lines the room occupied in a street by the guys becomes exceedingly objectionable, as in many cases the direction of the resultant is such as to necessitate a guy situated in such a manner as to interfere with traffic. Inasmuch as the method of guying is by far the cheapest expedient, it is still resorted to in all cases where it is possible for it to be successfully accomplished.

Various forms of pole-guys are indicated in Fig. 13, illustrating also the objectionable forms, such as tying the top of the pole by means of a guy, or in placing it directly below the cross-arms. The effect of such guying is to cause the pole to bend directly beneath

the arms, and, ultimately, to fail at this point. Probably the most valuable form of guy is that known as the "Y" guy, consisting in a tension member so arranged as to secure the pole directly at the top of, and immediately under, the cross-arms. In this way nearly all the stress of the line is transferred to the guy, in such a manner as not to cause sensible deflection of the pole. When there are more than two or three cross-arms, "Y" guying should always be adopted. Most frequently guys are made either of one or more strands of No. 8

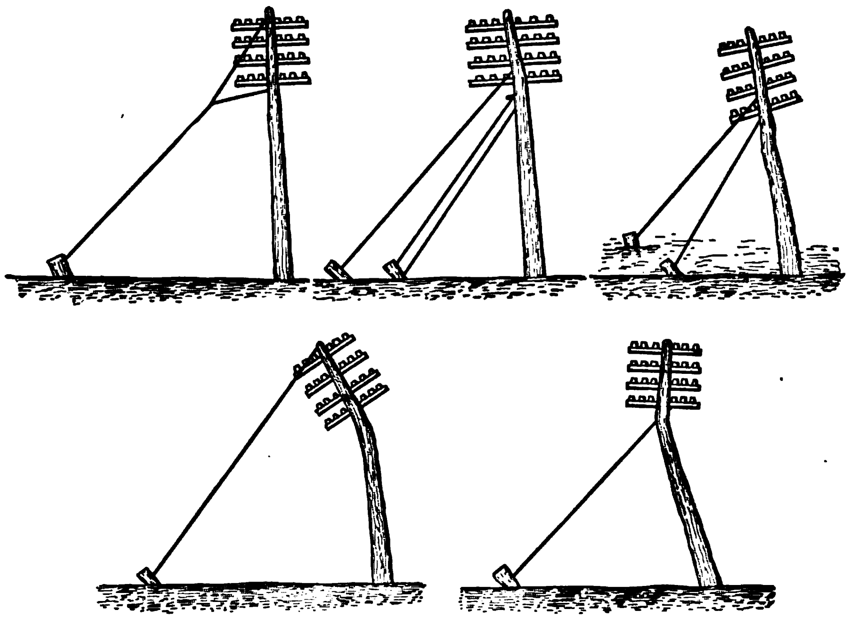


Fig. 13. Methods of Guying.

iron wire, or of ¼″ to ⅜″ steel wire strand. The wire strand is to be preferred, as it is more flexible, more easily drawn to the proper tension, and adapts itself more readily to the emergencies of each particular case. It is also customary, in straight line work, to frequently guy the top of each pole to the base of the next succeeding one for several poles. By this means the lateral vibration introduced by heavy wind storms may, in a great measure, be checked, and frequent instances are on record wherein this method of "head guying," as it is technically termed, has saved a line from destruction.

41. Anchor Poles. — In city lines, where an abrupt angle occurs, or where, for the purpose of entering underground conduits, it is essential to terminate a large and heavy line, it is necessary to provide

Fig. 14. Structural Iron Anchor Pole.

a pole of sufficient stiffness to assume the entire tension of all of the circuits. The neatest solution of the problem is to design a pole of structural iron of sufficient strength to withstand the stresses of all of the circuits, in such a manner as to make the pole entirely self-

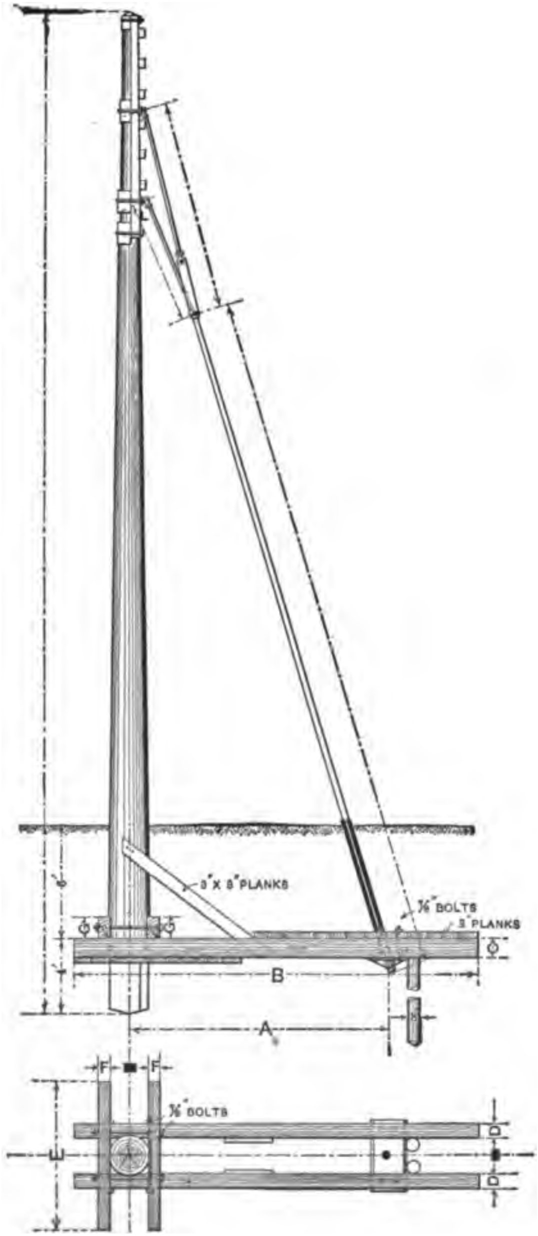


Fig. 15. Plan and Elevation of Combination Anchor Pole.
See TABLE 28.

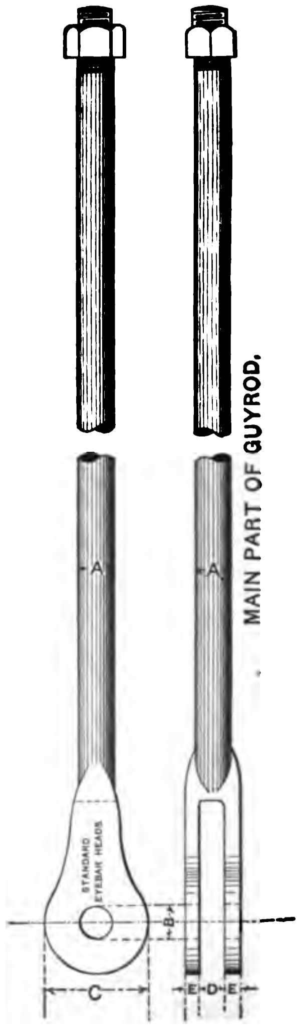


Fig. 16. Detail of Main Guy-Rod.
See TABLE 29.

sustaining. In ordinary line practice it is customary to stretch each wire to a tension of about 150 lbs., while the messenger strands supporting aerial cables usually have a tension of about 3,000 lbs. Thus it will be evident that in heavy lines, carrying say from four to five cables, and from 80 to 100 wires, the line stresses are by no means insignificant quantities, amounting in the above instance to some

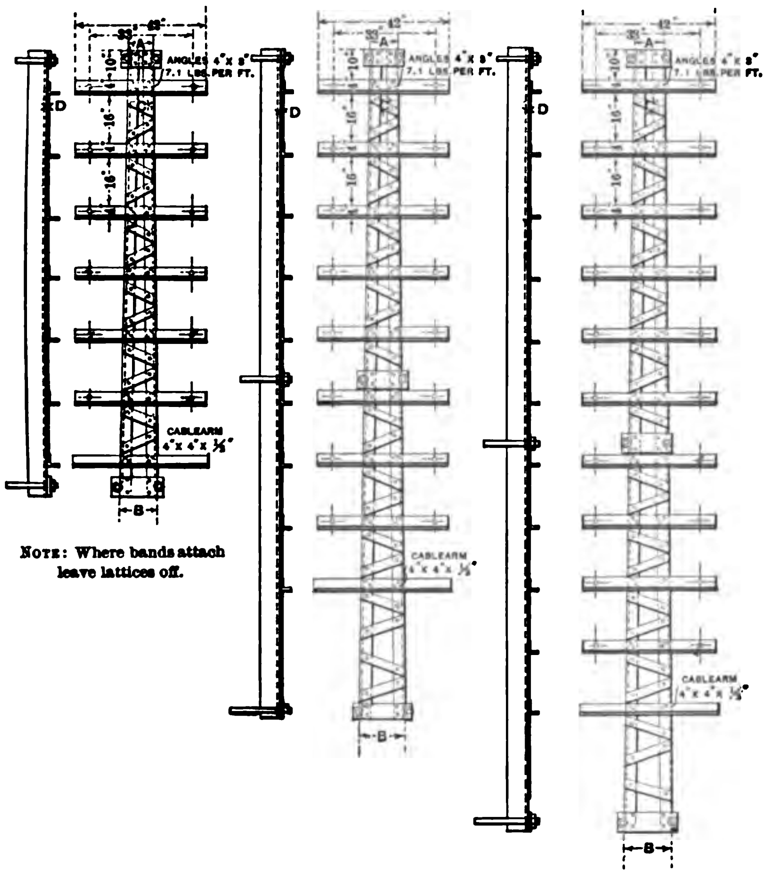
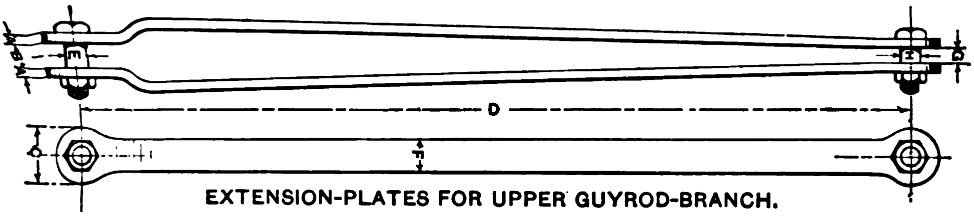


Fig. 17. Detail of Po'e Lattice. See TABLE 30.

30,000 lbs. upon the top of the pole. For lines of this magnitude, the pole is not less than from 50 to 60 ft. in height, and consequently the bending-moment at the base of the pole is exceedingly severe. There is no difficulty in manufacturing a structural iron or steel pole, setting the same in a heavy base of concrete, and thus

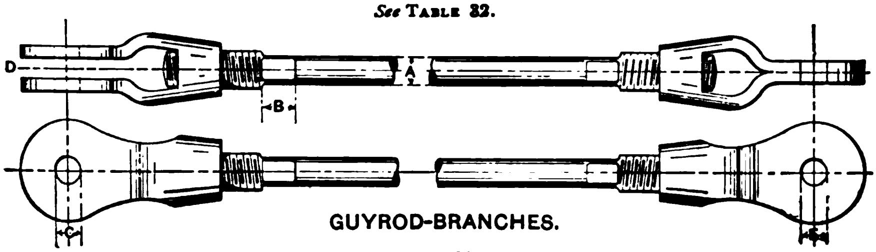
attaining the necessary strength. An illustration of such an anchor pole will be found in Fig. 14. The only objection to a structure of this kind exists in its initial expense, amounting, in the instance above cited, to a cost of some $600. As a compromise in the matter of expense, the plan has been adopted of constructing a composite pole, and anchoring the same by means of a guy-rod. The design for such a pole is indicated in Fig. 15, with the essential details in Figs. 16, 17, 18, 19, and 20. The pole is made by securing an appropriate wooden spar, about 24″ in diameter at the butt, and not less than from 10″ to 12″ at the top, the height of the pole conforming to the height of the line. The pole is then provided with a frame-work or anchor platform, as shown in plan and elevation in Fig. 15, by means of which the pole is solidly set into the earth. In setting the pole, the anchor platform is set in a direction away from the longitudinal stress of the line, in order that the pole may not fail by overturning, the weight of earth resting upon this platform being more than equal to the bending-moment arising from the line stress. The guy-rod extends from this platform to the top of the pole, so arranged as to take through the guy-rod branches the horizontal components of the line stress. The details of the guy-rod and branches are seen in Figs. 16 and 18.

The top of the pole consists of a lattice-work of angles, as shown in Fig. 17, that are fitted to the top of the pole, three sizes having sufficient range to be readily adapted to all ordinary lines. The lattice-work consists of two 3″ × 7″ steel angles latticed together to readily fit on to the top of the spar. At appropriate intervals light 3″ × 4″ angles are set, for the purpose of supporting the cross-arms. The lattice-work is secured to the pole by means of the bands shown in Figs. 19 and 20. At the proper points to balance the line tensions, guy-rod bands shown in Fig. 19 are placed, to which branch guy-rods extending to the main guy-rod, and thence to the anchor platform, are attached, by means of swivel clevises, so that any slack in the guy-rod may be taken up. A slight consideration of this design will show that the lattice-work is amply sufficient to carry, without sensible deflection, and transfer to the two parts of the guy-rod, all the horizontal components of the stresses introduced by the lines. As a result, the spar is entirely relieved from all bending-moment, being subjected solely to the vertical component of the stress in the



EXTENSION-PLATES FOR UPPER GUYROD-BRANCH.

See TABLE 32.



GUYROD-BRANCHES.

See TABLE 31.

Fig. 18. Detail of Guy-Rod Branches.

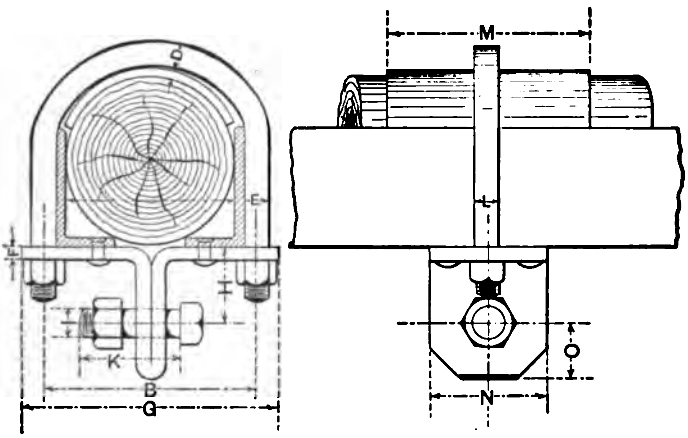


Fig. 19. Detail of Guy-Rod Bands. *See* TABLE 33.

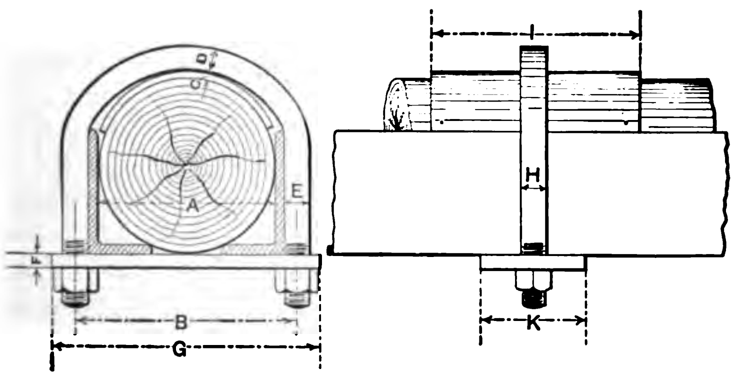


Fig. 20. Detail of Pole-Bands. *See* TABLE 34.

guy-rod. TABLES NOS. 28 to 34 contain full dimensions of all the sizes necessary for anchor poles of this description, capable of carrying from four to ten cross-arms, and from one to four cables. Blanks are left for the height of the pole, length of guy-rods, and size of opening in the top and bottom of lattice, as these dimensions will vary with each pole. The dimension letters in the illustration refer to the values to be found in the table. Anchor poles of this description have worked successfully, and may be introduced at about one-half the cost of the corresponding structural iron pole.

Fig. 21. Composite Anchor Pole.

The illustration, Fig. 21, is from a photograph of a composite anchor pole of this kind, designed for 100 wires and 4 cables. At the time of the photograph 70 wires were in place, but no cables. The spar forming the pole was a Norway pine stick 70 ft. long, 16″ at the top, and 22″ at the ground, and set 10 ft. below the surface.

The chief criticism to be passed is the certainty of the early rotting of the pole at or near its base, where the wood

TABLE No. 28. — DIMENSIONS FOR COMBINATION ANCHOR POLES.

Data for Anchor Platform. (*See Fig. 15.*)

NUMBER OF CROSS-ARMS.	NUMBER OF CABLES.	A	B	C	D	E	F	G
4	0	12′ 0″	20′ 0″	12″	8″	6′ 0″	6″	12″
6	0	12′ 0″	20′ 0″	12″	8″	6′ 0″	6″	12″
8	0	13′ 0″	22′ 0″	12″	10″	8′ 0″	8″	12″
10	0	13′ 0″	22′ 0″	12″	10″	8′ 0″	8″	12″
4	1	12′ 0″	20′ 0″	12″	8″	6′ 0″	6″	12″
6	1	13′ 0″	22′ 0″	12″	10″	8′ 0″	8″	12″
8	1	13′ 0″	22′ 0″	12″	10″	8′ 0″	8″	12″
10	1	14′ 0″	24′ 0″	12″	12″	10′ 0″	10″	12″
4	2	13′ 0″	22′ 0″	12″	10″	8′ 0″	8″	12″
6	2	13′ 0″	22′ 0″	12″	10″	8′ 0″	8″	12″
8	2	14′ 0″	24′ 0″	12″	12″	10′ 0″	10″	12″
10	2	14′ 0″	24′ 0″	12″	12″	10′ 0″	10″	12″
4	3	13′ 0″	22′ 0″	12″	10″	8′ 0″	8″	12″
6	3	13′ 0″	22′ 0″	12″	10″	8′ 0″	8″	12″
8	3	14′ 0″	24′ 0″	12″	12″	10′ 0″	10″	12″
10	3	14′ 0″	24′ 0″	12″	12″	10′ 0″	10″	12″

TABLE No. 29.

Data for Main Guy-Rod. (*See Fig. 16.*)

NUMBER OF CROSS-ARMS.	NUMBER OF CABLES.	A	B	C	D	E
4	0	1⅝″	2⅛″	7″	1½″	¾″
6	0	1⅞″	2⅛″	7″	1¾″	⅝″
8	0	2 ″	2⅛″	7″	1½″	⅞″
10	0	2 ″	2⅛″	7″	1½″	⅞″
4	1	1⅞″	2⅛″	7″	1¾″	¾″
6	1	2 ″	2⅛″	7″	1½″	⅞″
8	1	2 ″	2⅛″	7″	1½″	⅞″
10	1	2¼″	2⅛″	7″	1½″	⅞″
4	2	2 ″	2⅛″	7″	1½″	⅞″
6	2	2 ″	2⅛″	7″	1½″	⅞″
8	2	2¼″	2⅛″	7″	1½″	⅞″
10	2	2¼″	3 ″	7 13/16″	1¾″	1″
4	3	2 ″	2⅛″	7″	1½″	⅞″
6	3	2 ″	2⅛″	7″	1¼″	⅞″
8	3	2⅛″	3″	7 15/16″	1¾″	1″
10	3	2⅞″	3⅛″	9⅛″	2¼″	1″

TABLE NO. 30.

Data for Lattices. *(See Fig. 17.)*

No. of Cross-arms.	No. of Cables.	Size of Angle Iron.	Weight of Angle Iron in lbs. per foot.	No. of Angles.	Length of Angles.	A	B	Lattices. wide C	Lattices. thick D	Positions of Guy-rod Bands. Upper Band.	Positions of Guy-rod Bands. Lower Band.	Number of Clamping Bands.
4	0	6″ × 3½″ × ⅜″	11.7	2	12′ 0″	. .	. .	3″	¼″	Midway between 2d and 3d arm	. . . .	2
6	0	7″ 3½″ 7/16″	15	2	12′ 0″	. .	. .	3″	¼″	Midway between 3d and 4th arm	. . . .	2
8	0	6″ 3½″ ⅜″	11.7	2	18′ 0″	. .	. .	3″	¼″	Midway between 2d and 3d arm	Midway between 6th and 7th arm	3
10	0	7″ 3½″ 7/16″	15	2	21′ 0″	. .	. .	3″	¼″	Just above 3d arm	Just below 8th arm	3
4	1	7″ 3½″ 7/16″	15	2	12′ 0″	. .	. .	3″	¼″	Just below 3d arm	. . . .	2
6	1	6″ 3½″ ⅜″	11.7	2	12′ 0″	. .	. .	3″	¼″	Midway between 2d and 3d arm	Just below 6th arm	2
8	1	7″ 3½″ 7/16″	15	2	18′ 0″	. .	. .	3″	¼″	Just below 3d arm	Just below 8th arm	3
10	1	7″ 3½″ 7/16″	15	2	21′ 0″	. .	. .	3″	¼″	Midway between 3d and 4th arm	Midway between 9th and 10th arm	3
4	2	6″ 3½″ ⅜″	11.7	2	12′ 0″	. .	. .	3″	¼″	Just above 2d arm	Midway between 4th and cable-arm	2
6	2	6″ 3½″ ⅜″	11.7	2	12′ 0″	. .	. .	3″	¼″	Midway between 3d and 4th arm	Midway between 6th and cable-arm	2
8	2	7″ 3½″ 7/16″	15	2	18′ 0″	. .	. .	3″	¼″	Just below 3d arm	Just below 8th arm	3
10	2	7″ 3½″ 9/16″	24.9	2	21′ 0″	. .	. .	3″	¼″	Just above 4th arm	Just above 10th arm	3
4	3	6″ 3½″ ⅜″	11.7	2	12′ 0″	. .	. .	3″	¼″	Just above 2d arm	Just above cable arm	2
6	3	7″ 3½″ 7/16″	15	2	18′ 0″	. .	. .	3″	¼″	Midway between 3d and 4th arm	Midway between 6th and cable-arm	2
8	3	7″ 3½″ 9/16″	24.9	2	18′ 0″	. .	. .	3″	¼″	Midway between 3d and 4th arm	Just below 8th arm	3
10	3	7″ 3½″ 9/16″	24.9	2	21′ 0″	. .	. .	3″	¼″	Just above 4th arm	Just below 10th arm	3

TABLE No. 31.

Data for Guy-Rod Branches. *(See Fig. 18.)*

Number of Cross-arms.	Number of Cables.	A	B	C	D	E
4	0	1⅝″	2½″	7″	1½″	¾″
6	0	1⅞″	2½″	7″	1¾″	¾″
8	0	2 ″	2½″	7″	1½″	⅞″
10	0	2 ″	2½″	7″	1½″	⅞″
4	1	1⅞″	2½″	7″	1¾″	¾″
6	1	2 ″	2½″	7″	1½″	⅞″
8	1	2 ″	2½″	7″	1½″	⅞″
10	1	2¼″	2½″	7″	1½″	⅞″
4	2	2 ″	2½″	7″	1½″	⅞″
6	2	2 ″	2½″	7″	1½″	⅞″
8	2	2¼″	2½″	7″	1½″	⅞″
10	2	2½″	3 ″	7 13/16″	1¾″	1 ″
4	3	2 ″	2½″	7″	1½″	⅞″
6	3	2 ″	2½″	7″	1½″	⅞″
8	3	2½″	3 ″	7 13/16″	1¾″	1 ″
10	3	2⅞″	3½″	9½″	2¼″	1 ″

TABLE No. 32.

Data for Extension Plates. *(See Fig. 18.)*

Number of Cross-arms.	Number of Cables.	A	B	C	D	E	F	G
8	0	¾	3¼	7	54	2½	3	1½
10	0	¾	3¼	7	54	2½	3	1½
6	1	¾	3¼	7	48	2½	3	1½
8	1	¾	3¼	7	54	2½	3	1½
10	1	¾	3¼	7	54	2½	3	1½
4	2	¾	3¼	7	48	2½	3	1½
6	2	¾	3¼	7	48	2½	3	1½
8	2	¾	3¼	7	54	2½	3	1½
10	2	¾	3¾	7 13/16	54	3	4	1¾
4	3	¾	3¼	7	48	2½	3	1½
6	3	¾	3¼	7	48	2½	3	1½
8	3	¾	3¾	7 13/16	54	3	4	1¾
10	3	¾	4¼	9½	54	3½	5	2¼

TABLE No. 33.

Data for Guy-Rod Bands. *(See Fig. 19.)*

No. of Cross-arms.	No. of Cables.	UPPER BAND.														LOWER BAND.														Dia. of Rivets
		A	B	C	D	E	F	G	H	I	K	L	M	N	O	A	B	C	D	E	F	G	H	I	K	L	M	N	O	
4	0	..	..	3/8	1 3/8	1 3/8	3/4	..	4 3/4	1 3/4	5 3/4	1 3/8	12	7	3 1/2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	3/4
6	0	..	..	3/8	1 3/8	1 3/8	3/4	..	5 3/4	2	6 1/4	1 3/8	12	8	4	..	..	..	..	..	..	..	..	..	..	..	..	..	..	3/4
8	0	..	..	3/8	1 3/8	1 3/8	3/4	..	4 1/4	1 1/2	5 1/4	1 3/8	12	7	3 1/2	..	..	3/8	1 3/8	1 3/8	3/4	..	4 1/4	1 1/2	5 1/4	1 3/8	12	7	3 1/2	3/4
10	0	..	..	3/8	1 3/8	1 3/8	3/4	..	4 3/4	1 3/4	5 3/4	1 3/8	12	7	3 1/2	..	..	3/8	1 3/8	1 3/8	3/4	..	4 3/4	1 3/4	5 3/4	1 3/8	12	7	3 1/2	3/4
4	1	..	..	3/8	1 3/8	1 3/8	3/4	..	5 3/4	2	6 1/4	1 3/8	12	8	4	..	..	..	..	..	..	..	..	..	..	..	..	..	..	3/4
6	1	..	..	3/8	1 3/8	1 3/8	3/4	..	4 1/4	1 1/2	5 1/4	1 3/8	12	7	3 1/2	..	..	3/8	1 3/8	1 3/8	3/4	..	4 1/4	1 1/2	5 1/4	1 3/8	12	7	3 1/2	3/4
8	1	..	..	3/8	1 3/8	1 3/8	3/4	..	4 3/4	1 3/4	5 3/4	1 3/8	12	7	3 1/2	..	..	3/8	1 3/8	1 3/8	3/4	..	4 3/4	1 3/4	5 3/4	1 3/8	12	7	3 1/2	3/4
10	1	..	..	3/8	1 3/8	1 3/8	7/8	..	4 3/4	1 7/8	6	1 3/8	12	7	3 1/2	..	..	3/8	1 3/8	1 3/8	7/8	..	4 3/4	1 7/8	6	1 3/8	12	7	3 1/2	3/4
4	2	..	..	3/8	1 3/8	1 3/8	3/4	..	4 1/4	1 1/2	5 1/4	1 3/8	12	7	3 1/2	..	..	3/8	1 3/8	1 3/8	3/4	..	4 1/4	1 1/2	5 1/4	1 3/8	12	7	3 1/2	7/8
6	2	..	..	3/8	1 3/8	1 3/8	3/4	..	4 3/4	1 3/4	5 3/4	1 3/8	12	7	3 1/2	..	..	3/8	1 3/8	1 3/8	3/4	..	4 3/4	1 3/4	5 3/4	1 3/8	12	7	3 1/2	3/4
8	2	..	..	3/8	1 3/8	1 3/8	7/8	..	4 3/4	1 7/8	6	1 3/8	12	7	3 1/2	..	..	3/8	1 3/8	1 3/8	7/8	..	4 3/4	1 7/8	6	1 3/8	12	7	3 1/2	3/4
10	2	..	..	3/8	1 1/2	1 1/2	1	..	6	2 1/8	7	1 1/2	12	8	4	..	..	3/8	1 1/2	1 1/2	1	..	6	2 1/8	7	1 1/2	12	8	4	7/8
4	3	..	..	3/8	1 3/8	1 3/8	3/4	..	4 1/4	1 1/2	5 1/4	1 3/8	12	7	3 1/2	..	..	3/8	1 3/8	1 3/8	3/4	..	4 1/4	1 1/2	5 1/4	1 3/8	12	7	3 1/2	1
6	3	..	..	3/8	1 3/8	1 3/8	7/8	..	4 3/4	1 3/4	5 3/4	1 3/8	12	7	3 1/2	..	..	3/8	1 3/8	1 3/8	7/8	..	4 3/4	1 3/4	5 3/4	1 3/8	12	7	3 1/2	3/4
8	3	..	..	3/8	1 1/2	1 1/2	1	..	6	2 1/8	7	1 1/2	12	8	4	..	..	3/8	1 1/2	1 1/2	1	..	6	2 1/8	7	1 1/2	12	8	4	7/8
10	3	..	..	3/8	1 5/8	1 5/8	1	..	6 1/4	2 3/8	7 1/2	1 5/8	12	8	4	..	..	3/8	1 5/8	1 5/8	1	..	6 1/4	2 3/8	7 1/2	1 5/8	12	8	4	1

TABLE NO. 34.

Data for Pole-Bands. *(See Fig. 20.)*

No. of Cross-Arms.	No. of Cables.	UPPER BAND.										MIDDLE BAND.										LOWER BAND.									
		A	B	C	D	E	F	G	H	I	K	A	B	C	D	E	F	G	H	I	K	A	B	C	D	E	F	G	H	I	K
4	0	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	9	6	..	..	..	..	..	..	..	..	..	..	..	..	5/8	1 5/8	1 3/8	3/4	..	1 1/8	12	6
6	0	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	9	6	..	..	..	..	..	..	..	..	..	..	..	..	5/8	1 5/8	1 3/8	3/4	..	1 1/8	12	6
8	0	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	9	6	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	12	6	..	..	5/8	1 5/8	1 3/8	3/4	..	1 1/8	12	6
10	0	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	9	6	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	12	6	..	..	5/8	1 5/8	1 3/8	3/4	..	1 1/8	12	6
4	1	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	9	6	..	..	..	..	..	..	..	..	..	..	..	..	5/8	1 5/8	1 3/8	3/4	..	1 1/8	12	6
6	1	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	9	6	..	..	..	..	..	..	..	..	..	..	..	..	5/8	1 5/8	1 3/8	3/4	..	1 1/8	12	6
8	1	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	9	6	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	12	6	..	..	5/8	1 5/8	1 3/8	3/4	..	1 1/8	12	6
10	1	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	9	6	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	12	6	..	..	5/8	1 5/8	1 3/8	3/4	..	1 1/8	12	6
4	2	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	9	6	..	..	..	..	..	..	..	..	..	..	..	..	5/8	1 5/8	1 3/8	3/4	..	1 1/8	12	6
6	2	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	9	6	..	..	..	..	..	..	..	..	..	..	..	..	5/8	1 5/8	1 3/8	3/4	..	1 1/8	12	6
8	2	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	9	6	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	12	6	..	..	5/8	1 5/8	1 3/8	3/4	..	1 1/8	12	6
10	2	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	9	6	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	12	6	..	..	5/8	1 5/8	1 3/8	3/4	..	1 1/8	12	6
4	3	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	9	6	..	..	..	..	..	..	..	..	..	..	..	..	5/8	1 5/8	1 3/8	3/4	..	1 1/8	12	6
6	3	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	9	6	..	..	..	..	..	..	..	..	..	..	..	..	5/8	1 5/8	1 3/8	3/4	..	1 1/8	12	6
8	3	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	9	6	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	12	6	..	..	5/8	1 5/8	1 3/8	3/4	..	1 1/8	12	6
10	3	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	9	6	..	..	3/8	1 3/8	1 3/8	3/4	..	1 3/8	12	6	..	..	5/8	1 5/8	1 3/8	3/4	..	1 1/8	12	6

enters the ground. For this defect there is no cure, excepting the adoption of the structural iron pole, though a good coat of tar or asphalt will give the pole a life of ten or fifteen years.

42. Setting the poles. — Small and light poles should be set some six feet into the ground, and may be planted by simply excavating a hole a little larger than the butt of the pole, and then placing the same in position by lifting the pole bodily, with a sufficient gang of men, and dropping it into the hole. Where the ground is soft or marshy, or where the stresses brought upon the pole by the tension of the lines is excessive, a foundation should be formed by excavating a hole of sufficient depth, from four to six feet in diameter, and after the pole is planted filling the same with a concrete of broken stone, sand, and cement. A good mixture for this purpose may be made of one part of Rosendale cement, mixed with three parts of sand, and five parts of broken stone. The ingredients should be thoroughly mingled, and carefully moistened with about 25 per cent of water, and solidly rammed around the base of the pole. After the concreting is complete, the earth may be replaced and thoroughly tamped into position.

43. The "sand-barrel" is often used with success in soft locations. A stout barrel or cask is placed at the bottom of the excavation, into which the butt of the pole is set. A firm loam, clay, or sand, is then packed tightly into the barrel around the pole, thus forming a foundation. In sandy soils the "temporary sand-barrel" is a most valuable device. This consists of an iron cylinder about the size of a very large cask, but split in two parts, and provided with hinges and clasps. The cylinder is set at the bottom of the excavation, and the pole planted inside of it, and the earth carefully rammed around, completely filling the excavation. Then, by means of a fall, the iron cylinder is withdrawn from the earth, and, opening the clasps, is removed from the pole. For large poles it is customary to cut the ground away into a series of steps. The terraces thus made afford an opportunity to ease the pole into its position at the bottom of the hole; and then, with a working wagon or derrick and sufficient tackle, the pole may gradually be raised to, and sustained in, an upright position, while the earth or concrete is tamped around its base.

44. Insulators. — The number and form of line insulators, together with the materials proposed for their construction, have been

legion. In this country glass is almost universally used for telegraph and telephone work, and for the latter the tendency has been to make the insulator as small and light as possible. In England, however, the porcelain insulator is the most common, the difference in climate fully accounting for the English preference. As an insulating material, glass has several disadvantages. It is considerably more hygroscopic than porcelain, readily condensing on its surface a film moisture which rapidly lowers its insulating qualities. It is very

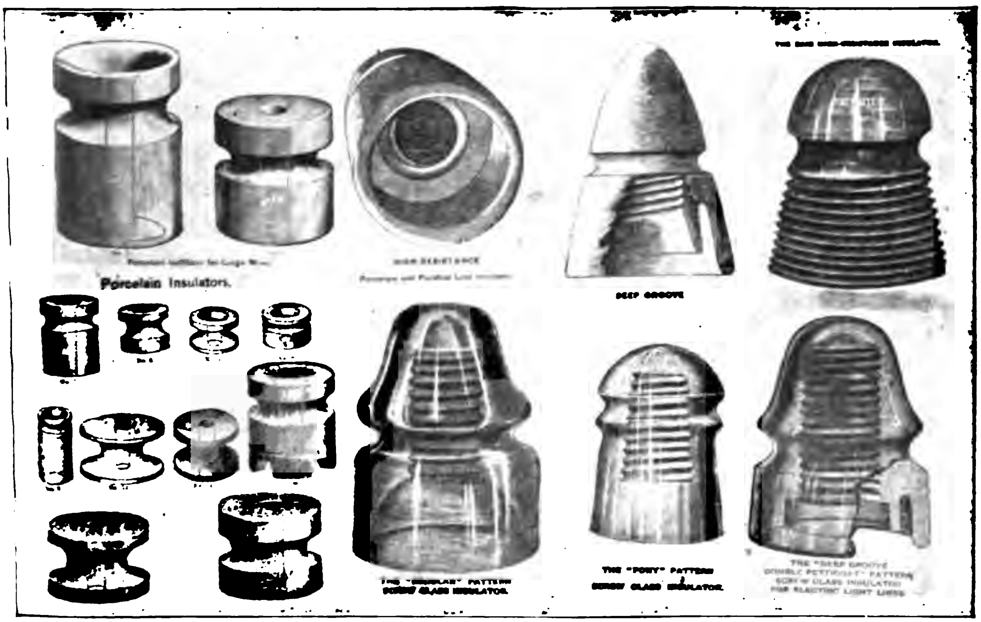


Fig. 22. Specimen Insulators.

brittle, decidedly more so than porcelain or earthenware. While blown glass is better in every respect than that which is cast, it is so much more expensive that molded insulators are almost universally used. The great advantage of the glass insulator lies in its transparency, which prevents the formation of cocoons under the petticoats of the insulator, that have a very marked effect in lowering the resistance of lines. Ebonite and india-rubber have been used to quite an extent for insulators; but as they quickly roughen by exposure to the weather, and are considerably more expensive, their

use has been almost exclusively confined to electric railway work. Brown stoneware forms an excellent substance for insulators, as it is strong, cheap, and durable, seldom cracks, and its color makes it inconspicuous. Thoroughly vitrified porcelain is probably the best insulator on the whole, and is used almost exclusively in England and on the Continent. The surface resists the formation of film moisture, and is easily washed clean by rain. The more common forms of line insulators are represented in Fig. 22 (p. 69).

45. The Value of Insulators. — Many experiments have been made to determine the value of poles, cross-arms, and insulators, to maintain line insulation. One test made on a telegraph line extending from New York to Boston, gave a result showing an insulation resistance of about 6,000 ohms per mile.

Some interesting figures derived from tests on cross-arms alone, erected in New York city, gave the following data: —

All four surfaces wet with sponge	3,120 ohms
Soaked one day, left to dry one day, and then wet	2,680 ohms
Painted three years before test	6,150 ohms
Same washed	9,166 ohms
Very dry 11,000 to	330,000 ohms
Newly painted	7,214 ohms
Unpainted for many years	4,300 ohms
Same after having been well washed	13,653 ohms
Same after having been well dried	80,000 ohms
Arms and pins together (wet)	3,686 ohms

From the same set of experiments the following figures are derived for the insulating power for dirty and soot-covered glass and pin insulators. The tests were made on 40 insulators, thus representing a mile of line: —

Dipped in water once	23,220 ohms
Dipped in water four times	56,400 ohms
New insulators and pins direct from supply dep't.	66,600 ohms

When the same insulators were carefully cleaned, their insulating power was raised to about three times the above value. These figures give in a striking manner the loss in insulation by exposure to smoke and dirt.

Some more recent experiments have been made by taking 50 of each of the typical forms of insulators, mounting them in the ordi-

nary way, and exposing them for some months to the action of the weather, the leakage over the insulators being carefully determined by the best-known electrical instruments, while a constant meteorological record was kept of the variations in atmospheric conditions. These experiments covered a period of nearly 150 days, observations being made at least once a day during the time. About half of the observations were made in clear weather, one-fifth in fair weather, 18% in cloudy weather, and 12% in foggy or rainy weather. The general results indicate that the greatest losses in insulation occurred during foggy or misty weather, when the insulators became coated with a thick beady film of moisture. During a heavy rain the insulation was somewhat higher; and after a storm, when sufficient time had elapsed for the drying of the insulator, the resistance of the line was considerably improved, owing to the cleaner condition of the insulating surface. The open double petticoat insulator was found to dry more rapidly than the close single petticoat; but during actual rainfall the loss in insulation of the double petticoat form is greater and more rapid than that of the single. In fine weather the large sizes of each form indicate parallel results, though the double petticoat form gave a much higher resistance than the single form of corresponding size. The true value of any form of insulator can only be properly computed when a consideration of the actual size of the insulating-bell has been eliminated, and attention concentrated entirely upon the possible cross-section of conducting material in the shape of moisture or dirt which may be deposited upon the exterior of the bell. To determine this, it is necessary to ascertain the mean circumference of the insulating material, divided by the conducting length between the point at which the wire is secured and the point of attachment of the insulator to the cross-arm. From this, the possible amount of conducting film may be determined by multiplying the mean circumference by the distance over the insulating surface, and evidently a form giving the greatest length in proportion to the mean circumference will have the highest insulating powers.

46. It is necessary to have the insulators closely and accurately fitted to the pins, and to plan the point of attachment of the wire as low down as possible, in order to give the smallest leverage upon the pin. With the growth of the transmission line the pin question has become of much importance. Mr. R. D. Mershon recently

proposed the following mechanical specifications,* which probably represent good engineering opinion as to dimensions of wooden pins:

Threaded End.—It is proposed to make the diameter of the small end of the pin 1 inch; the length of the threaded portion $2\frac{1}{2}$ inches; and the diameter at the lower end of the threaded portion 1.25 inches, so that the threaded portion will taper from 1.25 inches to 1 inch in a length of $2\frac{1}{2}$ inches. The threaded portion of the insulator should have the same dimensions and taper as that of the pin.

Shoulder.—It is proposed to make the shoulder $\frac{3}{16}$ inch on all pins. That is, the diameter of the pin just above the cross-arm will be $\frac{3}{8}$ inch greater than the nominal diameter of that portion of the pin in the cross-arm; it is proposed to carry this diameter $\frac{1}{2}$ inch above the cross-arm before tapering the pin.

Dimensions in Cross-Arm.—It is proposed to make the diameter of that portion of the pin in the cross-arm, just below the shoulder, $\frac{1}{32}$ inch less than the diameter of the hole in the cross-arm, and at the lower end of the pin $\frac{1}{16}$ less than the diameter of the hole in the cross-arm. It is proposed, also, to designate this portion of the pin as having a nominal diameter equal to that of the hole in the cross-arm into which the pin fits. Therefore, that portion of a pin which is to fit a $1\frac{1}{2}$-inch hole in a cross-arm will have a nominal diameter of $1\frac{1}{2}$ inches, but will have an actual diameter just below the shoulder of $1\frac{15}{32}$ inches, and at the lower end of the pin of $1\frac{7}{16}$ inches.

Thread.—It is proposed to use on all pins a thread having a pitch of $\frac{1}{4}$ inch, or four threads to the inch.

Designation.—It is proposed to designate that portion of the pin above the cross-arm as the "stem" of the pin; that portion in the cross-arm as the "shank" of the pin. It is proposed to designate a pin by the length of its stem, *i.e.*, a pin whose stem is 5 inches long will be designated as a "5"-inch pin, one 6 inches long as a "6"-inch pin, etc.

47. *Dimensions of Standard Pins.*—The following diagram (Fig. 23) and table give a number of sizes of pins, and their dimensions, which it is proposed to make standard. The diameter of the

* Trans. A. M. Inst. E. E., vol. xx., p. 415.

shank has in each case been fixed by making it approximately equal to (slightly larger than) the diameter of the theoretical pin corresponding to the length of the stem of the pin in question. The headings of the columns of the table refer to the lettering of diagram, which is a full-size unthreaded 5-inch (proposed standard) pin.

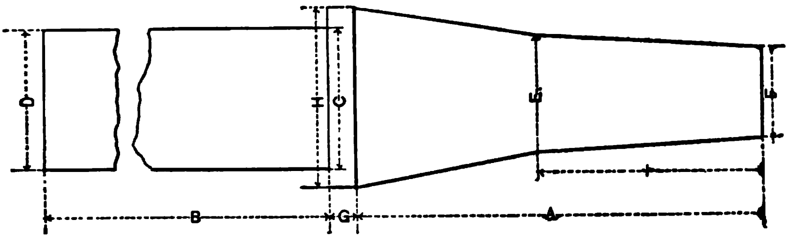


Fig. 23.

Size of Pin.	A	B	C Nominal.	C Actual.	D	E	F	G	H	I
5″	4¾″	4¼″	1½″	1 15/32″	1 7/16″	1¼″	1″	¼″	1⅞″	2½″
7	6¾	4¼	1¾	1 23/32	1 11/16	Same for all sizes of pins.	Same for all sizes of pins.	Same for all sizes of pins.	2⅛	Same for all sizes of pins.
9	8¾	4¼	1⅞	1 27/32	1 13/16				2¼	
11	10¾	4¾	2	1 31/32	1 15/16				2⅜	
13	12¾	4¾	2⅛	2 3/32	2 1/16				2½	
15	14¾	4¾	2¼	2 7/32	2 3/16				2⅝	
17	16¾	5¾	2⅜	2 11/32	2 5/16				2¾	
19	18¾	5¾	2½	2 15/32	2 7/16				2⅞	

48. To properly insulate a modern transmission line where cables may aggregate many hundreds of thousands of circular mils, carrying a potential of from 50,000 to 80,000 volts, taxes to the utmost all the resources of both electrical and mechanical engineering. From a mechanical standpoint the stresses both vertical and horizontal that the heavy wire or stranded cable necessary to a line transmitting hundreds or perhaps thousands of horse-power, are of no mean order, particularly when complicated with wind and snow loads, whose effect it is exceedingly difficult to calculate. All materials which have sufficient electrical resistance to be used in building insulators are mechanically weak and brittle, so that the engineer must not only deal with stresses of magnitude and of indeterminate amount, but is compelled

Fig. 24. 70,000-Volt Insulator.

DATA FOR FIG. 24.

Height..	5 inches
Diameter..	7 "
Top groove..	¾ inch
Weight..	8¼ pounds

Fig. 25. Double Petticoat, 100,000 Volt Insulator.

DATA FOR FIG. 25.

Height..............................	12 inches
Diameter..............................	9 "
Top groove..............................	1¼ "
Weight..............................	12.5 pounds
Test pressure..............................	100,000 volts

to use almost the worst possible material in an attempt to resist them. Nor is the problem an easy one from the electrical aspect. Lines

Fig. 26. 120,000 Volt Insulator.

DATA FOR FIG. 26.

Height	18¼ inches
Diameter	10 "
Top groove	1 inch
Weight	16 pounds
Test voltage	120,000

White or chocolate color. Article is three times length and width of illustration.

are actually working at 40,000 to 60,000 volts normal pressure, superimposed upon which is the possible exaltation of potential due to har-

monics produced by the properties of the line, or of the translating devices with which it may be loaded, or initiated by the sudden opening or closing of switches, an accidental short circuit, or other contingency. For heavy work glass and porcelain are the only materials now in current use, and opinion is divided as to which of the two is preferable. In a general way porcelain is admitted to be the best,

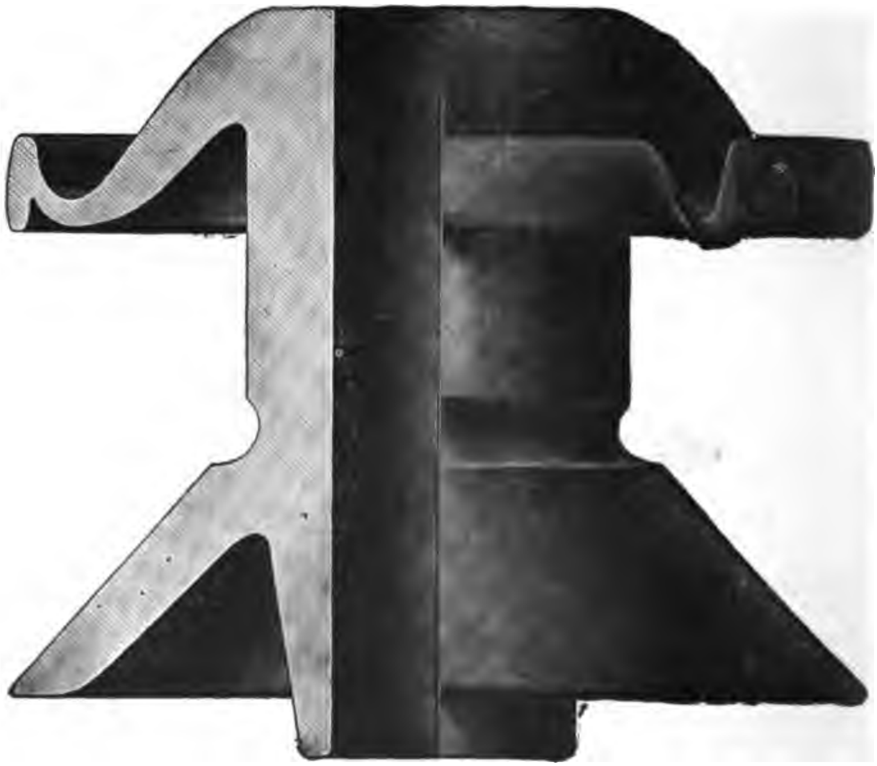


Fig. 27. Strain Insulator. For Corners.

provided it is *good*, but it is equally conceded that it is comparatively much more difficult to secure a good porcelain insulator than to get a good glass one. So, while it is intrinsically the better material, manufacturing difficulties are bringing glass more and more into use. Much ingenuity has been expended in the design of insulators for large transmission lines. Figs. 24, 25 and 26 show types that have been widely used, particularly with high voltages. Essentially insulators consist of three parts: an umbrella, an inner core that forms a petticoat, and

a pin-shield. The umbrella carries the line wire, which is held in a groove on the top and tied thereto in the usual manner. The umbrella is often provided with a rim surrounding its edge, called an "eave," as in Fig. 27, which serves to collect all the water which may collect on the insulator, and prevents it from either following the under side of the umbrella or being blown against the petticoat or pin-shield. A spout shown conveys away the drainage in any predetermined direction. The model of Fig. 25 is designed for 60,000 volts, while that in Fig. 26 is for 80,000 volts.

The designs of Figs. 25, 26, and 27 are built of porcelain. In Fig. 28 a combination insulator is shown which can be made of all

Fig. 28. Glass Petticoat Insulator.

porcelain, all glass, or either the petticoat or the umbrella can be made of glass or porcelain, and combined as may be desired.

Figs. 29 and 30 are illustrations of somewhat similar designs, but smaller in size and lighter, and intended to work at from 30,000 to 80,000 volts.

In view of the imperfections to which porcelain is liable, and to prevent excessive losses in burning, which are often occasioned by differences in thickness between the body of an insulator and the

thin edges of the petticoat, the artifice of making an insulator in sections, as shown in Fig. 31, is often adopted. As indicated, the insulators consist of three independent concentric bells designed to fit one on top of the other. Each bell is moulded and burned separately, and then tested, imperfect ones rejected; subsequently the three bells are coated with a fusible glaze and fired a second time, thus uniting all three into a single insulator. This illustration also shows a form of pin which is particularly desirable for transmission lines. The pin consists of a steel bolt from ½″ to ¾″ in diameter, depending upon size of wire which it is to support. Upon the top of the bolt a porcelain base is mounted, and above

Fig. 29. 80,000 Volt Insulator.

that a locust or oak block, upon which the proper screw-thread is formed for supporting the insulator. Evidently this pin may be made of any desired strength and yet necessitate but a relatively small hole in the cross-arm. The porcelain base provides an insulating incombustible and indestructible base, while the wooden thread secures sufficient elasticity to enable the pin to fit the thread of the insulator without being so rigid as to injure the vitreous material.

The type of glass insulator shown in Fig. 32 is a favorite with the electrical railway transmission lines, and is used extensively in Ohio and Indiana on the Interurbans. The line is bare copper from No. 4 to 00, depending on the power to be transmitted, and is tied

in a groove in the top of the insulator. Insulators of this kind are excellent for from 15,000 to 20,000 volts.

Fig. 80. 40,000 Volt Insulator. Niagara Type.

DATA FOR FIG. 80.

Height..............................	7½ inches
Diameter............................	7½ "
Top groove..........................	1 inch
Weight	8 pounds
Test voltage........................	70,000

White or chocolate color.

Much difference of opinion has existed as to the desirability of a multiplicity of petticoats. On one side several petticoats are claimed to give a longer surface over which leakage must take place. To offset this there is a greater surface for the collection of moisture and dust, shorter air space, and greater manufacturing difficulties. Latest designs incline to a large umbrella and very long pin shield, as is

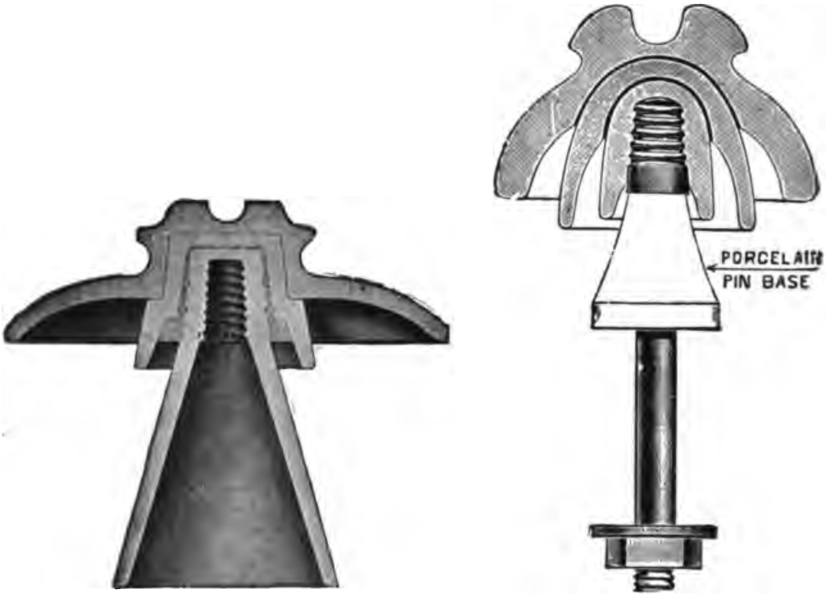


Fig. 31. Sectional Insulator and Pin.

DATA FOR FIG. 31.

Height	$11\frac{1}{2}$ inches
Groove	$1\frac{1}{4}$ "
Weight	150 pounds
Test pressure	150,000 volts

Fig. 32 20,000 Volt Railway Transmission Line Insulator.

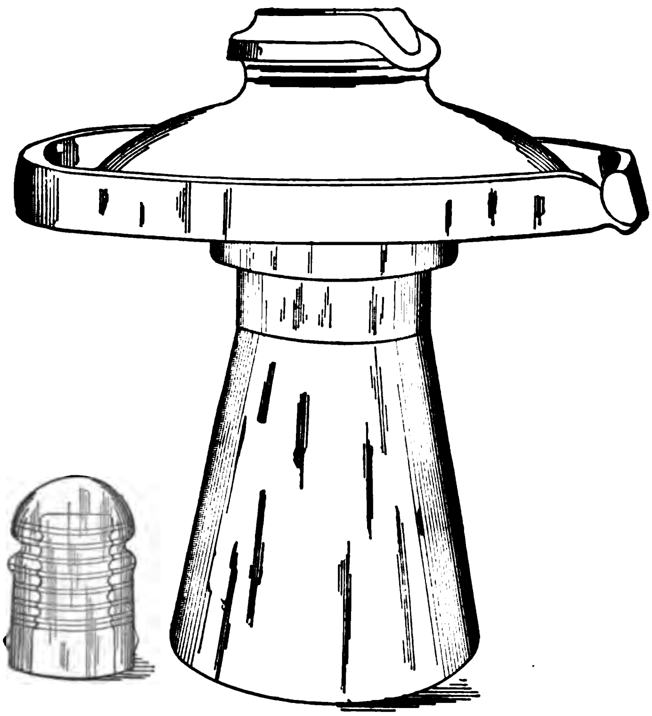


Fig. 33. Umbrella and Pin Shield Insulators.

illustrated in Fig. 33 and Plate I, in which relative sizes are shown by the insertion of an ordinary telegraph insulator.

The curves of TABLE NO. 35 (in Pocket) are interesting, as indicating the amount of leakage in watts as experimentally determined over various kinds of insulators. These curves are also instructive, as demonstrating that the material of which the insulator is constructed is of far less importance than the shape into which the material is formed.

Fig. 34. A Modern Transmission Line.

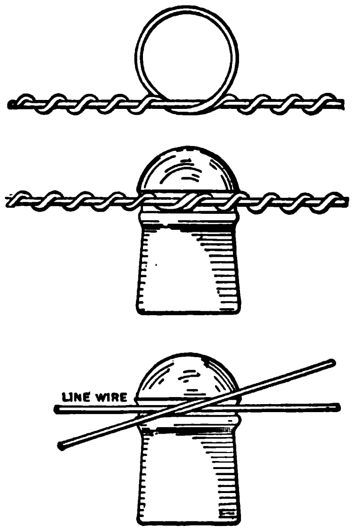


Fig. 35. Line-Wire Tie.

Relatively to telegraph and telephone construction, transmission lines carry very few wires; but as these are apt to be of large size, the line stresses may be increased, while those on the individual insulator and pin are largely augmented, requiring double arming at all corners, and often in straight-line work. Fig. 34 is a typical example of the modern transmission line.

49. Tying and "Dead-Ending."—To secure the line-wire to the insulator would seem a simple matter; yet to devise a tie that is secure, simple, economical, and effective has taxed the ingenuity of line men.

Transmission Line Insulator. 80,000 Volts Working-Pressure.

To face page 82.

The standard method now in use is shown in Fig. 35. The line-wire is laid in the groove of the insulator, and a soft copper wire from 8 in. to 24 in. in length, and from No. 12 to No. 6, depending on the size of the line wire, is placed in and around the insulator groove, in such a manner that one end of the tie-wire shall pass *down over* the line wire, and the other end *up over* it, as indicated in Fig. 35. The fastening is then completed by wrapping the tie-wire continuously around the line-wire. Much practice is needed to make this tie in the neatest and securest manner, without injury to the surface of the hard-drawn copper of the line. The strength of the insulator tie is usually supposed to be, when well made, about one-fourth to one-third of the line-wire. When a wire terminates, it must be "dead-ended," in order to secure it from falling, and transfer the tension of the wire to the pole. This is accomplished as shown in Fig. 36. The line-wire is carried entirely around the groove of the insulator, and either wrapped about itself, or fastened with a McIntire joint. In order to give service at any point on a line, the necessary circuit must be carried into the premises of the customer. To effect this the line is usually dead-ended on the nearest pole, and a loop carried to the building to be served. For this purpose brackets are necessary; the best forms for the multitude of cases that may arise in practice being indicated in Fig. 37, while in Fig. 38 the bracket in place is illustrated. For a grounded line, a single pin-bracket is sufficient, for only one wire is carried off of the pole. For a metallic circuit, or for a loop in a series circuit, a double pin-bracket is required. A favorite form, with the method of application, is sufficiently clearly illustrated in Fig. 38 to need no additional explanation. Other forms of double brackets are seen in Fig. 37.

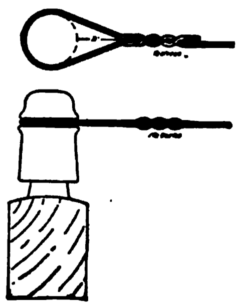


Fig. 36. "Dead-Ending."

50. Stringing Wires. — After the poles and insulators are set, the erection of the wire is to be undertaken. When there are a very few circuits, it is common to mount one or more reels containing the necessary wire upon a cart, and then to drive the cart slowly along, hoisting the wire up to its appropriate place as fast as the cart passes each pole. If a heavy line is in process of construction, the work can be greatly expedited by the use of what is termed a

"running-board." A number of reels of wire, usually ten or more, are mounted upon spindles, and a piece of wood, practically the same

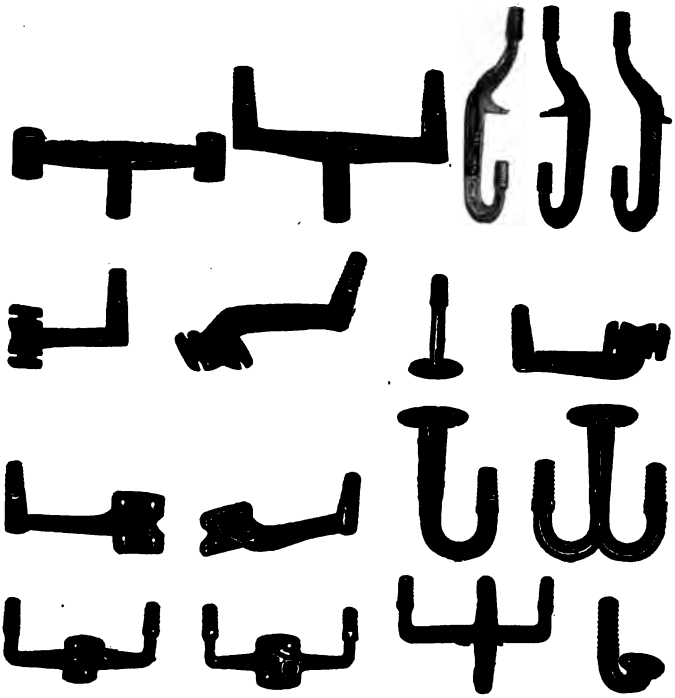


Fig. 37. Standard Brackets.

as a cross-arm, is arranged, to which ten or more wires are attached. Horses are then harnessed to the cross-piece, as the running-board

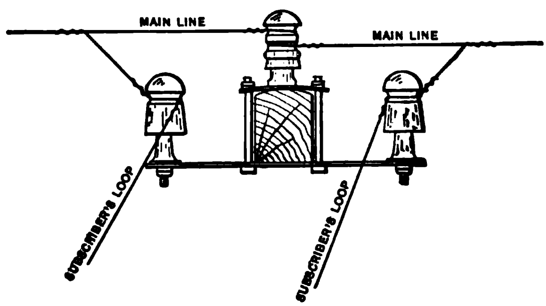
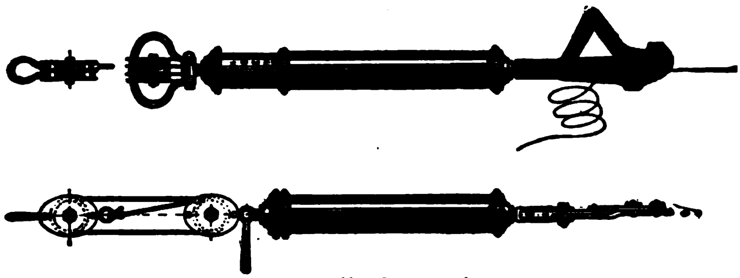


Fig. 38. A "Loop."

is termed, and as they "walk away," dragging the running-board after them, the wires are paid out from the reels, and, passing over

the appropriate cross-arm, may be immediately secured to the insulators by linemen stationed for the purpose. After the wire upon all the reels has been run out, each wire is pulled up to its



Line Dynamometer.

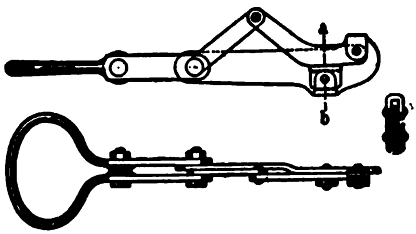


Fig. 39. A "Come-Along."

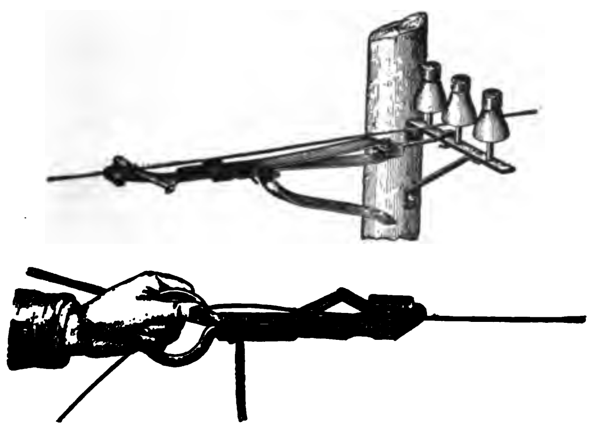


Fig. 40. The Come-Along in Service.

appropriate tension by means of a dynamometer, and a small portable vise, technically termed a "come-along," as illustrated in Figs. 39 and 40.

51. Wire Joints. — A legion of methods have been proposed for making splices in wire; but for all-round work, where slight inequalities in the line are not detrimental, the famous Western Union splice, illustrated in Fig. 41, has stood the test of many years' experience, and perhaps can hardly be excelled. For heavy circuits, such as electric railway feeds, the splices should be thoroughly soldered when made, and protected additionally by three layers of okonite tape thoroughly saturated with B. & P. paint. Line splices should not be made with soldering acid, but resin used as a flux, in order to guard against the possibility of future corrosion. In trolley wires, or other circuits in which the preservation of continuity is essential, without any enlargement of the wire, splicing is most successfully made by means of the tubular connector, into which the abutting ends of the successive coils may be slipped and brazed. This connector is indicated in Fig. 42. For telephone lines of hard-drawn copper, the McIntire splice, as illustrated in Fig. 43, is a favorite. This device forms a perfect connector; is as enduring as the wire itself; is made without the use of soldering, impervious to

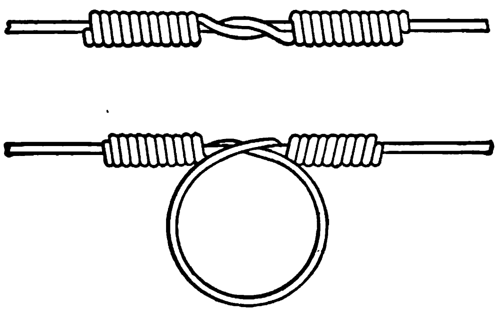


Fig. 41. Western Union Joint.

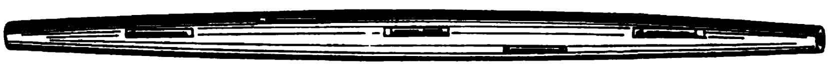


Fig. 42. Trolley Wire Splice.

moisture, and is equally strong as a hard-soldered joint. It moreover retains the inestimable advantage, especially in the use of hard-drawn metal, of retaining in the splice the full strength of the wire. As there is no soldering, the joint can be made with fewer tools, in less time, and does not anneal the wire.

52. The McIntire Splice consists of two tubes drawn side by side from one piece of copper, the interior diameter corresponding to the external diameter of the wires to be joined. The junction is effected by slipping the wires inside the two tubes and then twisting

the tubes on each other, thus by friction firmly binding the two wires together. In Fig. 43, various sizes, kinds, and applications of the McIntire joint are represented, with the special tools necessary to the completion of the joint. Nos. 1, 4, and 6 are completed joints. Nos. 2, 3, 5, 7, 10, 12, 13, 14, 15, 16, 19, 20, and 21 are various sized connectors fitting wire from No. 16 to No. 0. Nos. 17 and 18 are connectors used for joining two wires of different size. No. 8 indicates two wires thus united. No. 11 shows the McIntire joint used to take a branch from a circuit; 22, 23, and 24 are the styles of pliers employed to complete the splices.

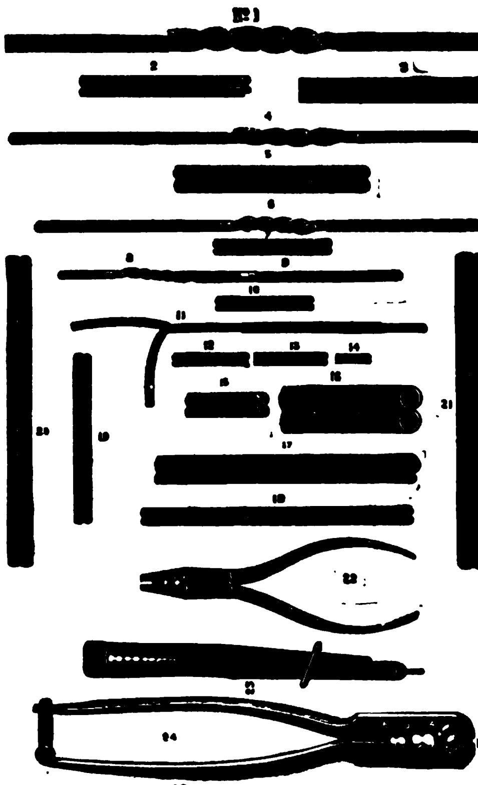


Fig. 43. McIntire Wire Joint.

53. Strength of Joints. — The strength of wire joints becomes an exceedingly important item in line construction, when it is considered that the weights of sleet and snow, with which aerial construction is frequently loaded in the winter time, introduces stresses that are dangerously near the elastic limit of the material. A series of experiments made by the Roeblings on copper wire having a strength of 520 lbs., indicate the following characteristics for the different forms of making wire splices: —

Western Union Joint, soldered, average of ten samples, 431 lbs. 83% of breaking strength of wire.

The McIntire Joint, average of nine samples, not soldered, 343 lbs. 66% of breaking strength of wire.

Western Union Joint, average of eleven samples, not soldered, 279 lbs. 53% of breaking strength of wire.

Western Union Joint, dipped and soldered with acid flux, 336 lbs. 68% of breaking strength of wire.

Western Union Joint, dipped and soldered with resin flux, average of ten samples, 339 lbs. 66% of breaking strength of wire.

Western Union Joint, soldered with iron and acid flux, average samples, 490 lbs. 94% of breaking strength of wire.

Western Union Joint, soldered with poured solder, resin flux, average of ten samples, 443 lbs. 85% of breaking strength of wire.

Western Union Joints, soldered with poured tallow flux, average of five samples, 477 lbs. 91% of breaking strength of wire.

Britannia Joint, two inches solder, average of ten samples, 488 lbs. 94% of breaking strength of wire.

54. The Suspension of Aerial Cables. — Very few of the cables that are used for aerial conductors have sufficient mechanical strength to be self-supporting over the ordinary spans adopted in pole-line construction, and it is necessary to arrange some means whereby the cable may be supported at frequent intervals, thus relieving it of any tension. To this end it is customary to run a suspending strand, usually composed of ½″ steel wire rope, between the poles, and hang thereto the cable. Methods for supporting the cables are indicated in Figs. 44 and 45, from which it will be seen that the cable is sustained by a clip, usually made of zinc, in order to obviate corrosion. This clip is passed around the cable, and sometimes secured to a hook, which is then attached to the strand of "messenger wire," as the supporting rope is technically called. In other cases, the clip itself forms a double hook, one part of which is devoted to supporting the cable, while the other is thrown over the messenger wire. The latter expedient is more simple, but not as satisfactory as the one first alluded to. It is usual to place the supporting-hooks on the cable at a distance of not less than 18″ to 24″ centers; as when longer spans are attempted, it is found that the lead sheath of the cable fails under

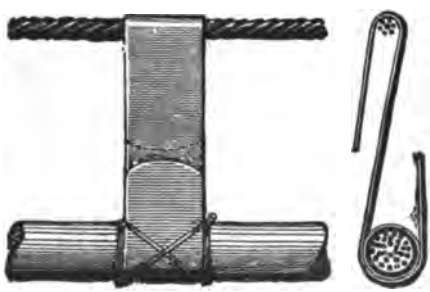


Fig. 44. Aerial Cable Suspension.

the tension and vibration to which the line is exposed, and, sooner or later, will admit moisture. Cable for messenger wire should be a good grade of stranded rope, which is as flexible as possible. The pole attachment is made by bolting to the pole a piece of angle iron, which forms the cable cross-arm. The messenger wire is attached to the cross-arm by means of a hook, or even more simply, passed directly through holes drilled in the cross-arm, and slotted out in such a manner as to prevent the messenger wire escaping.

Between successive poles the messenger wire should be drawn up tautly, in order that, when loaded with the cable, it may not present too great a deflection.

55. The Humming of Wires. — Considerable complaint has arisen from the loud humming sound that is occasionally produced by aerial lines, upon which the wind acts after the fashion of a gigantic Æolian harp. Difficulty from this source is more frequently experienced upon lines which are carried over housetops, for the roofs of buildings form a sounding-board that is capable of transmitting the sonorous vibrations throughout the entire structure. Much ingenuity has been expended in endeavoring to combat this difficulty with, it is to be regretted, rather poor success. The endeavors have been always in the direction of introducing something between the line and the insulator which would either absorb and annihilate the vibrations, or prevent them from being transmitted from the insulator and pole to the building. One device consists in terminating the line wire a short distance on either side of the insulator, and introducing between the insulator and each side of the line a spring having a sufficient stiffness to withstand the tension of the line, while, on the other hand, possessing sufficient elasticity to absorb and destroy the vibrations produced by the wind. This device is exceedingly expensive, not very successful, and introduces undesirable complications in the line.

Fig. 45. Cable-Hook.

Another attempt consists in lining the interior of the insulator with india-rubber, cork, or a similar substance, placing it between

the pin and the insulator. The elasticity of the india-rubber is supposed to be sufficient to take out the vibrations from the oscillating wire, and prevent them from being transmitted to the pole. Unfortunately, any substance of sufficient elasticity to act in this manner is hardly strong enough to withstand the severe stresses brought upon the insulator by the line; and, sooner or later, the insulator becomes loose.

Another method, indicated in Fig. 46, consists in enveloping the wire near the insulator with a piece of india-rubber tubing some eight inches in length, which is covered with a piece of sheet lead. The enveloped wire is then secured to the insulator, as indicated, by means of a second piece of wire, acting as a tie, which is similarly enveloped in india-rubber and lead. This device forms a cushion, which is fairly successful in absorbing vibrations, and allows the insulator and cross-arm to retain their original strength, and is, withal, exceedingly economical and speedy of application.

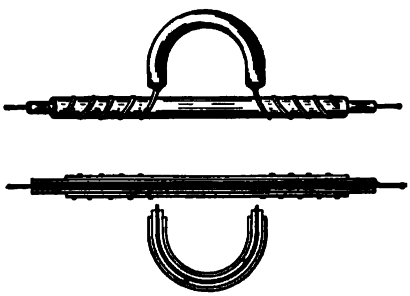


Fig. 46. Anti-Hummer.

56. The transposition of Telephone Lines.—Where aerial telephone lines are of considerable extent, and especially where upon the same pole-line other circuits are carried, it becomes essential to provide some means of eliminating the inductive disturbances that are initiated in the telephonic circuit. To accomplish this, the practice has been introduced of changing the position of each telephone circuit, with reference to all the other circuits, some five or six times in each mile. This is readily done by arranging the telephone circuits and the other circuits in such a manner that the circuits occupy respectively the four corners of a square. Supposing the corners of the square to be numbered from the upper left hand corner in a clock-wise direction, the telephone wires to occupy 2 and 4 and the other circuits 1 and 3, it is obvious that each half of each telephone circuit is contrarily affected by any induction from the corresponding halves of the other circuits. By frequently reversing the positions, so that in successive intervals the telephone circuits occupy the positions 1 and 3 and the other circuits 2 and 4,

while in succeeding intervals the telephone circuits occupy positions 2 and 4 and the other circuits 1 and 3, the inductive disturbances are annulled by reason of the transposition thus introduced The general arrangement of such a line is indicated in Fig. 47; the upper half of the illustration indicating the general appearance of the pole-line, while the lower half of the figure shows the placing of the circuits at the relative poles where the transpositions are effected. This method has been found to be an almost complete cure for inductive troubles, and is universally adopted upon all telephone lines of magnitude. Its use, however, renders the location of line troubles a little more difficult; but, after a short expe-

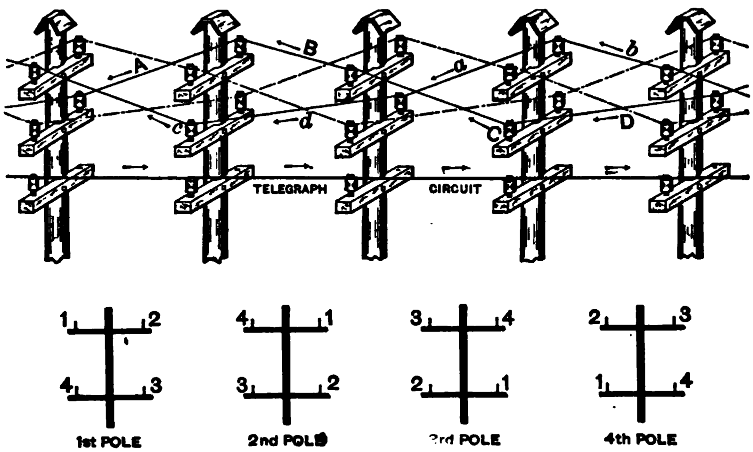


Fig. 47. Telephone Transposition.

rience, the linemen become so expert in the detection of trouble as to render this difficulty of little magnitude. In order, however, to locate a particular line, it becomes necessary to make the transposition according to some preconcerted system, which must be regularly carried out, or else to mark each wire at each successive pole. If transposition occurs at every fourth pole, there would be practically ten transpositions in a mile, and consequently, by numbering poles, it is easy to trace any particular line.

The transpositions are readily effected by dead-ending the wire at each insulator at which a change is to occur, with a McIntire joint, and then splicing across to the other side of the cross-arm.

Some common methods are shown in Fig. 48, the illustration being more lucid than any description.

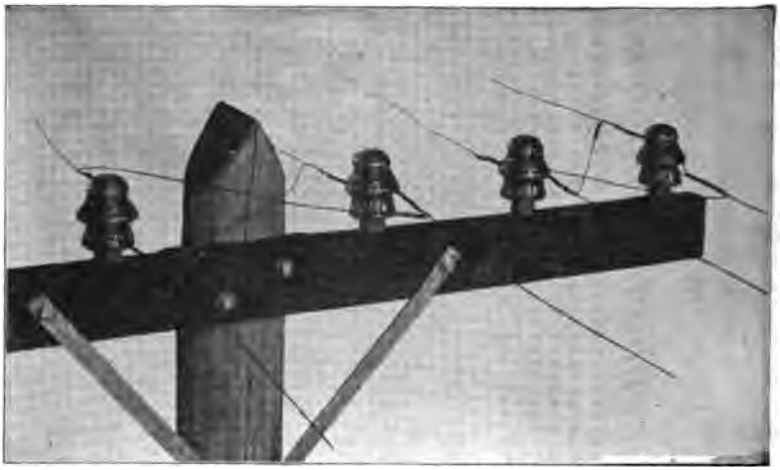


Fig. 48. Transposition Joints.

By the preceding plan each wire makes a complete twist around all the others, but when there are many cross-arms, each carrying many pins, this produces too much confusion of circuits, though it is by far the best preventive of inductive disturbance. Under ordinary circumstances quiet lines may be secured by transposing on each cross-arm the circuits in one plane. Fig. 49 shows the method of transposing when ten-pin cross-arms are used.

To locate transposition poles proceed as follows: Measure a distance of 1,300 feet from the first open-wire pole, and mark the pole nearest the point so located as "A." Measure a second distance of 1,300 feet from the first 1,300-ft. pole; mark the nearest pole "B." Proceed in the same manner to measure off poles at intervals of 1,300 feet for the entire length of line. Poles so marked are designated respectively "A," "B," "C," etc., in the diagrams, and are the poles upon which the wires shall be transposed, as shown by Figs. 49, 50, and 51. The diagram of Fig. 50 indicates the method of making transpositions on a twelve-wire line consisting of two six-pin cross-arms. Diagram in Fig. 51 indicates another method of arranging transposition on a four-arm forty-wire

line. These diagrams may be extended to any number of six-pin or ten-pin arm lines.

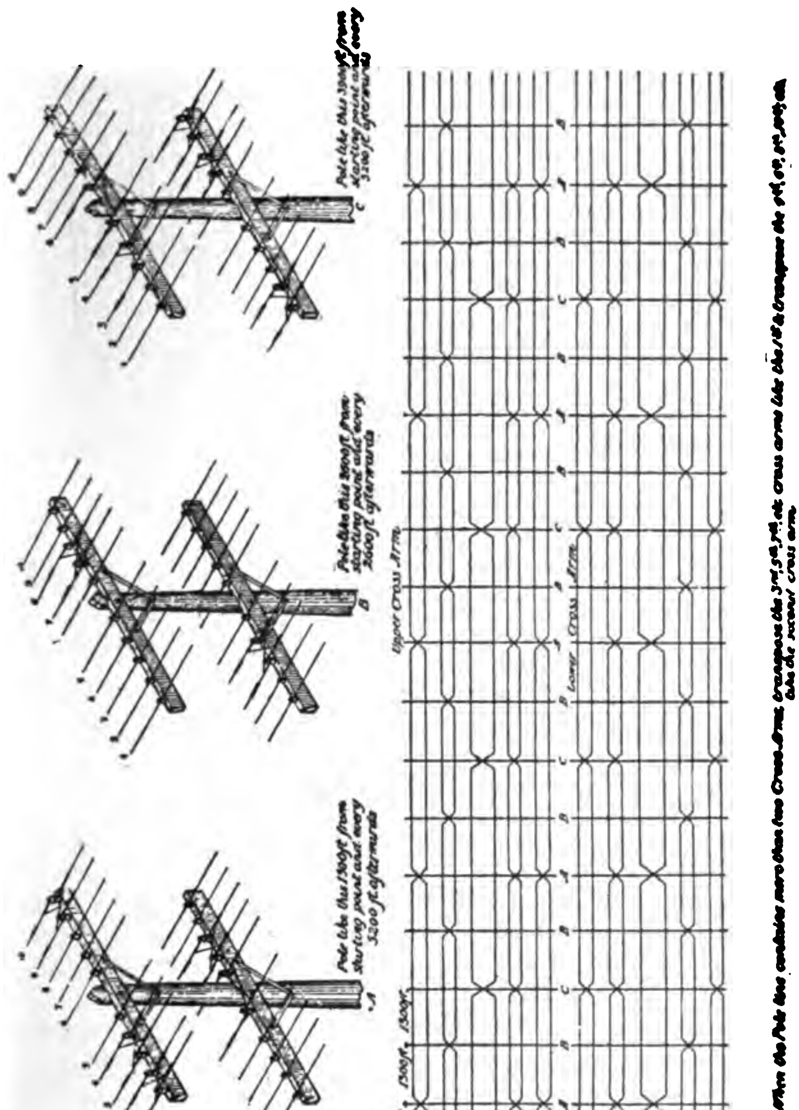


Fig. 49. Transpositions for 20-Wire Line.

The modern method of making transpositions is shown in Fig. 52. To make this transposition cut the wires "A" and "C" of such

a length as to enable them to be dead-ended on their insulators, slip over each wire a McIntire sleeve, and run it back about 2 feet; carry each wire to its insulator and dead-end it with a half-sleeve.

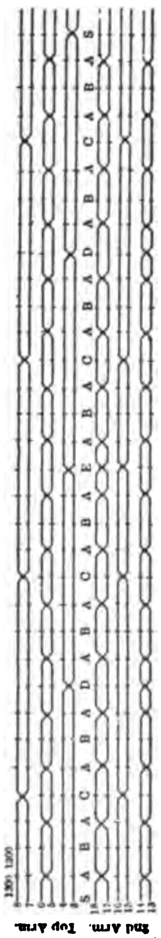


Fig. 50. Transpositions for 12-Wire Line.

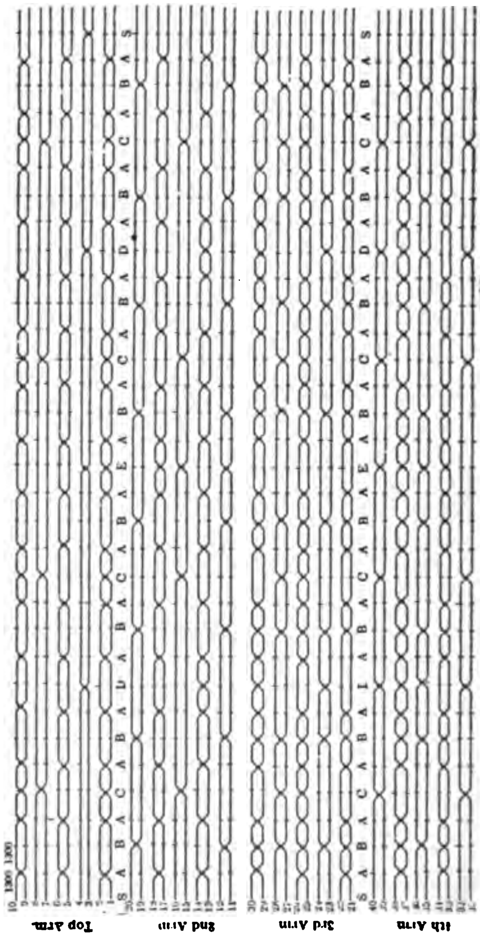


Fig. 51. Transpositions for 40-Wire Line.

Cut the wires "B" and "D" about 5 feet longer than is necessary to reach their insulators, carry each to its insulator, turn it around the insulator, and dead-end it with a half-McIntire; then carry the slack over to the wires "A" and "C" and fasten with the McIntire.

57. Pole-Line Specifications. — The best American practice is now constructing pole-lines under specifications, of which the following clauses are abstracts of the most important requirements.

58. Poles. — Standard poles should be of the best quality of live, green cedar, butt cuts, squared at both ends. They shall be reason-

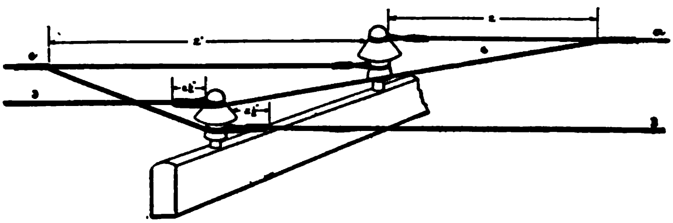


Fig. 52. Modern Method of Transposition.

ably straight, and well proportioned from top to butt, having the bark peeled, and the knots closely trimmed. The poles shall be of the following dimensions:—

TABLE No. 36.

Pole Dimensions.

MICHIGAN CEDAR.			CANADIAN CEDAR.		
Length in Feet.	Minimum Circumference at Top.	Minimum Circumference 6′ from Butt.	Length in Feet.	Minimum Circumference at Top.	Minimum Circumference 6′ from Butt.
20	18″	30″	20	18″	28″
25	18″	33″	25	18″	30″
30	20″	36″	30	20″	34″
35	23″	39″	35	21″	41″
40	23″	44″	40	21″	44″
45	23″	47″	45	21″	47″
50	23″	50″	50	21″	50″

A variation in the circumference of the butt of 1″ will be allowed, but the above circumference of top must be insisted upon. All poles shall be subjected to inspection by a representative of the purchasing company, at points of shipment. The tops of the poles shall be carefully roofed, by chamfering the top to equal angles of 45° on either side of the pole center. The roof shall be painted with three coats of best white lead. Each pole shall be gained with the appropriate number of gains to carry the required number of cross-arms. The center of the upper gain shall be 10″ from the apex of the pole roof. Each gain shall be cut square and true with the axis of the pole and with all other gains, and shall be cut to accurately fit

the cross-arms. All gains shall receive two coats of best white lead previously to the introduction of the arm.

59. Guy-Stubs and Anchor-Logs. — The quality of wood shall conform to pole requirements. Guy-stubs shall not be less than 24″ in circumference at the top. Anchor-logs shall be 10″ in diameter and 4 ft. to 8 ft. long.

60. Cross-Arms. — Cross-arms shall be of thoroughly sound, straight-grain timber, and made of Norway pine or Southern pine, as specified in particular instances. The arms shall be from 3 feet to 10 ft. long, 3¼″ thick, and 4¼″ deep. They shall be sawn true and square, fully up to the dimensions specified. The two 4¼″ sides shall be sawn parallel and at right angles to one of the 3¼′ sides. The other 3¼″ side shall be chamfered throughout the whole length of the arm with the exception of 10″ in the center, which shall be left square to fit into the gain upon the pole. This chamfering shall be done to the radius of a circle about 40″ in diameter. All cross-arms shall receive two good coats of mineral paint put on with a brush.

61. Iron Steel Fittings. — All iron steel fittings shall be of good quality of best refined wrought iron, that would be conformable to good bridge specifications, and shall be thoroughly galvanized.

62. Galvanizing. — All galvanizing may be tested by selecting samples, which shall be plunged in a saturated solution of sulphate of copper for seventy seconds, and then wiped clean. This process will be repeated four times. If, at the end of the fourth trial, the sample appears black, the galvanizing will be accepted; but if any deposit of copper is shown, giving an indication that the iron has been exposed, the sample will be rejected.

63. Cross-Arm Braces. — Each cross-arm shall be braced with two galvanized iron braces 1¼″ wide and ¼″ thick by 20″ to 30′ long. Each pair of braces shall be screwed to the pole by one galvanized iron carriage-bolt. All braces shall be attached to the cross-arm by means of ⅜″ galvanized iron carriage-bolts, of sufficient length to go through the braces and the arm. A galvanized iron washer shall be placed under the head and nut of each bolt.

64. Cross-Arm Bolts. — Each cross-arm shall be screwed to the pole by one ⅝″ galvanized iron bolt, extending entirely through the arm and pole. Under the head and nut of each bolt a galvanized iron washer, not less than 2½″ in diameter, shall be placed.

65. Pins. — All pins shall be of the best quality of sound, clear, split locust, free from knots and sapwood. The standard pin shall be 1¼″ in diameter for the shank in the cross-arm, and 4″ in length. The top of the pin shall be 1⅝″ in diameter, where it rests upon the cross-arm, and then shall be tapered and threaded to fit the insulator for which it is intended. The threading and tapering shall be neatly and accurately cut, showing the full thread, and shall accurately fit the insulator. Each pin shall be secured to the cross-arm by one six-penny galvanized iron wire nail driven straight through the shank of the pin.

66. Insulators. — Standard white glass insulators shall be used, which shall be sound and strong, free from fins and sharp edges, having threaded holes accurately molded and of uniform size.

67. Guy-Rods. — Anchor guys shall be attached to galvanized iron guy-rods. These rods shall be 6 ft. to 8 ft. long, ⅝″ in diameter, provided with a square galvanized iron washer, ⅜″ in thickness and 3″ square, with a ¾″ hole for the reception of the rod.

68. Wire-Rope Fittings. — All wire-rope fittings, such as thimbles, guy-clamps, rings, sockets, etc., shall be of first-class quality of wire-rope fittings, equivalent, in every respect, to those manufactured by the Roebling Company, or Washburn & Moen.

69. Lightning-Rods. — Every tenth pole shall be supplied with a lightning-rod, made of No. 6 galvanized iron wire, carried at least one foot above the top of the pole, and secured to the same with heavy galvanized steel wire staples, made of No. 4 B. & S. wire. These staples shall be 2¼″ in length. The wire shall be carried down the pole, and thoroughly buried in the ground at the base of the pole in moist earth.

70. Guy-Rope. — Guy-rope shall be of a good flexible quality of steel rope, preferably of seven-strand. Each strand shall be Siemens-Martin steel No. 10 B. & S. wire. The wire shall be cylindrical, free from scales, inequalities, and other imperfections. The wire shall be capable of elongating 4 per cent in 1 ft. lengths, and shall stand at least 15 twists in a length of 6″ without breaking. The tensile strength of the wire must be at least 4.8 times its weight in pounds per mile. The seven strands shall be laid up with a right-hand lay, not exceeding 3½″ in length. The galvanizing of the strands must be subjected to the same test as previously specified. Strand-rope

shall be furnished in coils of such length as to weigh between 150 and 200 lbs.

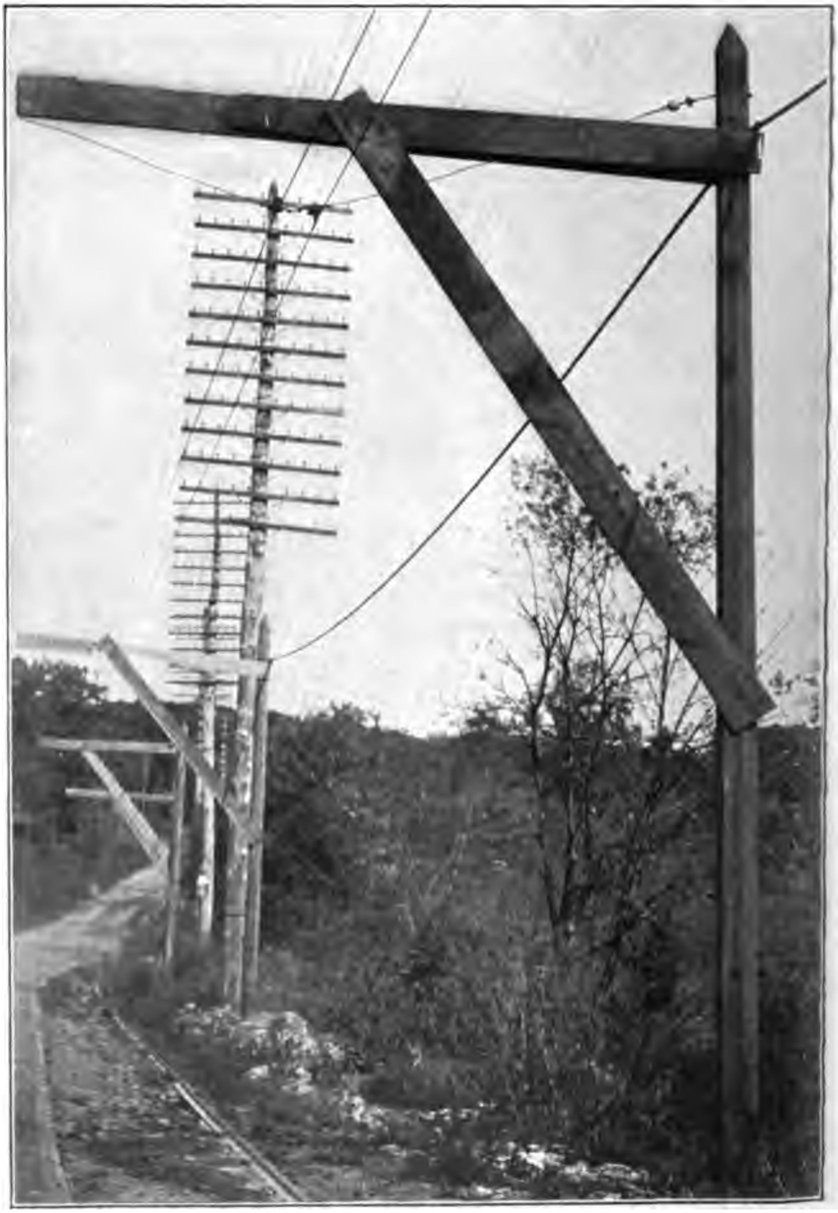
71. Construotion Details. — The line shall be located by measuring off, and placing stakes for pole location at distances of one 130 ft. average. In case of obstacles, the poles should be located as near the stakes as possible. In the distribution of the poles, the strongest and heaviest poles shall be placed on line corners, while the best looking shall be distributed throughout towns and cities, or in front of residences. The length of the pole shall be proportioned to the contour of the country, so that the line wire may be strung without abrupt changes in level. On straight lines, all poles shall be set in the ground to a depth of 6 ft., unless otherwise particularly specified. All poles shall be set perpendicularly on straight line work. On curves poles should be set with an outward rake. The holes shall be dug sufficiently large to admit the butt of the pole without hewing; and after the pole is set, the earth shall be returned and thoroughly tamped around the base of the pole. Tamping shall be done in the proportion of three tampers to one shoveler. Upon curves, the poles shall be set to a depth of at least 6½ feet. Where the soil is particularly soft, artificial pole foundations of concrete or timber shall be used.

72. Placing of Cross-Arms. — On straight line work, the cross-arms shall be placed on alternate sides of succeeding poles. On long spans, the cross-arms of terminal poles shall be placed opposite the long section. At the end of lines, the arms of at least the last two poles shall be placed on the side facing the terminal of the line. On curves, the cross-arms shall face toward the middle of the curve. Long spans of 200 feet shall be head-guyed, and, if possible, side-guyed in both directions.

73. Tying of Wires. — Line-wires shall be tied in the manner shown in Fig. 35. On curves, all wires shall be located upon the side of the insulator away from the center of the curve. On straight lines, all wires to be located on the side of the insulator next the pole, excepting the two wires nearest the pole, which are to be on the outside of the insulator.

74. Joints. — The joints shall be made with McIntire sleeves, each having three complete twists.

PLATE II.



How not to Build a Railway Line.

To face page 99.

CHAPTER IV.

CONSTRUCTION OF AERIAL CIRCUITS.

ELECTRIC RAILWAY CIRCUITS.

75. Electric Railway Circuits. — The marvelous extension of the electric railway systems, leading, during the past few years, to an investment in this country of nearly two and a half billions, has caused the development of a special branch of engineering, presenting problems in line construction which are unique to this particular department of the art. At present, with but very few exceptions, the electric railway circuit is an aerial line; yet it must be able to carry very large quantities of electrical energy, at sufficiently high potentials to become a source of danger, provided the very best workmanship and materials are not used. Usually the railway circuit consists of a series of conducting wires, called feeds, extending from the power station over the route of the railway, from which, at various points along the line, energy is supplied to the trolley wire placed over the center of the track. Two forms of railway lines are in use, respectively designated as "*center*, or *side pole*," and "*span wire*" construction, depending upon whether the poles for supporting the trolley and feed wires are extended along the street, between, or just at one side of the tracks, supporting the trolley wire on brackets, or whether they are placed in a double row along the curbs of the street, the trolley wire being carried upon span wires extended across the street from the tops of opposite poles, while the feeds are carried directly on the poles. These methods of construction are indicated in Figs. 53, 54, 55, 56, and 57.

76. The Railway Return Circuit. — With the exception of a few of the early double trolley roads and some of the modern conduit lines, the electric railway line has always been a grounded return, the current passing from the station through the feed wire to the trolley wire, thence through the car motor into the rails, and back to the station through the ground. So long as railway systems were small, this practice answered well

enough; but with increasing magnitude, the amounts of energy thus discharged into the ground have given rise to very serious and perplexing problems. The first noticeable effect was the production of

Fig. 53. Center Pole Construction.

earth currents of such importance as to seriously interfere with telephonic and telegraphic service; then a wide-spread electrolytic action made its appearance, affecting in a most serious manner all metallic underground structure, such as gas and water pipes, and the lead

Center Pole Line, Boston & Worcester Ry.

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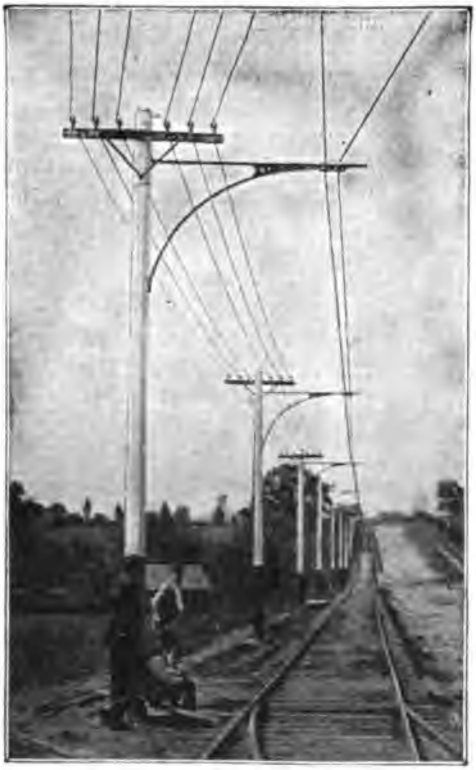


Fig. 54. Side Pole Construction.

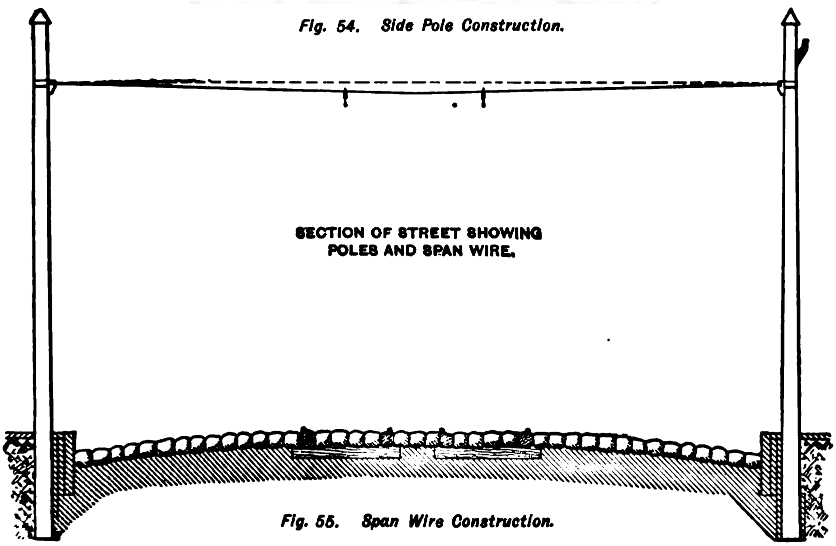


Fig. 55. Span Wire Construction.

sheathes of underground cables; and lastly, the poor quality of the earth as a conductor makes itself manifest, necessitating a very considerable fall of potential, and consequent wasteful expenditure of energy in this part of the circuit. On account of these difficulties, the larger roads are now aiding the ground return by re-enforcing it with copper wire "return feeds," looking in the near future to a more or less complete metallic circuit for the railway system.

77. Experience has shown that it is entirely fallacious to place any reliance upon the conducting power of earth to form a return for trolley current, but that in all cases it is necessary to provide a sufficient metallic path over which electricity may return to the power station. For this purpose the rails may be utilized and in the majority of instances will be found sufficient. A railway track consists of suc-

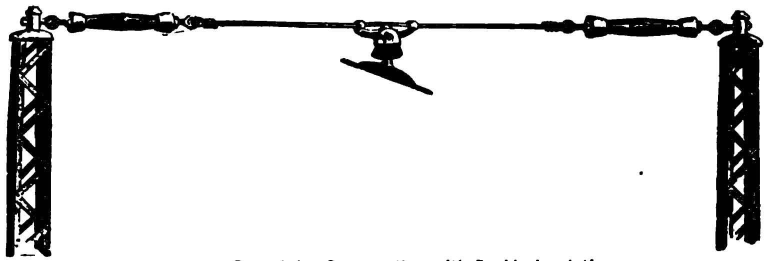


Fig. 56. Span Wire Construction with Double Insulation.

cessive rails that are electrically discontinuous, as the joints are formed by splice-plates bolted to each rail. While this fastening has been from a mechanical standpoint more or less satisfactory, it is utterly inadequate as an electrical conductor. The most widely adopted plan of providing electrical continuity is to bond the track. This operation consists in spanning each joint by pieces of copper wire of proper conductivity, termed bonds. One end of each bond is metallicly connected to the end of one rail, while the other is fastened to the succeeding one. Provided bonds are given sufficient section and properly secured, a metallic conducting system is obtained which is usually sufficient to return the current from even large and extensive systems, without excessive fall of potential. Experience has shown, however, that it is exceedingly difficult to secure adequate bonding. From motives of economy, there is a constant tendency to make bonds too small. This has been carried to such an extent that rail bonds have been known to run so hot as to burn ties in two, producing derail-

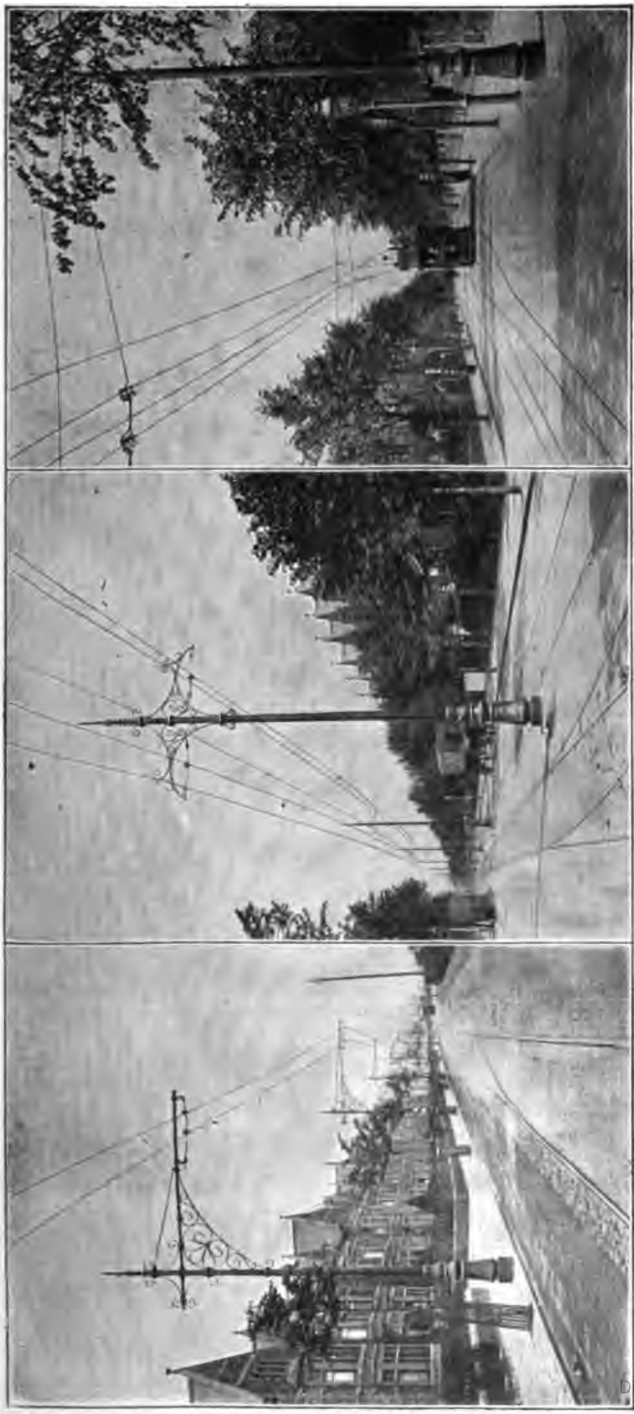


Fig. 67. Urban Electric Railway Construction.

ment by the spreading of the rails. It is necessary to attach the ends of the bonds to the flange or web of the rail. Here the iron section is thin, and unless great pains are taken a considerable resistance is introduced at the junction. In moist situations copper and iron form a weak galvanic couple, and unless the joint between bond and rail is hermetically sealed electrolytic action sooner or later takes place, corroding and destroying the electrical continuity.

The cross-section in square inches of any rail is approximately equal to its weight per yard in pounds divided by 10. Thus a rail weighing 50 lbs. per yd. would have a cross-section of 5 sq. in.; so if W be the weight per yard of rail, $\frac{W}{10}$ is the sectional area of a single rail, and $\frac{W}{5}$ the total area of a single track in square inches. It is easy to express the copper equivalent of the track by multiplying the preceding expression by the conductivity ratio of copper to iron. For pure metals this is about 1 to 6, but steel rails contain notable percentages of carbon, silicon, manganese, phosphorus, and sulphur, all of which tend to exalt the conductivity ratio. For, as is shown in Art. 13, very small quantities of extraneous elements markedly affect electrical conductivity. Mr. F. H. Parshall* has made one of the most exhaustive investigations upon return circuits, from which the data regarding rail conductivity given in TABLE No. 37, Sec. I, are abstracted.

TABLE No. 37, Sec. I.

Relation between Chemical Composition and Electrical Resistance of Steel Rails

Carbon, Per cent.	Manganese, Per cent.	Silicon, Per cent.	Phosphorus, Per cent.	Sulphur, Per cent.	Resistance compared with Copper at 20° F.	Resistance of 1 Mi. 1 Sq. In. Sectional Area at 20° F., Ohms.
0.378	0.550	0.181	0.040	0.041	10.8	0.468
0.446	0.568	0.188	0.046	0.044	11.1	0.482
0.536	0.592	0.201	0.051	0.059	11.3	0.490
0.568	0.608	0.204	0.053	0.061	11.4	0.495
0.588	0.632	0.214	0.056	0.065	11.5	0.499
0.610	0.650	0.220	0.062	0.071	12.9	0.560

* Earth-returns for Electrical Railways. Journal of Institute of Electrical Engineers, Vol. 27, No. 135.

TABLE No. 37.

Chemical Composition and Conductivity of Iron Alloys.

SECTION II.

GROUP.	MAKER'S MARK.	PERCENTAGE COMPOSITION.				ELECTRIC CONDUCTIVITY.		SPECIFIC RESISTANCE.
		Fe	C	Mn	Si	Iron = 100	Copper = 100	
Carbon Series "A"	S. C. I.	99.89	0.028	....	0.07	100.0	16.8	10.2
	B.	99.71	.03	0.036	.14	93.4	15.7	10.9
	L. S. S.	99.72	.05	.18	.02	90.4	15.2	11.3
	1166	99.50	.14	.20	.08	80.4	13.5	12.7
	1392 H	99.02	.78	.10	.10	70.3	11.8	14.6
	" I	98.86	.83	.25	.06	67.3	11.3	15.2
	" B	98.78	.84	.18	.20	63.7	10.7	16.1
	" A	98.66	.85	.32	.17	62.5	10.5	16.4
	" L	98.42	1.09	.32	.17	58.9	9.9	17.4
	" G	98.51	1.23	.14	.12	58.3	9.8	17.6
"B"	611	98.335	.58	.58	.49	49.4	8.3	20.7
	613	97.93	1.00	.58	.49	45.8	7.7	22.3
	614	97.67	1.25	.62	.46	43.5	7.3	23.6
Manganese Series "A"	48	99.30	.20	.50	....	70.8	11.9	14.5
	4147	98.76	.24	1.00	....	43.5	7.3	23.6
	53	97.34	.41	2.25	....	35.1	5.9	29.2
	1379 B	96.29	.08	3.50	.13	34.5	5.8	29.7
	39	95.64	.26	4.00	....	35.7	6.0	28.7
	34	94.89	.33	4.75	....	31.9	5.86	29.4
	32	94.53	.32	5.15	....	27.4	4.6	37.4
	1323 C	94.46	.15	5.40	....	30.4	5.1	33.7
	1338 B/2	86.74	.26	13.00	....	16.7	2.8	61.8
	1379 D/2	84.64	.15	15.20	....	15.8	2.65	64.9
"B"	1381	95.41	.78	3.81	....	23.2	3.9	44.1
	945 A	91.80	1.20	7.00	....	18.5	3.1	58.7
	1379 D	89.11	.16	10.10	.63	16.1	2.7	63.7
	1310 B	86.84	1.66	11.50	....	16.7	2.8	61.5
	1010	85.77	1.23	13.00	....	16.1	2.7	63.7
	30	83.25	1.50	15.25	....	15.5	2.6	66.2
	598	80.96	1.54	18.50	....	14.9	2.5	69.0

As germane to this subject, TABLE No. 37, Secs. II and III, are compiled, Sec. II gathered from the researches of Barrett, Brown, and Hatfield.* Sec. III gives extended data, from tests made by the General Electric Co.,† as to the relation between electrical conductivity and chemical composition of a large variety of irons. From these data it appears that the conductivity ratio of irons likely to be used in the manufacure of rails will vary from 8 to 15, with a tendency towards the lower ratio.

* Researches of Electric Conductivity and Metallic Properties of 100 different Alloys of Iron. Journal of Institute of Electrical Engineers, Vol. 31, No. 156.

† Trans. Am. Inst. M. E.: Tests on Conductor Rails, J. A. Clapp, Oct. 1903.

Table No. 37.

Section III.

Serial Number.	Specific Resistance. Microhms per Cm Cm²	Specific Resistance. Temp., Deg. C.	Conductivity. Mat-thiessen Standard	Resistance. Cu = 1.	Percentage Composition. C.	Mn.	P.	S.	Si.	Total. Not Fe.	P + S + Si.
1	22.72	19°	7.58	13.20	0.33	1.27	0.09	0.05	0.05	1.79	0.19
2	20.90	20°	8.27	12.12	0.17	1.09	0.09	0.05	0.004	1.404	0.144
3	21.29	25°	8.27	12.09	1.40	0.222	0.01	0.020	0.082	1.734	0.112
4	19.87	19°	8.65	11.55	0.20	0.95	0.10	0.08	0.05	1.38	0.23
5	19.80	19°	8.68	11.51	0.43	0.77	0.10	0.04	0.066	1.406	0.206
6	19.80	19°	8.69	11.51	0.36	0.80	0.10	0.04	0.047	1.347	0.187
7	19.81	20°	8.69	11.51	0.22	1.08	0.10	0.05	0.06	1.510	0.210
8	19.69	19°	8.73	11.44	0.74	0.58	0.043	0.036	0.20	1.599	0.279
9	18.95	25°	9.29	10.76	1.61	0.147	0.015	0.018	0.092	1.882	0.125
10	18.17	19°	9.46	10.56	0.41	0.72	0.039	0.041	0.11	1.32	0.190
11	17.27	19°	9.96	10.04	0.36	0.87	0.08	0.09	0.04	1.44	0.21
12	17.10	19°	10.06	9.94	0.37	0.73	0.09	0.04	0.06	1.29	0.19
13	17.10	19.5°	10.06	9.94	0.23	0.80	0.016	0.033	0.016	1.095	0.065
14	16.96	19°	10.14	9.86	0.30	0.95	0.063	0.01	0.01	1.333	0.083
15	16.95	19.5°	10.14	9.86	0.29	0.99	0.084	0.01	0.01	1.384	0.104
16	16.32	19°	10.55	9.48	0.23	0.89	0.058	0.01	0.005	1.193	0.073
17	16.25	19.5°	10.59	9.44	0.26	0.83	0.053	0.01	0.004	1.157	0.067
18	16.21	20°	10.62	9.42	0.28	0.65	0.083	0.06	0.05	1.123	0.193
19	16.09	19°	10.69	9.36	0.22	0.68	0.077	0.07	0.05	1.097	0.197
20	16.09	19°	10.69	9.36	0.16	0.66	0.074	0.030	0.014	0.938	0.118
21	15.32	19°	11.24	8.90	0.33	0.49	0.068	0.05	0.02	0.958	0.138
22	14.57	19.5°	11.82	8.46	0.31	0.45	0.10	0.04	0.026	0.926	0.166
23	14.49	20°	11.88	8.42	0.25	0.41	0.10	0.04	0.03	0.83	0.17
24	14.73	23.5°	11.88	8.42	0.144	0.46	0.09	0.08	tr.	0.774	0.17
25	14.62	23.5°	11.96	8.36	0.188	0.48	0.09	0.08	tr.	0 83	0.17
26	14.15	19°	12.17	8.22	0.22	0.56	0.024	0.34	tr.	0.838	0.058
27	14.03	19°	12.26	8.16	0.192	0.57	0.024	0.34	tr.	0.82	0.058
28	13.86	19°	12.41	8.06	0.16	0.48	0.091	0.04	0.01	0.781	0.144
29	13.83	19.5°	12.44	8.04	0.10	0.55	0.08	0.05	0.024	0.804	0.154
30	13.80	19°	12.57	8.02	0.14	0.41	0.11	0.05	0.009	0.719	0.169
31	13.67	19°	12.58	7.95	0.23	0.48	0.024	0.01	0.023	0.767	0.057
32	13.64	19°	12.61	7.93	0.24	0.57	0.029	0.01	0.003	0.850	0.042
33	13.90	24°	12.63	7.92	0.10	0.25	0.04	0.02	0.05	0.46	0.11
34	13.31	19°	12.92	7.74	0.25	0.37	0.04	0.03	0.018	0.708	0.088
35	13.30	19.5°	12.94	7.73	0.23	0.49	0.024	tr.	0.004	0.748	0.028
36	13.27	19°	12.97	7.71	0.19	0.37	0.09	0.05	0.01	0.71	0.15
37	13.25	19°	12.99	7.70	0.27	0.41	0.024	0.01	0.001	0.715	0.035
38	13.18	19°	13.05	7.66	0.28	0.28	0.027	0.034	0.04	0.661	0.111
39	13.18	19°	13.05	7.66	0.07	0.40	0.08	0.07	0.013	0.633	0.163
40	13.07	19°	13.16	7.60	0.28	0.42	0.022	0.04	0.008	0.770	0.070
41	12.87	20°	13.27	7.48	0.16	0.38	0.08	0.04	0.009	0.669	0.129
42	12.73	20°	13.52	7.40	0.15	0.45	0.011	0.033	tr.	0.644	0.044
43	12.69	19°	13.55	7.38	0.19	0.21	0.025	0.04	0.034	0.499	0.099
44	12.53	19°	13.74	7.28	0.215	0.22	0.051	0.113	......	0.599	0.164
45	11.01	19°	15.63	6.40	0.05	0.19	0.054	0.059	0.03	0.383	0.143

Using the ratio 10, the copper area equivalent to any rail is equal to the weight in pounds per yard divided by 100, while the copper area equivalent to single track is equal to the weight per yard of *rail* divided by 50 for single track or divided by 25 for double track. For example: 80-lb. girder rails are common in paved streets; if adequately bonded, the equivalent copper area of a *rail* would be 0.8 square inches or

about 1 025,000 cm., that for one *track* would be 1.6 sq. in. or 2,050,000 cm., and that for *double* track 3.2 sq. in. or 4,100,000 cm. An engineer would display considerable hardihood to advise the installa ion of 4,000,000 cm. of copper in the return system of an electric railway; so this simple example indicates the conducting power of the track and emphasizes the necessity of utilizing it to its full extent, which can only be done by electrically eliminating the joints. This is the object of bonding.

78. The successful bond must not introduce objectionable resistance, due to imperfect contact between metal of bond and that of rail, and must provide a joint that will not deteriorate. Three methods are now in u e: The most common consists in providing a copper conductor of sufficient length to span the rail joint, the ends of which are supplied with large copper studs or rivets. Holes are drilled in either the flange or web of the rail. The copper rivets are driven into the holes and expanded to make a tight fit. Rail joints are always designed to provide for the expansion and contraction due to annual changes of temperature. Obviously the bond must be flexible, or the motion of the rail will rupture the bond or tear out the rivet. To secure elasticity bonds are often built of a number of small copper wires as shown in Fig. 58. To attach the bond, holes are drilled in the rail by a portable drilling-machine shown in Fig. 59. It is customary to place the bonds under the fish-plates as shown in Fig. 60 to prevent thieving, for experience has shown that exposed bonds are liable to be stolen and sold for old copper. Bonds thus located are usually termed "*protected bonds.*" Another plan applicable to "T" rails is to place the bonds in web and underneath rail, as in Fig. 61. While this locality is more exposed than that beneath fish-plates, it is usually covered by ballast to be reasonably secure. To obtain sufficient copper section and adequate area between the bond terminals and the rail it is easy to multiply the number of bonds as in Fig. 61, where two are shown. The same end is attained by bonds designed as in Fig. 62, in which each end is provided with two terminals. The unprotected bond with method of application is indi-

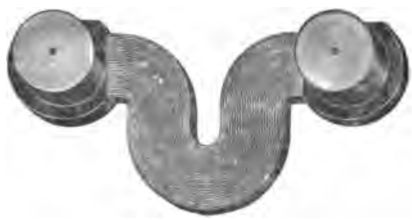


Fig. 58. Horse Shoe Bond.

Fig. 59. Portable Rail Drill.

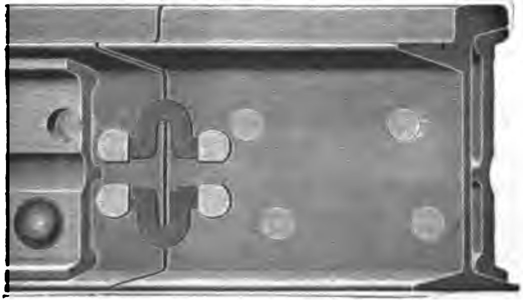


Fig. 60. Protected Bonds.

cated in Fig. 63. This merely consists of a piece of copper wire provided with terminals riveted (Fig. 64) to the rails surrounding the fish-plate.

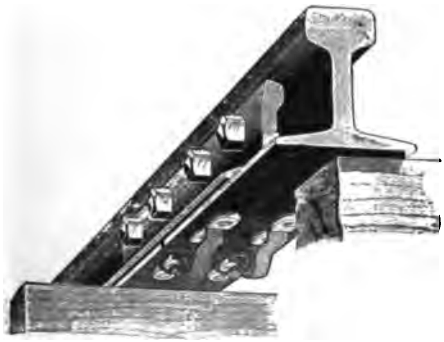


Fig. 61. Bonds in Rail Flange.

Fig. 62. Multiple Terminal Bond.

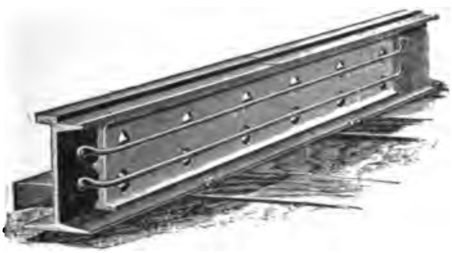


Fig. 63. Unprotected Bonds.

79. If it were possible to secure electrical contact between the fish-plates and rails, sufficient conductivity could be obtained and the expense of copper saved. The "*Plastic Rail Bond*" is an attempt to solve this problem and one from which favorable experience has been secured. This device is illustrated in Fig. 66. It consists of a plastic alloy largely composed of mercury, forming a putty-like compound, whereby metallic contact between rail and fish-plate is secured, and an elastic cork case to hold the alloy in its proper place between rail

and fish-plate. This case is made about twice as thick as the distance between web of rail and plate, so it is compressed about one-half when the fish-plate is bolted in its normal position. To install the plastic bond a spot upon the end of each rail two or three inches in diameter is cleaned from rust by means of a portable emery wheel shown in Fig. 65, and similar spots cleaned upon the fish-plate. When the iron is bright it is rubbed with the so-called "*Edison Alloy,*" a compound of mercury somewhat the same as plastic alloy, and by this means the iron is thoroughly silvered. The cork case containing plastic alloy is then placed against the silvered spot on the rail

Fig. 64. A 5/0 Solid Copper Bond.

and wired in place by means of a bit of No. 22 wire as in Fig. 66. The fish-plate is then bolted into position. Care must be taken to see that the amalgamated spots on rail and fish-plate are matched when assembled. Some tests made upon plastic rail bonds by the Boston Elevated are shown in Table 38.

An ingenious modification of the plastic bond is known under the name of Plastic Plug Bond, illustrated in Figs. 67 and 68. One or more holes are drilled through each end of the fish-plate into the flange of rail. These holes are filled with plastic alloy and then a copper plug is driven or threaded into the holes in such a manner as to dip into the plastic alloy.

80. The third method of securing conductivity between bond

and rail consists in soldering the bond to the rail, as is illustrated in Fig. 69. For this purpose a very short "U"-shaped bond is used and soldered to either the web or flange of the rail. The wire of the abutting rails is cleaned with the emery wheel, and the tinned

Fig. 65. Portable Emery Wheel.

ends of the bond are sweated to the rail by means of a blow-lamp.

81. To avoid the defects inherent in bonding, to attain greater mechanical strength, and secure more perfect rail alignment two plans have been suggested which endeavor to provide track which is con-

Fig. 66. Plastic Rail Bond Ready for Application.

Fig 67.

Table No. 38.

Tests of Rail Bonds, made by Roger W. Conant.

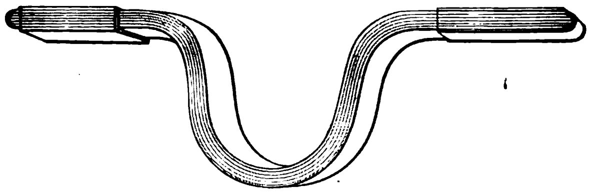
Ninety-pound Rail Joint with Plastic Rail Bonds under both Angle Plates. 12 inches between centers		Ninety-pound Rail Joint with 2 West End No. 0000 Copper Bonds. Rail and plates new and rail ends touching. 12 inches between centers.	
Amperes.	Volts Drop.	Volts Drop.	Volts Drop with angle plates removed and rails separated.
500	.....	.....	.0250
600	.....	.....	.0300
650	.....	.....	.0330
750	.....	.0138	
800	.....	.0138	
850	.0058	.....	.0450
900	.0063	.0160	.0495
1000	.....	.0166	.0530
1100	.....	.....	.0640
1200	.....	.....	.0700
1400	.....	.....	.0810
1500	.0118	.0284	.0860
1600	.0128	.0300	.0950
1700	.....	.0325	Bonds too hot to permit further tests
2200	.0175	.0430	
2300	.0185	.0450	
2400	.0190	.0470	

tinuous from end to end. One plan is to electrically weld the adjacent ends of rails and thus construct track which is essentially in one

Fig. 68. Plastic Plug Bond.

piece. Such a radical proposition seemed almost ridiculous to previously conceived notions of track construction, which had assumed

that a multiplicity of joints was necessary to provide room for expansion due to annual temperature changes. But many experiments proved this necessity to be overestimated and that at least girder rails, enclosed and supported by street pavement, can thus be safely joined; so the projectors of the weld process claim that it provides a track mechanically and electrically perfect. Some experience has shown that the weld is by no means objectionless. The process is expensive, requires elaborate machinery, can only be performed when large quan-



Soldered Flexible Rail Bond.

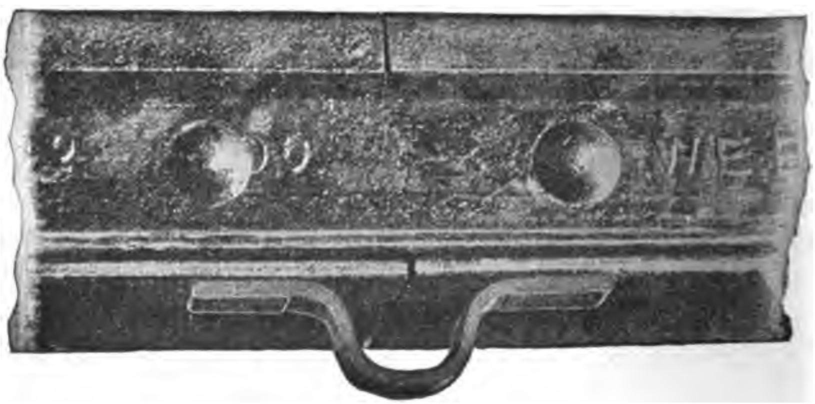


Fig. 69. Soldered Flexible Rail Bond Applied to Rail.

tities of electrical energy are available, anneals the ends of rails, makes joints soft in spots, invites wear, alters the chemical composition of the track so that the weld presents greater resistance than other portions of rail, and causes it to become brittle, so that track frequently ruptures where welds have been made. For this reason the welding process did not attain at first the currency desired by its inventors.

More extended experience has indicated that by placing a splice-plate on each side of the rail, as shown in Fig. 70, and by arranging a powerful press to squeeze the abutting ends of the rail-joints forcibly

together and to retain them under pressure while the joint is cooling, largely reduces the number of broken joints, and tends to avoid the deterioration which the steel undergoes when heated without subsequent working. When the welding process was first introduced statistics show that upwards of 1 per cent of the joints failed by breaking. In 1903 there were something like 300 miles of electric-railway

Fig. 70. Splice-bar Electrically Welded Joint.

track with welded joints, and of this number a little less than one-tenth of 1 per cent of the joints have proved troublesome.

The apparatus for electrically welding rails is cumbersome and expensive, and is shown in Fig. 71. There are four essential features: 1st. A sand-blast outfit with which the rails and splice-bars can be cleaned. 2d. A rotary transformer to change the trolley current into an alternating one. 3d. The welder and clamp. 4th. An emery grinder to smooth the completed joint.

The cost of introducing welded joints upon existing track will depend very largely upon the nature of the surrounding paving and

the age of the rails. When applied to new track, as it is originally laid the cost may be kept within $1.50 to $2.00 per joint, while with old track the expense rises to from $3.00 to $6.00.

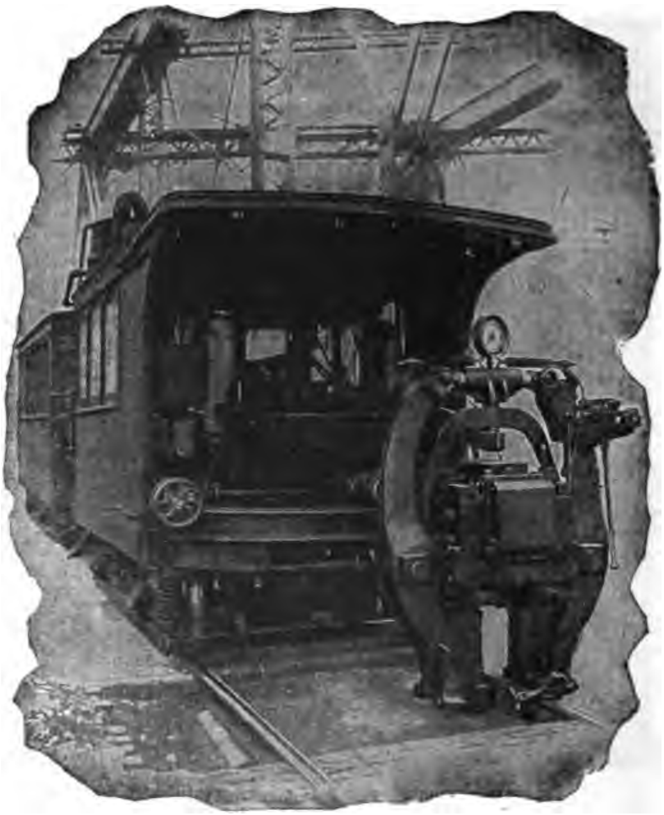


Fig. 71. Electric Welder.

82. The other plan proposes to envelop the ends of the rails in a mold which shall be filled with molten iron or other metal, thus inclosing the rail ends in a solid casting. It has been assumed that if the rail ends were properly cleaned prior to casting, adequately heated, and the metal poured at a sufficiently high temperature, the melted bath would fuse itself to the rails and form complete electrical and mechanical contact. Experience shows that this expectation is rarely if ever realized, for while cast joints undoubtedly closely hug rail ends, there is little question but that rails are often never fused to the

casting, but are separated therefrom by a thin film of oxide or other impurity. While this separation destroys the electrical perfection of the joints, it undoubtedly has a compensating value in affording a possibility for expansion. Cast weld joints, therefore, must be bonded in exactly the same manner as that adopted for those made with fish-

Fig. 72. Cast Weld Joint Girder Rail.

Fig. 73. Cast Weld Joint Tee Rail.

plates. Figs. 72 and 73 are illustrations of two forms of cast weld joints, while Fig. 74 is a section made by cutting through the center of the casting.

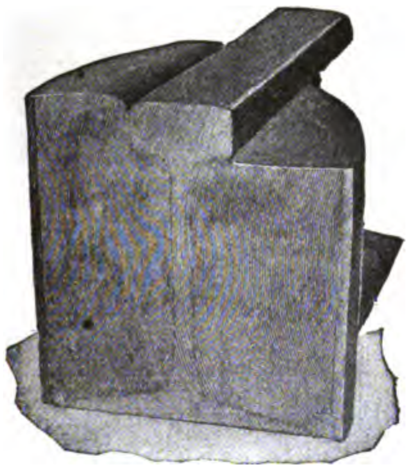


Fig. 74. Section of Cast Weld Joint.

Cast iron is usually employed as the metal for forming cast weld joints. A portable cupola is arranged either upon a car or cart so that it may be transported along the track, the furnace being fired by oil. During daytime rail-joints are exposed and molds set in place, and then at night, in order to avoid interference with street traffic, the joints are poured.

The Philadelphia Traction Co. has adopted a form of cast weld

joint in which zinc is employed as metal for forming weld. A pair of angle-plates are used which do not come in immediate contact with rails, but are riveted thereto with spacing-washers. The openings at ends of angle-plates are corked with asbestos, and then molten zinc is poured between the rail and plates.

A method recently devised in Germany, known as the ***thirmite*** process, is advocated as requiring little or no expensive paraphernalia. This method depends upon the fact that powdered aluminum when mixed with certain other metallic oxides will burn violently and reduce the oxide to a metallic state. The requisite quantity of powdered aluminum is mixed with oxide of iron and placed in a crucible and then ignited. A violent chemical action ensues, reducing the iron to a

Fig. 75. Thirmite Weld Joint.

metallic molten liquid, which may be poured from the crucible around the joint. The apparatus required merely consists of a pair of clamps with which to compress the rail-joints, and a crucible of sufficient capacity to hold the requisite quantity of material. A complete joint made by this process is illustrated in Fig. 75.

The cost of the thirmite process is stated to be about one-fourth that required for either electrical or cast weld for the actual welding of joints. Naturally the cost of preparing rails and of necessary paving is the same by either process.

The success of any form of continuous rail will depend very largely upon the nature of surrounding paving. To use continuous rails upon an ordinary "T"-rail track, exposed upon sleepers, would be folly, as, owing to the wide variations in the temperature and the lack of longitudinal support, it would be impracticable to keep the track in line; but with girder rails supported upon solid concrete foundation inclosed

in rigid Belgium pavement this method may be adopted with entire success, because the longitudinal support of the pavement is so great that it is impracticable for a rail to sensibly deflect. The inclosure offered by paving limits variations in temperature, and finally the friction of the paving prevents the track from creeping.

Where other forms of paving, such as asphalt, brick, wooden block, etc., are used, more or less difficulty with change in alignment of track will be found and the pavement will be correspondingly more or less crowded out of place by expansion. In general, therefore, no form of continuous rail should be adopted excepting when accompanied by the most substantial pavement.

83. Even with the best care bonds may break or be stolen, or the juncture with the rail grow defective. To guard against this con-

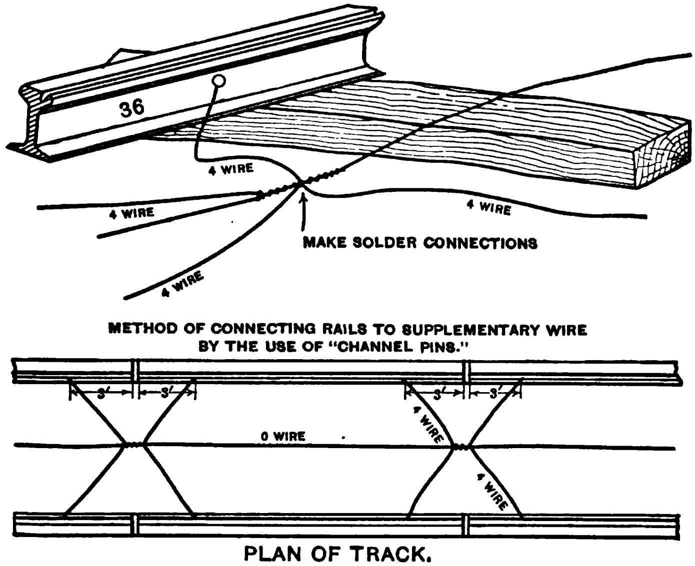


Fig. 76. Cross-Bonding and Ground Wiring.

tingency it is customary to frequently "*cross-bond*" the rails of a track or even connect in a similar manner all four rails of a double track, and this is particularly desirable at frogs, switches, turn-outs, and other special work. This is accomplished by drilling into the web or flange of the rails and cross-connecting with bonds long enough to extend from rail to rail and track to track. To still further insure continuity and to reinforce the conductivity of the track a supplementary wire called a "*ground* wire" is often placed between the rails and attached

to the cross-bonds. "Cross-bonding" and "*ground* wiring" are shown in Fig. 76, while Fig. 77 shows the method of attaching feeder cables to the rail.

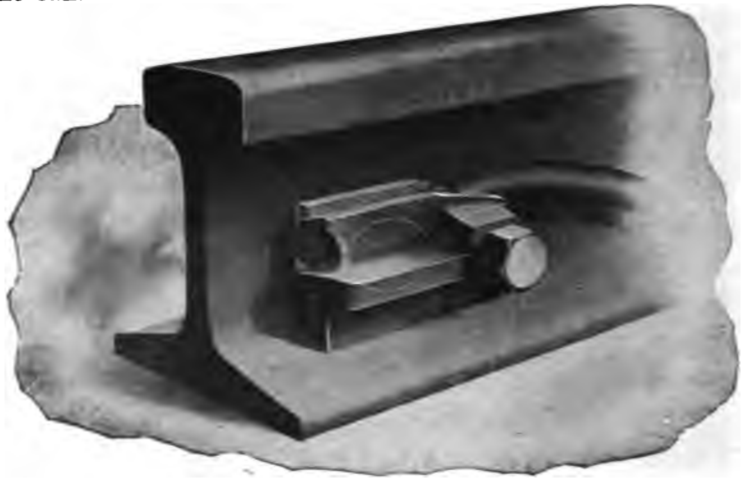


Fig. 77. Feeder Terminal.

In designing the return circuit, care should be taken in installations where the rail lines do not pass close to the power station, to introduce a sufficient amount of return feeds from the station to the rails as shall be fully equal to the cross-section of the return circuit as obtained through the rails and return feeds.

84. Electrolytic Action.—The first complete investigation upon the electrolytic action of underground currents has been made by Mr. I. H. Farnham, of the New England Telephone & Telegraph Co.[1] Mr. Farnham's attention was drawn to the matter some ten years ago, by the appearance of injurious corrosion in the lead sheaths of the underground telephone cables. Investigation traced the cause of the corrosion directly to the return current in the ground from the electrical system of the West End Railway, and further showed that the difficulty could be traced to points along the cable where the underground currents tended to leave the cable for a path of less resistance to the station. At the beginning of these investigations, the negative pole of the generating-station was connected to the overhead trolley system, while the opposite pole was

[1] See transactions of American Institute of Electrical Engineers, April, 1894.

put to earth. Many volt-meter measurements were made, to obtain the difference in potential between the underground cables, water-pipes, and gas-pipes, and the surrounding earth, by means of which it was possible to map out the entire city of Boston, showing where the corrosive action was likely to be expected, thus inclosing an area appropriately termed "Danger District." It was suggested to reverse the poles of the dynamo, placing the positive pole to the trolley, and the negative pole to earth. This suggestion was carried into effect, and a second set of measurements made, showing that, under the new conditions, the amount of the danger district was very much decreased, and indicating that the area in which corrosive action might be expected was practically confined to the immediate neighborhood of the power stations, and thus brought very much more under control. It has been pointed out that by restricting the danger district, the intensity of electrolytic action may be much increased. Such an effect as this is a logical consequence, but the restriction of the district to comparatively small areas renders repairs very much easier of execution. To protect underground structures within the danger district, it has been proposed by Mr. Farnham that large copper conductors should be extended from the grounded side of the generators, entirely through the district, and should be connected, as often as possible, to all metallic structures which are exposed to electrolytic action. This experiment was tried in Boston, subsequent volt-meter measurements indicating that the protection thus afforded was sensibly complete.

Mr. Farnham's investigations further indicated that a very small difference in electrical potential was sufficient to initiate the corrosive action.

85. In the discussion of Mr. Farnham's paper, Prof. D. C. Jackson, of Madison University, gives exceedingly interesting results from experiments to determine the minimum amount of electrical pressure likely to be injurious, and the chemical effects which are produced in the soil through the action of the current. Professor Jackson concludes that a difference of potential as small as one-thousandth of a volt, constituting mere directive force, may be sufficient to initiate and continue sufficient corrosive action to be injurious, provided it extends over a considerable period of time. Professor Jackson also says that, in most cases, the action may be considered

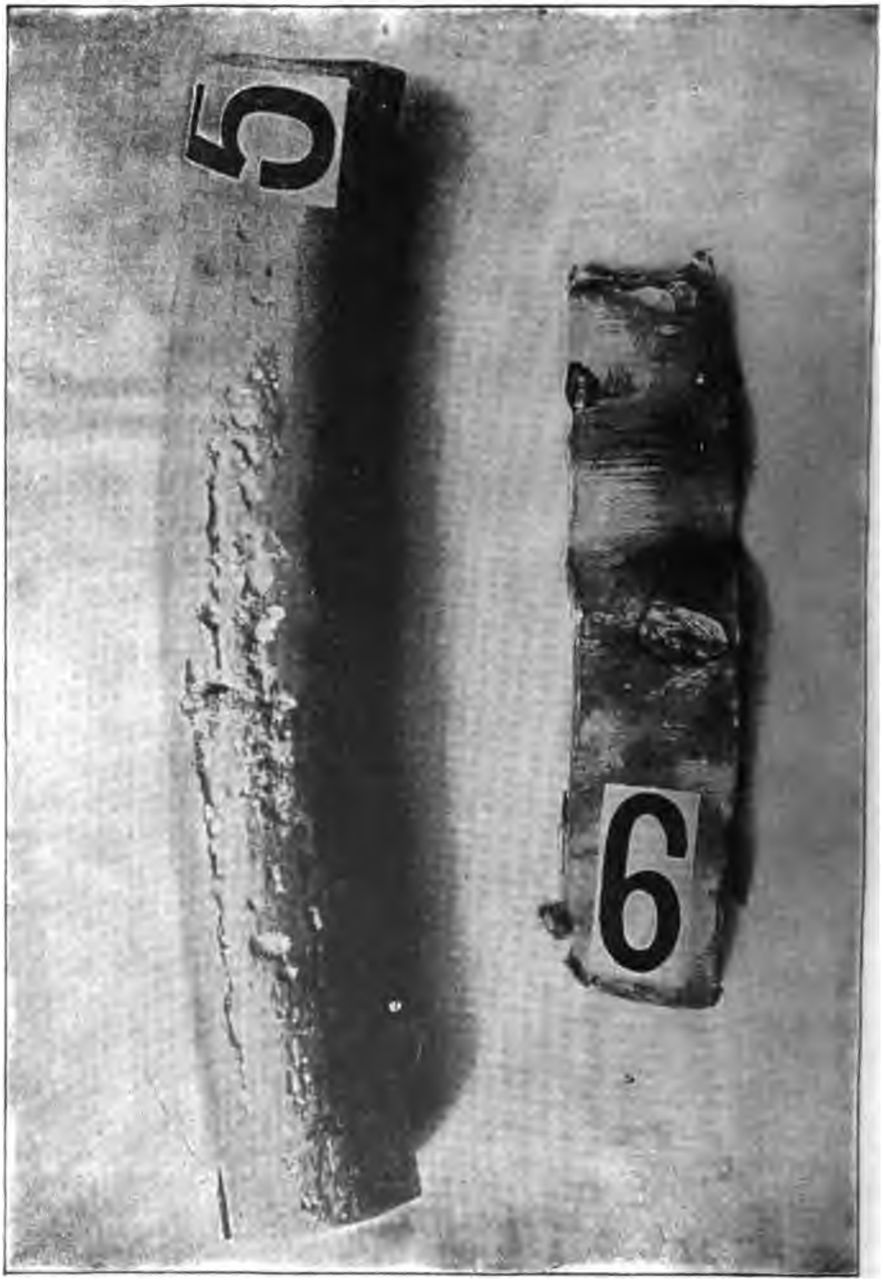


Fig. 78. Corroded Water-Pipe.

to be that of an electrolytic cell with iron electrodes, having an electrolyte of the various salts of the alkaline metals, or earths, which would be naturally found in the street soils. These alkaline salts are electrolyzed by the current, the acid radicals attracted by the anode and forming an iron salt, while the metals pass over to the cathode, forming with water a hydroxide, liberating hydrogen. The ferrous salt thus formed diffuses toward the cathode, while the alkaline hydroxide, in a similar manner, diffuses toward the anode. Where these salts meet in the soil, ferrous hydroxide is precipitated, and the original salt re-formed.

Fig. 79. Corroded Water-Pipe.

Assuming the correctness of this theory, it is evident that the actual corrosion is due to an attack by the acid radical of the salt in the electrolyte, which is set free by the passage of the current.

This investigation still further emphasizes the necessity in street railway work of providing a metallic return circuit which shall be amply sufficient to convey to the power-house all the energy required to operate the railway system. The appearance of corroded water-pipes is illustrated in Figs. 78 and 79.

86. **Railway Poles.** — Nearly all of the timber woods have been pressed into service for electrical railway construction. Poles of pine, cedar, chestnut, cypress, spruce, and tamarack are most frequently in demand. Of these spruce, cypress, and tamarack, and occasionally poplar, have been used, particularly near the localities in which the various kinds of timber are found native. Spruce poles make handsome lines, and are strong and elastic, but have a very short life, usually rotting out in from two to three years. Cypress and tamarack have great durability, and are largely used in the Southern States. Cedar, pine, and chestnut are abundant, and in the North poles are usually selected from one of these woods.

White cedar and chestnut are frequently selected, both for cheapness and prompt delivery. Pine, whether Michigan, Oregon, Norway, Georgia, or North Carolina yellow pine, is usually used for manufactured poles.

By means of a little mill-work, railway poles may be made in a wide variety of shape and finish, according to the choice of the designer. Commonly the butt of the pole is left round, while from the ground up it is sawed either square or octagonal. A manufactured pole, without question, when carefully made and tastefully painted, makes a line of unexceptional appearance. The tops of the poles should be carefully chamfered to a neat point, and should be thoroughly painted with at least three coats of best white lead and oil. In crowded localities, the butts of the poles should always be protected by an iron wheel-guard, to prevent injury by collision with the wheels of vehicles.

87. Wooden Poles should be not less than 6″ at the top, and at least 28 or 30 ft. in length. For center-pole construction the poles are set at intervals of from 100 to 150 ft. longitudinally, between the rails in case of double track, or just outside of the rail in a single track. The poles may be either round or octagonal. They should be true, straight, and fully up to the size specified, free from knots and shakes, and sound in every respect. Typical railway poles are shown in Fig. 80.

In setting the poles the base should be thoroughly tarred for a distance of 5 or 6 ft., and firmly planted in the earth, special care being taken to ram the earth solidly around the pole. Where soft ground is encountered, a concrete foundation must be used.

88. For center-pole construction the poles are supplied with brackets as shown in Fig. 54, to which the insulator supporting the trolley wire is attached.

Care should be taken to see that the brackets are sufficiently firm and strong, and that they are solidly attached to the pole, as frequent accidents have occurred by the fall of the bracket due to a blow from a passing trolley.

89. For span-wire construction, two poles are required for each span, one set on either side of the street close to the curb-line. To counterbalance the tension of the span, it is customary to set the poles with a rake, outward away from the center of the street about

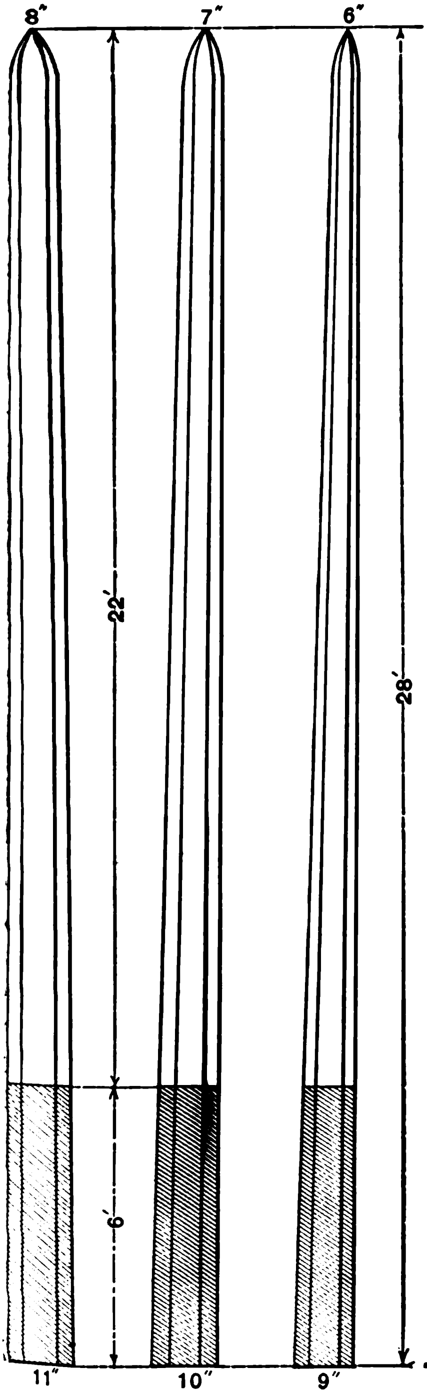


Fig. 80. Wooden Railway Poles.

Fig. 81. Pole Ratchets.

18". By this means the tension of the span-wire gradually pulls the pole to a straight line. On account of this tension, span-wire construction must be exceedingly solid. Specially good foundations must be provided for the poles, and they must be amply stiff to resist the bending-moment due to the span wire.

90. The span-wire should be made of galvanized iron or steel cable about ⅜" to ½" in diameter, depending upon whether the line is a double or single line. The span-wire should be attached to the poles by means of a ratchet shown in Figs. 81 and 82, in order that requisite adjustment of tension or location of insulators may at any time be made. If rigid economy is desired, the span-wire may be fastened by means of an eye-bolt extending through the pole, the

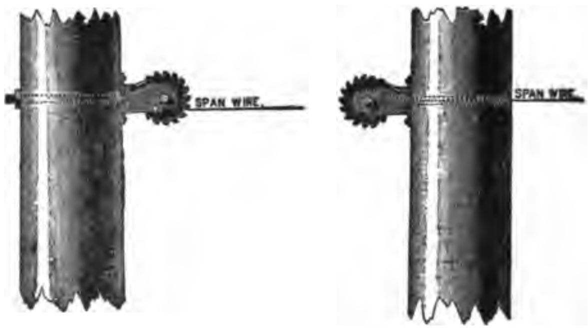


Fig. 82. Pole Ratchets in Place.

tension being adjusted by means of the nut on the shank of the bolt. Each span-wire should be supplied with two strain insulators, one set near to each pole, as a protection against any leakage from the trolley wire, as in Fig. 56. The strain insulators are introduced into the span-wire by forming eyes in the cable by whipping one end of the cable over on itself with annealed copper wire, carefully turning in all of the wire ends. It is not advisable to use iron wire for any purpose of this kind, as it sooner or later rusts.

91. **Iron Poles.** — Iron poles are made either of successive lengths of wrought-iron pipe shrunk together at the joints, or of some of the various forms of structural iron. A great variety of designs may be found in the market, of which the examples in Figs. 83, 84, 85, and 86 may be considered as typical of the best forms.

The Lattice-pole, Fig. 83, is an excellent form, and may be designed to present a very ornamental appearance in the street. Unfortunately, in the early designs,

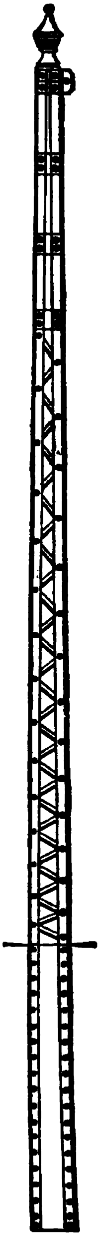


Fig. 83.

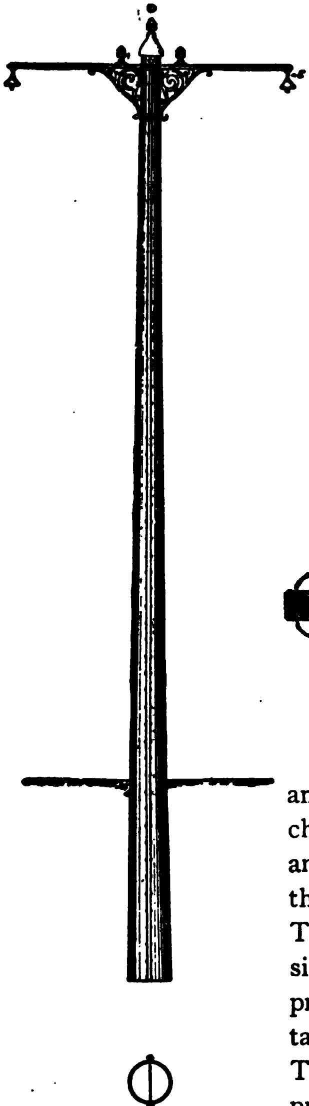
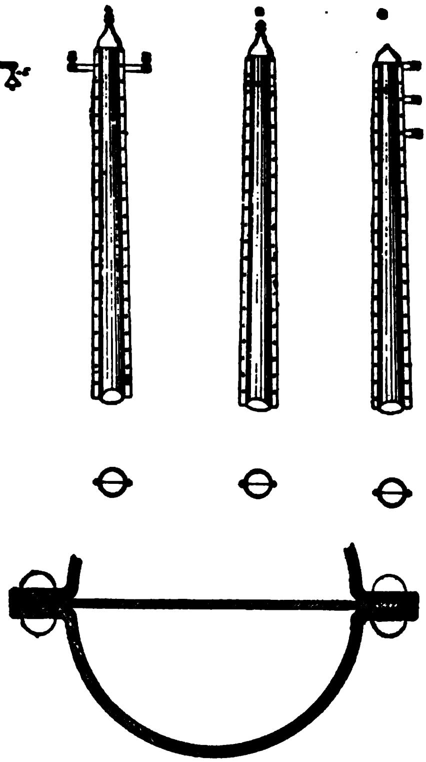


Fig. 84. Tubular Pole.



an unwise attempt to produce too cheap a pole led to many failures, and has caused a prejudice against this style that should be unfounded. The lattice-pole, being open on all sides to inspection and painting, presents in this respect an advantage over other designs. The Tubular Steel pole, Fig. 84, is probably the lightest, stiffest, and theoretically the best designed pole. It is rather the most expensive, and, the inside being inac-

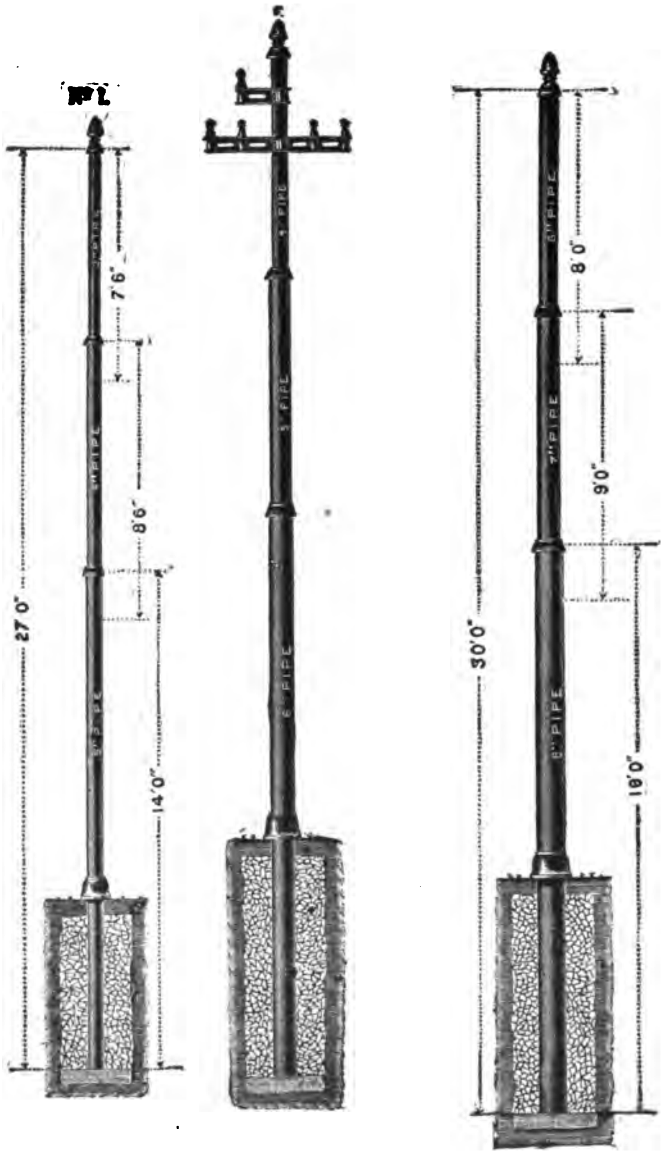


Fig. 85. Iron Pipe Poles.

Fig. 86.

cessible, the thin metal of the shell is likely to suffer from rust. The Iron Pipe pole, Figs. 85 and 87, was the earliest design on the market, and is deservedly a favorite, as it may be obtained in any

desired weight or strength. As usually made, it consis three or four lengths of iron pipe of different siz nected together by shrinking each piece over the next smaller one. In selecting pipe-poles, particular attention must be paid to the joints, to see that the pieces are properly joined, securely shrunk, and that a sufficient length

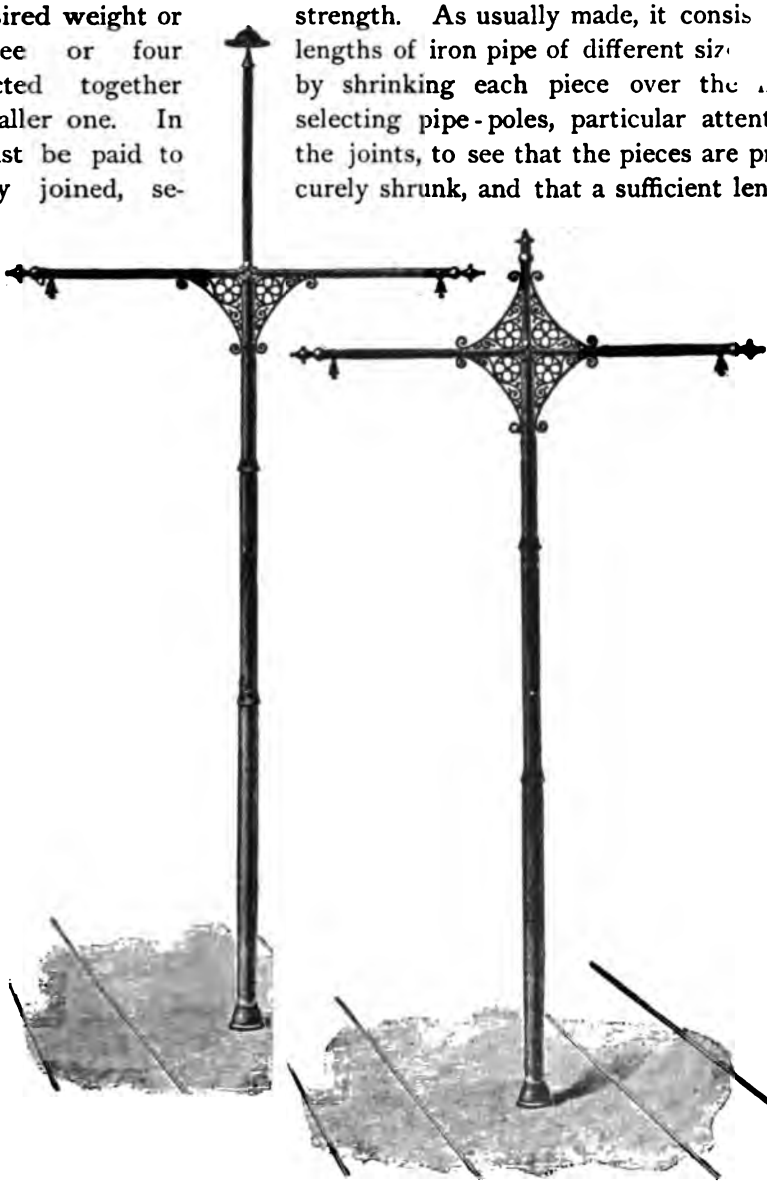


Fig. 87. Pipe-Poles for Center Pole Work.

of joint (at least 18″ to 24″) is used, in order that the respective pieces may not loosen under the vibration of the passing trolley.

The Tripartite Pole, Fig. 86, is a new and novel design that would seem to possess much merit.

92. The poles used for span-wire railway lines must be stiff enough to resist the tension due to stretching the span wire loaded with the trolley-wire insulators and other appliances between their tops. Mr. E. A. Merrill shows that under usual conditions the deflection of the span wire and the tension on the pole tops for various widths of street, will be as in Tables 39 and 40. Both tables are calculated on the assumption that the longitudinal distance along the road between spans is 125 ft.

Table No. 39.

Sag of Span Wire and Strain on Side Poles for Single Trolley Wire.

Span in Feet.	Strain on Poles in Pounds and Sag.						
	500	800	1000	1500	2000	2500	3000
	Inches	Inches	Inches	Inches	Inches	Inches	Inches
30	7.8	4.9	3.9	2.6	1.9		
40	10.6	6.5	5.3	3.5	2.7		
50	13.6	8.5	6.8	4.5	3.4	2.7	
60	16.7	10.4	8.3	5.6	4.2	3.3	2.8
70	19.9	12.4	9.9	6.6	4.9	4.0	3.3
80	23.2	14.5	11.6	7.7	5.6	4.6	3.9
90	26.7	16.7	13.4	8.9	6.6	5.3	4.5
100	30.3	18.9	15.2	10.1	7.6	6.1	5.1
110	34.0	21.3	17.0	11.3	8.5	6.8	5.7
120	37.9	23.7	18.9	12.6	9.5	7.6	6.3

Table No. 40.

Sag of Span Wire and Strain on Side Poles for Two Trolley Wires 10 Ft. apart.

Span in Feet.	Strain on Poles in Pounds and Sag.						
	500	800	1000	1500	2000	2500	3000
	Inches	Inches	Inches	Inches	Inches	Inches	Inches
40	15.4	9.6	7.7	5.1	3.9	3.1	
50	20.8	13.0	10.4	6.9	5.2	4.2	
60	26.3	16.4	13.1	8.8	6.6	5.3	4.4
70	31.9	19.9	15.9	10.6	8.0	6.4	5.3
80	37.6	23.5	18.8	12.5	9.4	7.3	6.3
90	43.5	27.2	21.8	14.5	10.9	8.7	7.3
100	49.5	30.9	24.8	16.5	12.4	9.9	8.3
110	55.6	34.7	27.8	18.5	13.9	11.1	9.3
120	61.9	38.7	30.9	20.6	15.5	12.4	10.3

TABLE No. 41. — Iron Poles.

Sizes of Poles.						Weight of Pole.	Deflections for Given Loads. With Butt Planted 6 Feet in Ground, Weight Applied, and Deflection Measured 18 Inches from Top. Load in Pounds.										
Bottom Section, 17′ Long.		Middle Section, 9′ 6″ Long.		Top Section, 7′ 6″ Long.			100	300	500	700	900	1200	1600	2000	2800	3600	4500
Outside Diam.	Thickness.	Outside Diam.	Thickness.	Outside Diam.	Thickness	Pounds.	Deflection in Inches.										
10¾	⅝	9	½	7⅝	⅜	1798	.133	.399	.665	.931	1.197	1.596	2.128	2.660	3.724	4.788	5.985
10¾	9/16	9	7/16	7⅝	5/16	1604	.146	.438	.730	1.022	1.314	1.752	2.336	2.920	4.088	5.256	
10¾	½	9	⅜	7⅝	9/32	1424	.163	.489	.815	1.141	1.467	1.956	2.608	3.260	4.564	5.868	
10¾	7/16	9	5/16	7⅝	¼	1242	.186	.558	.930	1.302	1.674	2.232	2.976	3.720	5.208		
10¾	⅜	9	9/32	7⅝	3/16	1067	.212	.636	1.060	1.484	1.908	2.544	3.392	4.240	5.936		
10	⅝	8⅝	½	7⅝	⅜	1694	.163	.489	.815	1.141	1.467	1.956	2.608	3.260	4.564	5.868	
10	9/16	8⅝	7/16	7⅝	5/16	1510	.180	.540	.900	1.260	1.620	2.160	2.880	3.600	5.040		
10	½	8⅝	⅜	7⅝	9/32	1342	.199	.597	.995	1.393	1.791	2.388	3.184	3.980	5.572		
10	7/16	8⅝	5/16	7⅝	¼	1171	.226	.678	.130	1.582	2.034	2.712	3.616	4.520	6.328		
10	⅜	8⅝	9/32	7⅝	7/32	1023	.256	.768	1.280	1.792	2.304	3.072	4.096	5.120			
10	11/32	8⅝	¼	7⅝	3/16	927	.279	.837	1.395	1.953	2.511	3.348	4.464	5.580			
9⅝	9/16	8⅝	7/16	7⅝	5/16	1472	.196	.588	.980	1.372	1.764	2.352	3.136	3.920	5.488		
9⅝	½	8⅝	⅜	7⅝	9/32	1299	.218	.654	1.090	1.526	1.962	2.616	3.488	4.360	6.104		
9⅝	7/16	8⅝	5/16	7⅝	¼	1141	.246	.738	1.230	1.722	2.214	2.952	3.936	4.920			
9⅝	⅜	8⅝	9/32	7⅝	7/32	998	.281	.843	1.405	1.967	2.529	3.372	4.496	5.620			
9⅝	5/16	8⅝	¼	7⅝	3/16	853	.325	.975	1.625	2.275	2.925	3.900	5.200				
9	½	8	13/32	7	5/16	1252	.267	.801	1.335	1.869	2.403	3.204	4.272	5.340			
9	7/16	8	11/32	7	9/32	1099	.301	.903	1.505	2.107	2.709	3.612	4.816	6.020			
9	⅜	8	5/16	7	¼	967	.335	1.005	1.675	2.345	3.015	4.020	5.360	6.700			
9	5/16	8	¼	7	3/16	792	.403	1.209	2.015	2.821	3.627	4.836	6.448				
8⅝	½	7⅝	13/32	6⅝	5/16	1193	.305	.915	1.525	2.135	2.745	3.660	4.880	6.100			
8⅝	7/16	7⅝	⅜	6⅝	9/32	1069	.339	1.017	1.695	2.373	3.051	4.068	5.424	6.780			
8⅝	13/32	7⅝	5/16	6⅝	¼	966	.371	1.113	1.855	2.597	3.339	4.452	5.936				
8⅝	⅜	7⅝	9/32	6⅝	7/32	885	.400	1.200	2.000	2.800	3.600	4.800	6.400				
8⅝	5/16	7⅝	¼	6⅝	3/16	756	.462	1.386	2.310	3.234	4.158	5.544	7.392				
8	7/16	7	11/32	6	9/32	962	.442	1.326	2.210	3.094	3.978	5.304	7.072				
8	⅜	7	5/16	6	¼	846	.497	1.491	2.485	3.479	4.473	5.964	7.952				
8	11/32	7	9/32	6	7/32	770	.539	1.617	2.695	3.773	4.851	6.468					
8	5/16	7	¼	6	3/16	694	.595	1.785	2.975	4.165	5.355	7.140					
7⅝	⅜	6⅝	5/16	5 9/16	¼	804	.583	1.749	2.915	4.081	5.247	6.996					
7⅝	11/32	6⅝	9/32	5 9/16	7/32	729	.634	1.902	3.170	4.438	5.706	7.608					
7⅝	5/16	6⅝	¼	5 9/16	3/16	658	.694	2.092	3.470	4.858	6.246	8.328					
7⅝	9/32	6⅝	7/32	5 9/16	5/32	584	.769	2.307	3.845	5.383	6.921						
7	11/32	6	9/32	5	7/32	663	.835	2.505	4.175	5.845	7.515						
7	5/16	6	¼	5	3/16	600	.913	2.739	4.565	6.391	8.217						
7	9/32	6	7/32	5	5/32	532	1.009	3.027	5.045	7.063	9.081						
6⅝	11/32	5 9/16	9/32	4½	7/32	616	1.026	3.078	5.130	7.182	9.234						
6⅝	5/16	5 9/16	¼	4½	3/16	558	1.107	3.321	5.535	7.749							
6⅝	9/32	5 9/16	7/32	4½	5/32	498	1.225	3.675	6.125	8.575							
6⅝	¼	5 9/16	3/16	4½	⅛	435	1.373	4.119	6.865	9.611							
6	5/16	5	9/32	4	7/32	524	1.482	4.446	7.410	10.374							
6	9/32	5	¼	4	3/16	470	1.625	4.875	8.125								
6	¼	5	7/32	4	5/32	415	1.814	5.442	9.070								
6	7/32	5	3/16	4	⅛	360	2.063	6.189	10.315								

The elastic limit of the steel is reached under the various weights where the Table of Deflections stops.

Tables 41 and 42 give the deflection which iron and wooden railway poles may be expected to show under different tensions applied at the top 28 ft. above the ground.

TABLE No. 42.

Constants for Wooden Poles.

Wooden Railway Poles.			American Yellow Pine or Cedar.	
Length.	Diameter. Top. Butt.	Section.	Approximate Weight in Pounds.	Stress to Deflect Pole 7 Inches.
Feet	Inches			
27	6 × 8	Circular	360 to 450	350
27	7 × 9	"	450 to 560	500
27	7 × 9	Octagonal	500 to 620	500
28	7 × 9	Circular	490 to 600	500
28	7 × 9	Octagonal	520 to 650	500
28	8 × 10	Circular	620 to 750	750
28	8 × 10	Octagonal	650 to 800	750
30	7 × 9	Circular	530 to 670	450
30	7 × 9	Octagonal	560 to 700	450
30	8 × 10	Circular	660 to 820	700
30	8 × 10	Octagonal	700 to 850	700
30	9 × 12	"	900 to 1150	850

In pipe-poles the earth is the weak point. Iron poles should always be set in concrete, as the butt of the pole presents too small a surface to secure a permanent bearing against mere earth.

93. While the iron pole presents the advantage in appearance, and is undoubtedly of greater durability, the metal of which it is formed being an excellent conductor, it presents the disadvantage of exposing the public to much greater danger from leaky trolley wires. The wooden pole is so good an insulator that no accidents have as yet been reported from leaks through the pole. With the iron pole, however, there are many cases reported where either animals or men have received severe shocks from leaky span-wires.

94. Feed-Wire Insulators, and Pole-Tops. — In addition to providing a support for the trolley wire, the railway pole must carry feed-wire, guard-wire, lightning arrester, cut-out switches, and electric light fixtures, and perhaps a lighting circuit. The pole-top becomes an important feature in the circuit. Typical pole-tops are

shown in Figs. 88, 89, 90, and 91. The device illustrated in Fig. 88 consists of a casting into which any number (up to seven) of iron supports may be placed. Each support carries a locust pin to which any desired form of insulator may be attached for sustaining the feed-wires.

In Fig. 90 a variety of pole-tops are shown, all of which are applicable to the pipe-pole, the designs being arranged to meet the usual requirements of street-railway work. These tops are made to

Fig. 88. Adjustable Iron Pole-Top.

be insulated, if desired, a precaution well worth the slight extra expense, as leaky trolley wires have already caused sensible damage on iron-pole lines.

95. For wooden poles there is no better top than that given in Fig. 89, especially where heavy feeds are to be carried. A cheaper expedient is found in bolting a cross-arm on to the pole, supplied with wooden pins and glass insulators, in a manner precisely similar to that of the ordinary telegraph construction.

Fig. 91 represents the pole-top used in Philadelphia. In this case the feeder system is underground, the tap running up through the centre of the pole. The span-wire is provided with a special break insulator close to the pole, and provision is made for guard-wire and

lightning arrester, the whole device being worked out in an exceedingly mechanical manner.

96. Trolley Insulators.—Insulators for supporting trolley wire have presented a difficult problem to mechanical inventors, owing to the severity of the service to which they are subjected. Insulators are usually made by forming a bell of some insulating substance, which is hung from the span-wire or pole-bracket. The trolley wire is suspended by a clip inserted into the insulating material forming

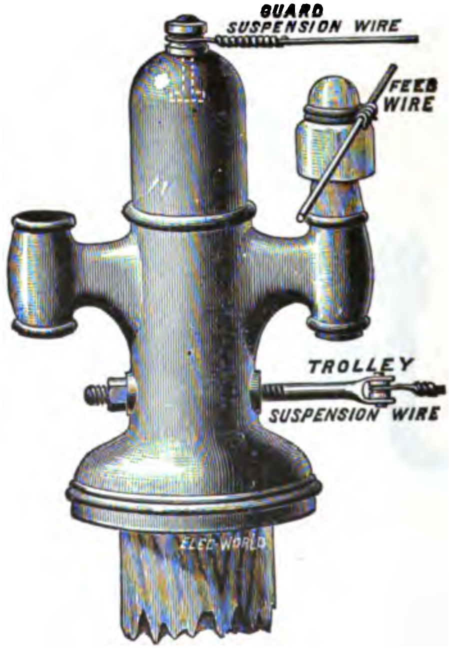


Fig. 89. Wooden Pole-Tops.

the bell. The general arrangement of standard forms of trolley insulators is indicated in Figs. 92 to 98 inclusive. The attachment of the trolley wire to the insulator has presented one of the greatest difficulties in the problem. In the early forms, the connection between the trolley wire and insulator was always accomplished by soldering the wire to a semicircular brass or bronze support, which was screwed to the under side of the insulating bell. This practice was exceedingly objectionable, from the fact that it annealed the hard-drawn copper used for the trolley wire, and from the difficulty which arose in making

a change whenever the insulator was worn out or destroyed. To obviate these defects, a multitude of devices arose, whereby the trolley wire was inserted into a clip split through the center, clasping the trolley

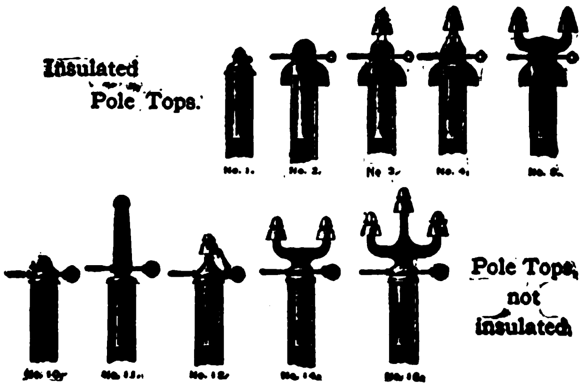


Fig. 90.

wire, which was prevented from falling out of the clip by a screw, or nut, which locked the two halves of the clip holding the wire in position.

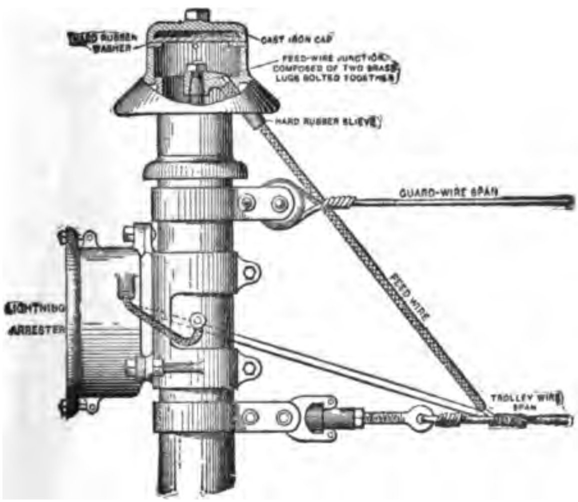


Fig. 91. Pole-Top of Philadelphia Traction Company.

The difficulty of soldering was thus obviated, and by loosening the lock-nut, the insulator could at any time be set free from the wire. Such an arrangement always presented the difficulty of offering a slight

Fig. 92. Standard Trolley Wire Insulator, with Soldered Ear.

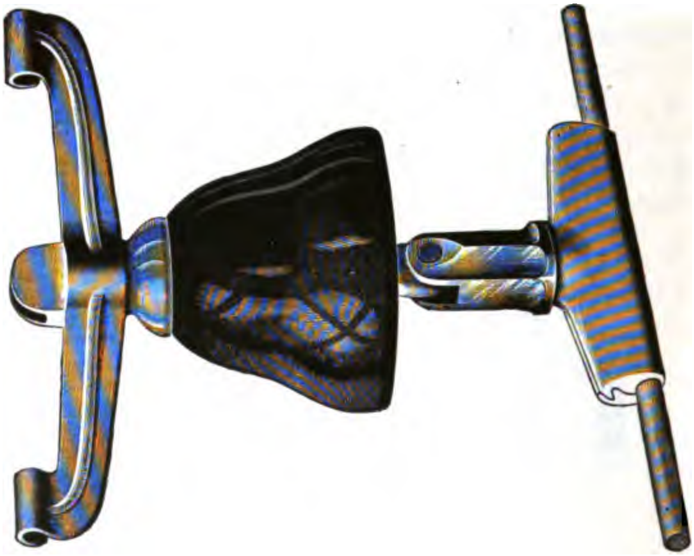


Fig. 93. Trolley Wire Insulator, with Adjustable Wire Clip.

impediment to the passage of the trolley, and concentrating the wear of the trolley wheel upon the clips in such a manner as to rapidly destroy them, allowing the wire to sooner or later fall from the insulator. As these objections are less serious than those presented by the method of soldering, the latter design is often adopted. As trolley insulators must be adapted to all kinds of line construction, a variety of forms must be provided to meet the different methods of suspension. Thus, in Figs. 92 and 93, designs are indicated for span-wire construction, in which the span-wire is placed through ears on top of and around the insulator. Fig. 94 is a bracket insulator designed for a bracket of iron pipe to be secured by the bolt on the left hand; Fig. 95 is a bracket insulator for angle-iron brackets; Fig. 96 is a straight-line insulator for two trolley wires; Fig. 97 is a double-curve insulator, the extended ears of which receive the pull-offs that hold the curve in position; Fig. 98 shows two sectional views indicating the interior construction of insulators, and the method of securing insulation. The lower design is an approved form for 500 volt D. C. circuits; the upper one can be used on single-phase A. C. lines. By varying the size of the outer shell any thickness of insulation can be secured. To secure an insulating substance from which to form the bells, which should possess sufficient insulating properties to be safe upon a five hundred volt circuit, and strong enough to withstand the blows of the trolley wheel, has presented the greatest difficulty in this problem. Glass, porcelain, and india-rubber have been tried with but little success. Also various other substances, such as mixtures of mica, india-rubber, asbestos, etc., have been experimented with, and each engineer has his own pet insulator. It is probable, however, that the best trolley insulator has yet to be invented.

97. **Railway Line Work.**—The railway circuit is chiefly an aerial line. If the trolley wire were extended in a single straight line, its erection and maintenance would be relatively a simple matter. On the contrary, it must accommodate itself to curves of every description, intersect other lines, and afford proper methods of switching; and, at each of its many supports, it must be thoroughly electrically insulated, with sufficient mechanical strength to withstand the severe usage of the trolley

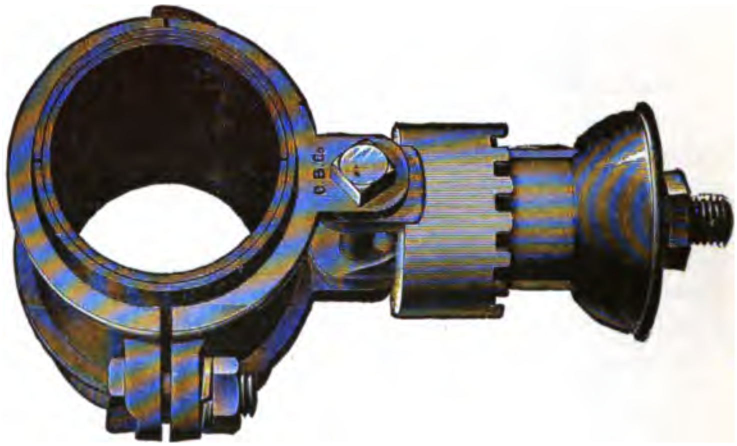


Fig. 94. Bracket Insulator.

Fig. 95. Angle Iron Bracket Insulator.

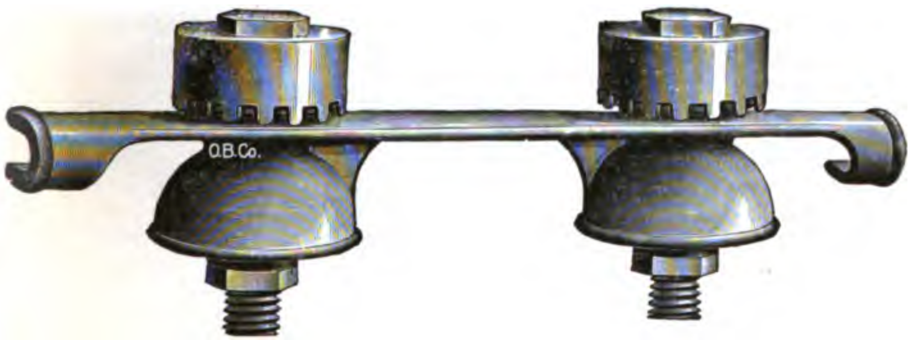


Fig. 96. Straight-Line Insulator.

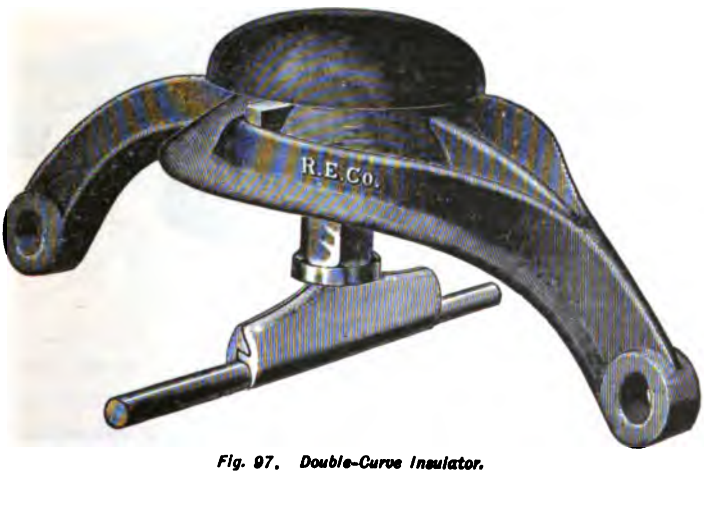


Fig. 97. Double-Curve Insulator.

wheel. There are three points in every trolley line deserving of special attention, namely, *curves*, *switches*, and *cross-overs*. To sustain the trolley wire at each curve, it is customary to plant special guy-poles, to which the entire curve is anchored by means of strain insulators or pull-offs; and a similar method is adopted wherever there occurs any change in direction of the trolley wire, as, for

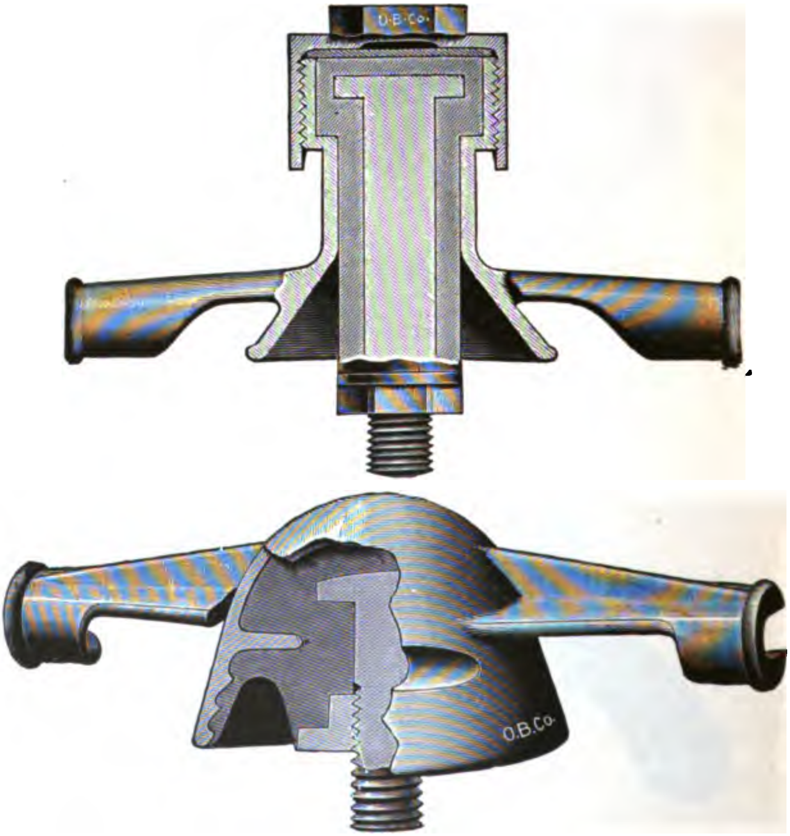


Fig. 98. Sections of Insulators.

example, in the case of turn-outs. The solution of all problems in railway line construction may be readily solved by careful application of the principle of the parallelogram forces. It is essential to study the location of each point on the line where a change in direction occurs, and determine the resultant of the forces which act upon the trolley wire. As all electric railway work is held in place by means of wire guys, all forms of construction are always in tension, and

must be designed to meet this form of stress in every particular. It is hardly practicable to give diagrams illustrating all the possible forms of wire construction which might be met with. In Figs. 99,

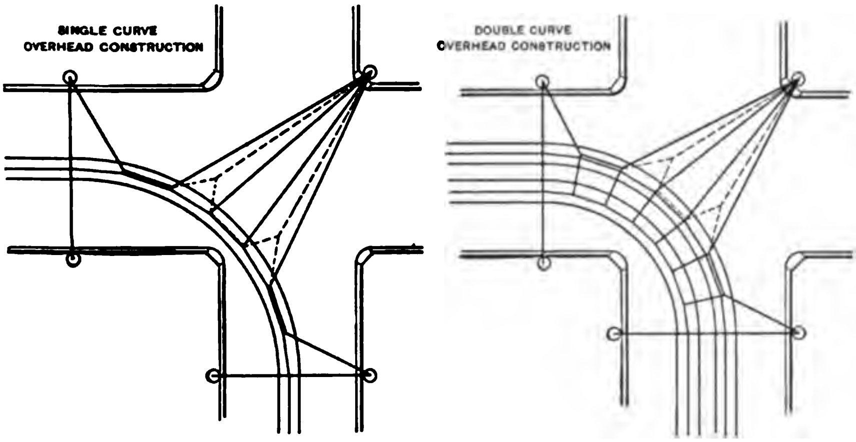


Fig. 99. Curve Construction.

100, 101, and 102 are given the general methods which are in common use on the best roads for curves and turn-outs, in which the location of anchor poles and method of staying may be readily seen. To ascertain the correct location for the pull-off pole, or poles, for a curve,

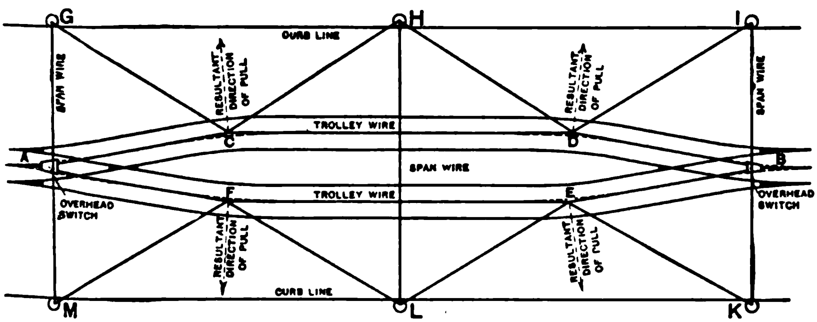


Fig. 100. Turnout Construction.

draw an equilateral triangle, having for its base the cord of the arc formed by drawing a line between the tangent points of the curve. The last two span-wires should be located at the end of the tangents, which also gives the location of the base of the triangle

AA′C, Fig. 101. The apex of this triangle gives the best location for the pull-off pole at C. If it is inconvenient to locate the pole here, it is permissible to move it slightly forward or backward, or a little to the right or left, though point C is the only correct location. An expedient sometimes adopted for situations of this kind consists in making the pull-offs up on an iron ring, and carrying a pendant over from the ring back to the pole, which, under these circumstances, may be located at any distance along the line CD. If possible, the guy-poles E and F are advisable. Right and left curves may be built, as indicated in Fig. 102, in which A is the point of

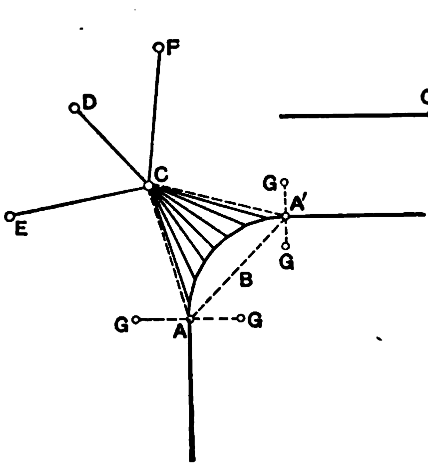


Fig. 101. Curve Guy-Poles.

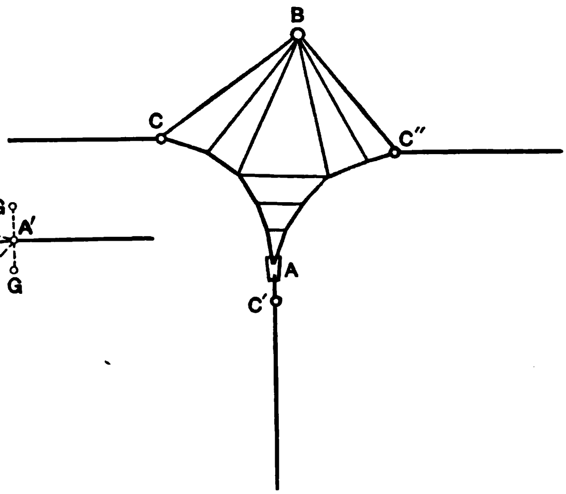


Fig. 102. Right and Left Curve Construction.

location of the switch, and AC and AC″ the two curves extending respectively to the right and left. The pull-off pole is located in a straight line at B, which is a prolongation of AC′, each half of the double curve being treated in precisely the same manner as indicated for a single curve.

98. Strain Insulators. — As the trolley wire must be retained by tension in its appropriate place along the curve, a form of insulator must be designed which shall sustain a severe lateral pull. Such an insulator has already been alluded to in Fig. 97. Another form of a similar device is shown in Fig. 103. The trolley-wire clip is supported by a goose-necked rod, the end of which is protected by

an insulating cover of vulcanized rubber, which is attached to the pull-off wire by means of a bail-shaped swivel, that surrounds the head of the goose-neck. Designs for pull-off insulators are as numerous as those for trolley insulators, and have met with corresponding success. It is frequently necessary to stay trolley wire with lateral guys, which must be carefully insulated; and for this

Fig. 103. Pull-Over or Curve Hanger.

purpose it is necessary to insulate the guy from the trolley wire with a strain insulator. The strain insulator is always subjected to severe tension, often rising to several thousand pounds, and is, further, under constant vibration from the passage of the trolley wheel. Strain insulators in the past have been defective from weak construction. A good form is indicated in Fig. 104, which consists of a strong iron bolt carefully overlaid with vulcanized india-rubber, as

Fig. 104. The Strain Insulator.

an insulator. A disk in the center serves the purpose of a break, preventing the lodgment of a continuous coating of snow and ice. Pull-offs are attached to either end of the guy-wire by means of bail-shaped swivels.

The pull-off wires should be made of $\frac{3}{8}''$ or $\frac{1}{2}''$ steel cable, carefully and neatly secured to the insulators and other attachments. All guy-poles to which strain insulators are attached must be specially heavy, and exceedingly securely set with an extra amount

of outward rake, with especial attention directed to the security of foundation. Moreover, insulators, turn-buckles, pull-offs, and other fixtures upon which special stress is concentrated, should receive particular attention, in view of the severity of service which they are called upon to endure.

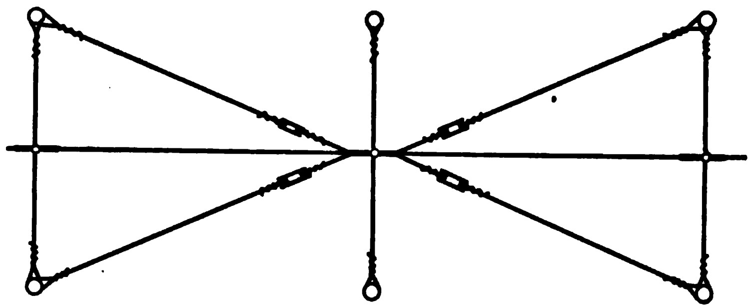


Fig. 105 Method of Anchoring Single Line.

99. Anchors. — At the end of every curve and turn-out, and as often as every 2500 ft. in the tangents, the trolley line should be anchored in both directions by means of ears which are soldered to the wire. The object of the anchoring is to sustain the line in both directions, so that in case of rupture of the trolley wire, only

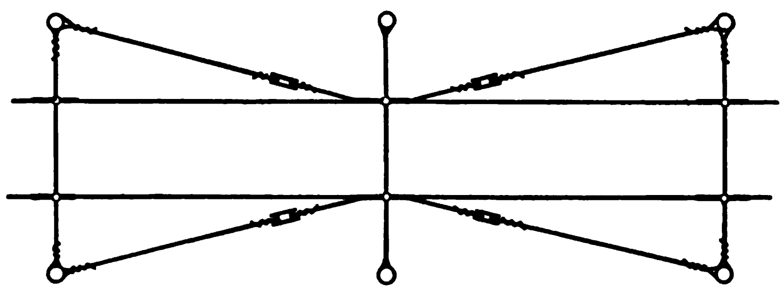


Fig. 106. Method of Anchoring Double Line.

a comparatively short length of line will be pulled down and thrown out of service. In default of the provision of anchoring, instances have been known when the overhead system of an entire railway fell flat to the earth, owing to the rupture of the trolley wire at a single point. The method of constructing the anchors is indicated in Figs. 105 and 106.

A metallic clip, similar in general shape to a trolley line clip, is soldered to the wire, from which four guys are extended to the nearest line-poles in such a way as to sustain the longitudinal tension of the trolley wire in both directions. These guys must be carefully insulated by appropriate strain insulators. Diagrams 105 and 106 indicate clearly the methods to be adopted in both the case of a single- and a double-track road.

100. Line Sections. — As short circuits are a matter of frequent occurrence in street railway work, it is customary to split the trolley wire and the feeder system into a number of independent sections, by introducing section insulators into various parts of the trolley

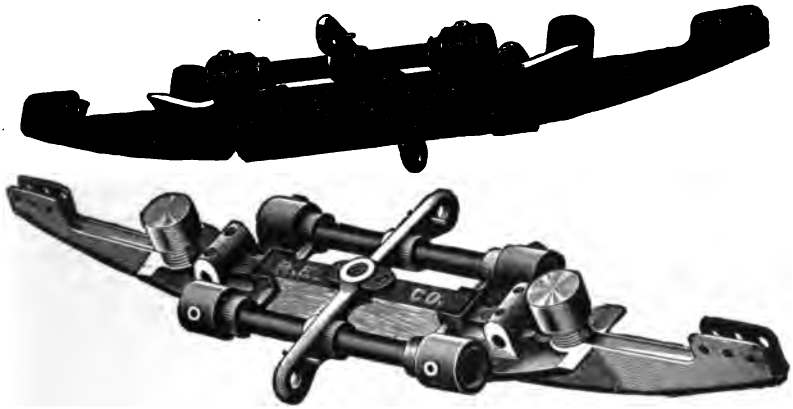


Fig. 107. Section Insulator.

wire, so that a ground may interrupt the traffic on only a small portion of the line. An approved form of section insulator is shown in Fig. 107 and consists of two trolley wire clips which are separated by a heavy strain insulator. The ends of two adjacent sections of the trolley wire are carried into the metal clips and firmly soldered into place, the strain insulator serving as an electrical break between the sections, and at the same time ensures mechanical continuity, so that the trolley wheel may pass the section insulator without leaving the wire.

101. Switches. — The perfect overhead switch, like the perfect trolley insulator, has yet to be devised. All the arrangements so far provided for this purpose have been, almost invariably, open to the objection that they cause the trolley to run off the wire. Two of

the best forms which have yet been placed on the market are shown in Figs. 108 and 109.

From the illustrations, the device is seen to be a metal casting furnished with ears for the reception of the trolley wires. On one end the switch is provided with a single ear for securing one section of the wire, while on the other side the switch is split into two, three,

Fig. 108. Two-Point Switch.

or more parts, depending upon the number of diverging lines which radiate from the switch. The setting of the switch on the overhead line is a matter of considerable care; and it is only by the most skillful placing of the switch that the trolley wheel, under any circumstances, can be coaxed to remain upon the line. The proper location of the switch is shown in Fig. 110, in which the dotted line

Fig. 109. Three-Point Switch.

indicates the true path described by the trolley wheel, while the heavy central line indicates the correct position of the switch, which should be located at the center of the arc described by the trolley. Double-track roads never require switches, excepting where two or three lines radiate from a single point; and as such intersections are rare, the car conductor can, at these points, be expected to give especial attention to keeping the trolley wheel in proper place. In a

single track it is necessary to have two switches at every turn-out. To avoid the difficulties introduced by switches, it is now customary to run a double trolley wire over single-track roads. While at first this might seem a useless expenditure for wire, it must be recollected that the additional amount of copper employed in a double trolley wire may be deducted from the feed-wire; and, as the trolley wire is uninsulated, it is cheaper in proportion than the feeds. The trolley insulators are more expensive, but the extra cost entailed in this direction is more than compensated by the decreased expense in annual maintenance of trolley wheels and switches.

102. Line Crossings. — Where two different railways intersect each other that do not use the same power station, it is necessary to so arrange the overhead lines on all crossings as to render the roads electrically separate from each other. Each road must have an independent trolley right of way, that is electrically entirely distinct from

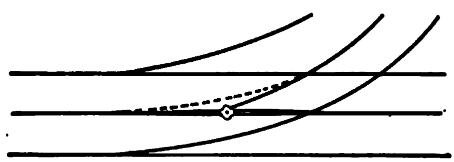


Fig. 110. Method of Setting Frog.

that of its neighbor, and yet neither road must place any mechanical obstacles that will interfere with the free passage of the other trolley. A number of automatic devices have been proposed as solutions of the cross-over problem, a typical form employing a light but stiff horseshoe-shaped casting suspended by the usual hanger from the span wire or bracket. One trolley wire is hung in the upper part of the inverted U thus formed, by the usual form of trolley wire insulator, and always runs continuously through the cross-over. The other line enters either end of the U-shaped casting, and normally is open at this point, thus always affording a free passage for the trolley wheel of the first or upper line. To close the lower line a swinging bridge is arranged, that, on the approach of a trolley wheel belonging to the lower line, swings into position across the gap in the U, providing a complete path for the trolley wheel. While inventions of this class are exceedingly ingenious, and are mechanically success-

ful under ordinary circumstances, they are likely to fail in stormy weather, just at a time when the road is most heavily loaded, and their service most in demand.

Fig. 111. Cross-overs and Switches in Place.

The illustration, Fig. 111, is a representation of the overhead system of two intersecting and two branch railway lines. The locations of the switches and cross-overs are readily seen.

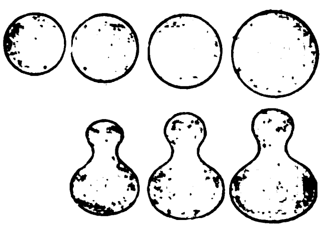


Fig. 112. Trolley Wire.

103. Prevailing current practice adopts either 0, 00, or 000 B. & S. wire for the trolley. Some efforts have been made to manufacture a wire more or less in the shape of the figure 8, upon the theory that such a section would be more readily and securely fastened in the insulator-clips. Trolley wires of this section are illustrated in Fig. 112, and in many respects the experience therewith has been satisfactory. Many efforts have been made to use some of the bronzes for trolley wire upon the theory that the harder metal would wear longer. The increased resistance and expense of all such attempts have been so great as to discourage engineers from further efforts in this direction.

CHAPTER V.

CONSTRUCTION OF AERIAL CIRCUITS.

PROTECTION.

104. Every electrical circuit is presumably designed to safely carry (with a fair margin) a certain definite current and pressure. If this normal current and pressure be seriously exceeded, either the circuit or apparatus attached thereto may be injured, or may become a menace to its environments. There are three sources of danger: An overload may occur from derangement of the apparatus attached to the circuits, or by crossing with another line carrying a greater current or pressure, or by forcing the circuit to a greater duty than it was designed to perform.

Aerial lines are exposed to incursions of electricity from atmospheric causes, or, in vernacular, may be "struck by lightning."

Alternating circuits carry electricity in the forms of waves, and under special conditions the waves emitted by the alternators may so synchronize with the natural period of vibration of the circuit as to cause what is termed (from its analogy to a similar acoustic action) *resonance*. This resonance effect may develop a potential so much greater than that for which circuit was designed to carry as to become injurious. Similar results occasionally are due to the sudden opening of switch or an accidental short circuit, which produces a rush of electricity whose operation is exceedingly mischievous unless provided for. To avoid injury to lines and apparatus and to secure continuity of action it is desirable to protect circuits in as thorough a manner as possible from the contingencies described.

105. Protective devices can be divided into two classes.

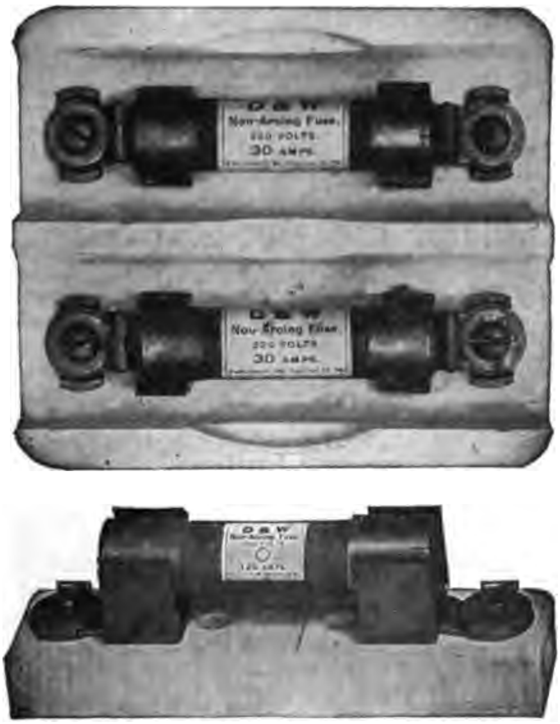
1st. Overload protectors or those which are designed to guard circuits against the imposition of too great a burden of current.

2d. Static protectors, those devices which are designed to release the circuit from the electric stress due to sudden impulsive

rushes of electricity that may arise either from atmospheric causes or resonant action.

106. Overload apparatus may be divided into two classes—fuses and automatic circuit-breakers. The fuse is the earliest, simplest, and cheapest form of protection. It consists of a short piece of conductor, designed to melt when the current has reached a predetermined value, inserted in the circuit at desirable points. The fusing of this portion of the circuit causes a gap in the line which opens the circuit, and for the time being relieves it of further duty. The laws governing the heating and fusing of conductors are discussed in Chapter XI, so that here only the mechanical and protective features of fuses will be considered. The first desideratum is reliability, that is to say, a fuse must be able to constantly and uniformly carry the normal load for which the circuit is designed, and it must equally surely fail when the circuit is subjected to predetermined overload. Second, when the fuse fails it must promptly open the circuit and destroy the arc which at once forms across the gap which was bridged with the fuse. Lastly, fuses must be so designed that they may be easily and rapidly replaced after failure, in order that the operation of the plant may not be interfered with for an unreasonable period of time. Usually fuses are constructed of a wire or strip of some lead-tin alloy, which has a low melting-point. The simplest form of fuse is a lead wire which is secured under the head of a couple of bolts and spans to a gap in a porcelain block. For circuits carrying very small amounts of energy such a device is fairly sufficient, but owing to inequality in alloys, variations in size of the fuse wire, and changes in the temperature of the environment, fuses of this description are exceedingly unreliable, and may either be injurious because they blow at too low an overload, or fail to protect by transmitting a sufficient amount of current to harm other portions of the circuit. When a fuse of this description is placed in a circuit carrying a large amount of energy, the arc which is necessarily formed at least for a moment at the instant of rupture volatilizes the metal of the fuse, which then forms a conducting vapor across the gap, and continues the arc sometimes with almost explosive violence. To remedy these difficulties protection by fuses is very much improved by making the fuse-gap long and enclosing the fuse inside of an insu-

lating incombustible tube, which may be packed with various compositions tending to prevent or destroy the formation of an arc. The D. & W. fuse is representative of this type of protector and is illustrated in Fig. 113. On a porcelain block a pair of brass clips are secured, provided with terminals for the reception of the line wires. The fuse case consists of a piece of fiber tube, furnished with metallic caps at its ends, through the center of which the appropriate



ig. 113. Inclosed Fuse.

size fuse-wire is stretched, while the interior of the tube is filled with a composition designed to prevent the formation of an arc. This device aims to fulfill all desired conditions, for the tube protects the wire from air currents and maintains it at nearly a constant temperature, so its blowing is assured within reasonable variations of load. As the fuse is enclosed the melted metal formed by the rupture of the wire does no harm, while the length of gap and arc-

suppressing compound are sufficient to prevent the continuation of an arc and consequent possibility of fire. Lastly, the spring-clips provide a means whereby the fuse may be quickly replaced after failure. A little mechanical ingenuity will adapt this form of protection to

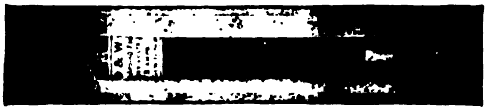


Fig. 114. 10,000 Volt Fuse Cut-out.

almost any kind of circuit. In Fig. 114 a high-tension cut-out is shown that is designed to protect 10,000 to 20,000 volt alternating circuits. In Fig. 115 a set of telephone protectors are shown. These may be of either the single-pole, as at *A*, or double-pole type, as at

A

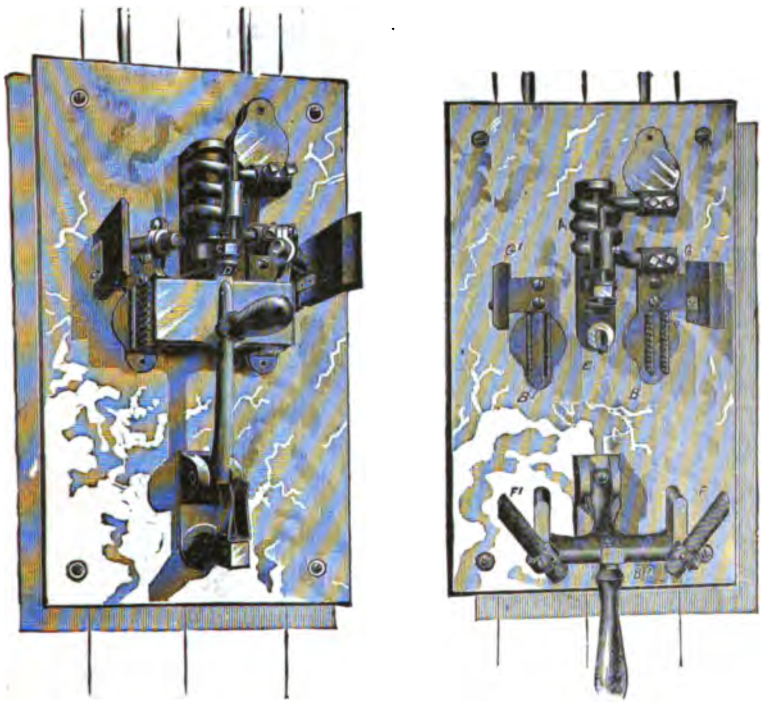
B

Fig. 115. Telephone Protectors.

B. There is the marble base upon which two fuses, similarly designed to those of Fig. 113, are mounted. In addition the protector is supplied with a heat-coil, the office of which is to open the line upon the incursion of too small a current to affect the fuse-wire. In the brass cap in

the center of the instrument two carbon plates are located, separated by a thin strip of mica which forms a short spark-gap and protects the line from high-tension discharges. Thus on a single base plate a lightning, or static, arrester, an overload fuse, and a sneak current protector are combined.

107. The term "automatic circuit-breaker" includes a great variety of switches, which contain a releasing apparatus usually operated electro-magnetically, so designed that when an abnormal current



Circuit Breaker Closed. *Fig. 116.* *Circuit Breaker Open.*

traverses the line an electro-magnet is excited, that operates a trigger, releasing a spring, whose office is to open the blades of a switch. While the automatic circuit-breaker is much more expensive than the fuse, it is correspondingly more desirable, for it is capable of working upon a very much closer margin of overload; and it can be adjusted over very wide ranges of load, is not destroyed when it operates, and can be replaced, closing the circuit with but a moment's loss of time. The technical elements of the circuit-breaker are shown in Fig. 116,

an early type of the Westinghouse breaker. The line wire is brought to a marble base and connected there to a coil that surrounds an iron plunger which may be weighted to any desired amount. Normally the plunger rests upon a stop, but when an overload occurs the magnetism of the coil is increased until it is able to lift the plunger from its stop, causing it to strike a trigger, which releases a lever that opens the blades of the switch. In order to obviate a destructive arc between these switch-blades two carbon blocks F F′ are arranged

Fig. 117. General Electric Automatic Circuit-Breaker.

to rub along the plates G and G′. The arc, instead of taking place between the switch-blades, forms between the carbons, which can be easily replaced when consumed. With the development of electrical circuits the automatic circuit-breaker has grown proportionally, until now it is a device capable of safely handling thousands of amperes and working with clock-like precision. Fig. 117 represents a well-known type of circuit-breaker manufactured by the General Electric Company, and may be obtained in various sizes capable of handling up to 10,000 amperes.

108. Circuit-breakers built on the general plan outlined may be designed for both continuous- and alternating-current circuits. The breaker illustrated in Fig. 118 is of the type manufactured by the Westinghouse Company, designed for a three-phase alternating-current circuit. The apparatus is mounted upon a substantial marble block, with marble shields placed between each of the poles, and, though built upon a large scale and capable of handling an enormous

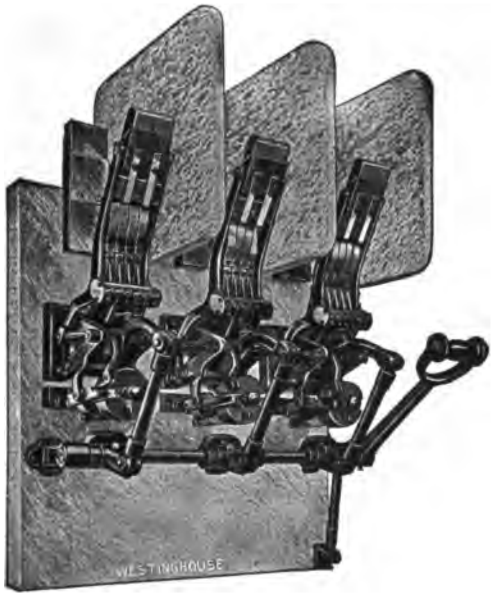


Fig. 118. Westinghouse Triphase Automatic Circuit-Breaker.

amount of current, does not differ in intrinsic principles from the early type of Fig. 116.

For circuits of high voltage and heavy currents it is difficult to handle the arc which is formed at the moment of rupture. For such circuits it is customary to immerse the blades of the circuit-breaker in oil and thus destroy the arc that otherwise would be destructive. Fig. 119 shows the Westinghouse design for this type of apparatus arranged for a triphase circuit. There are three fire-proof brick chambers, in each of which an oil-reservoir is placed in which the switch-blades are immersed. The switches are controlled by a system of levers operated by springs placed upon the

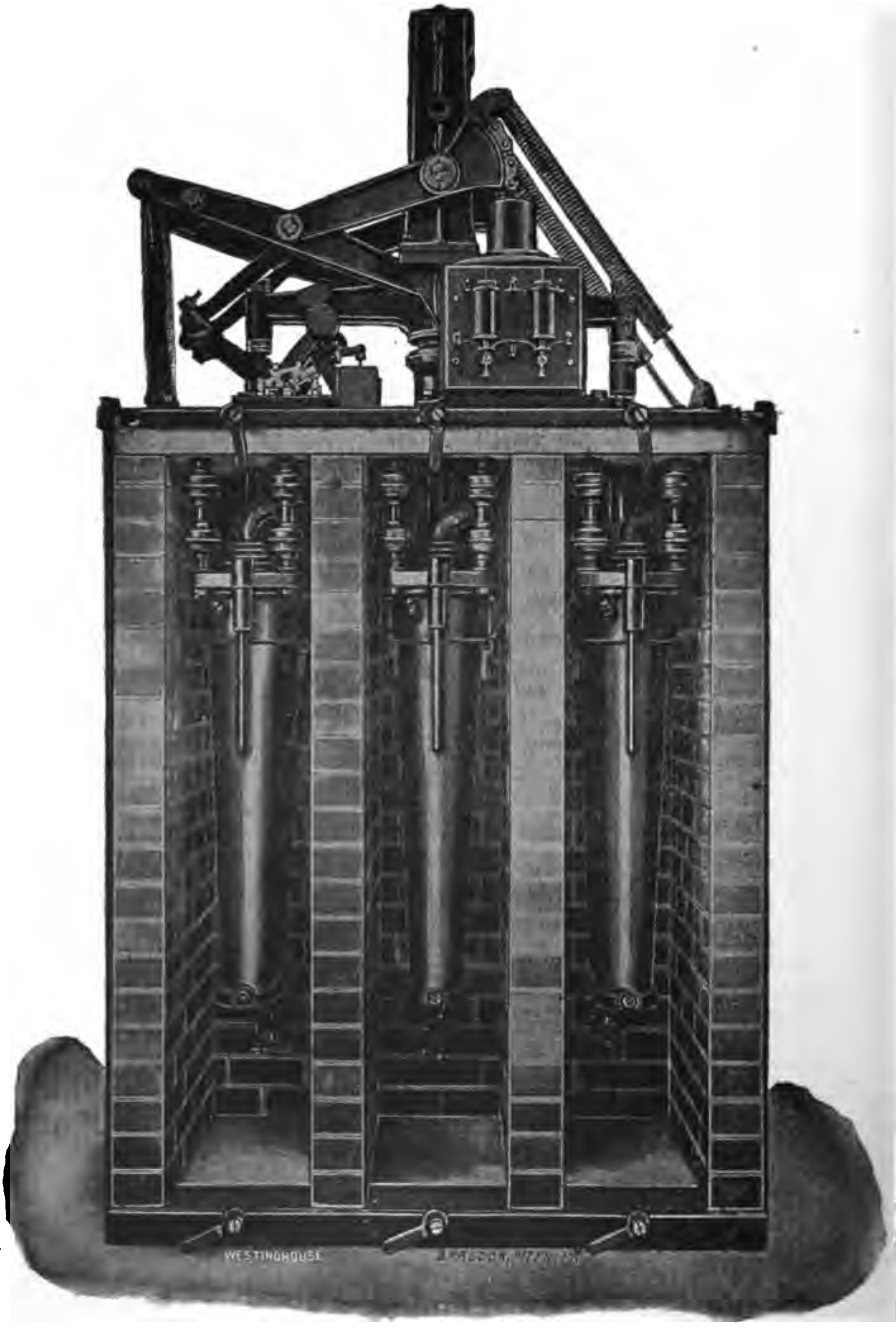


Fig. 119. Westinghouse Electrically Operated Oil Circuit-Breaker.

top of the oil-chamber. Sometimes compressed air is employed as the motive power for such circuit-breakers and the switches worked much after the fashion of a car-brake. But in all cases an electro-magnetic mechanism is provided, capable of adjustment so that the circuit-breaker can be arranged to open at any desired amperage. So great electrical and mechanical perfection has been developed that automatic circuit-breakers are now commonly manufactured, so that they may be adjusted to open upon a current f om one-half to twice the normal load of the circuit for which they are designed.

109. All electrical circuits, and particularly those fo: electric railways, are often subjected to momentary overloads, and if the circuit-breaker opened with every fluctuation of current, the interruptions to service would become intolerable. As such overloads last but for a few seconds, the circuit, unless the burden is exceptionally severe, is not injured thereby. To obviate this difficulty circuit-breakers are often supplied with what is called a *time-limit relay.* This consists of a spring-motor which can be so set that it will not allow the circuit-breaker to open unless the overload continues for a predetermined time, say from one to ten seconds. So the constant opening of the circuit due to a momentary overload (which if continued would become disastrous) is avoided, and yet the circuit is amply protected.

110. For circuits carrying small currents at very high potentials the magnetic circuit-breaker is not well adapted, and the device shown in Fig. 120 has proved itself superior and is largely used upon transmission lines carrying from 10,000 to 80,000 volts.

The circuit-breaker consists of two hardwood poles, one longer than the other, mounted upon a marble base, to which the terminals of the mains are connected. The poles are connected by a hinge, so that they may drop into the positions shown in Figs. 120 and 121. The main arm is hinged at the bottom and carries on top two zinc jaws which clamp one end of a fuse. The upper end carries the switch-plate which fits into the jaws to which the terminals of lines are attached. The fuse extends from the top of the main arm through an insulated tube containing a non-conducting, non-arcing powder, and is attached to the top of the second arm. In case of overload the fuse is ruptured and at the instant after failure is drawn through the tube and thus the arc is extinguished. When the fuse fails the arm drops into the lower

position shown in Fig. 120, whereby an enormous gap is provided through which it is impossible for the arc to follow. In the normal posi-

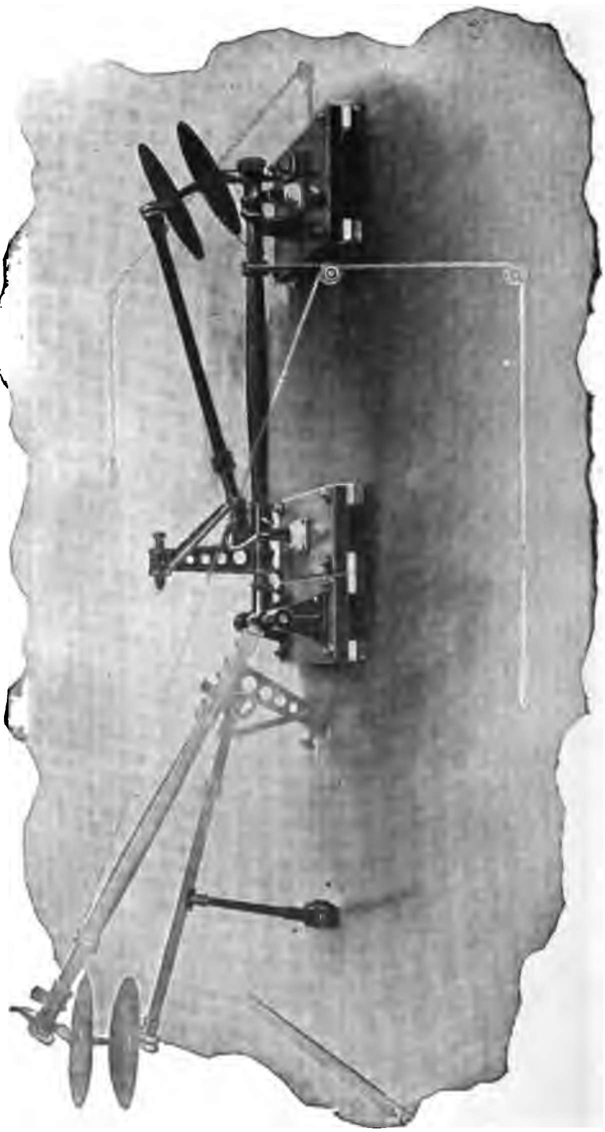


Fig. 120. High-Tension Circuit-Breaker. 1st and 3d Position.

tion, current entering at the upper terminal passes to the zinc jaw, then through the fuse to the auxiliary arm through a cable contained therein to the lower blade on the main arm, and thence through the lower terminal.

To release the switch intentionally, a pull upon a rope opens the jaws, setting one end of the fuse free, which is drawn through the tube, the auxiliary arm falling to the horizontal position, as shown in Fig. 121. This breaks the circuit, and the main arm may now be unlocked and swung to the position of Fig. 120.

The switch of Fig. 120 is designed for voltages of 50,000 to 80,000. Fig. 122 is one for potentials from 20,000 to 30,000, and

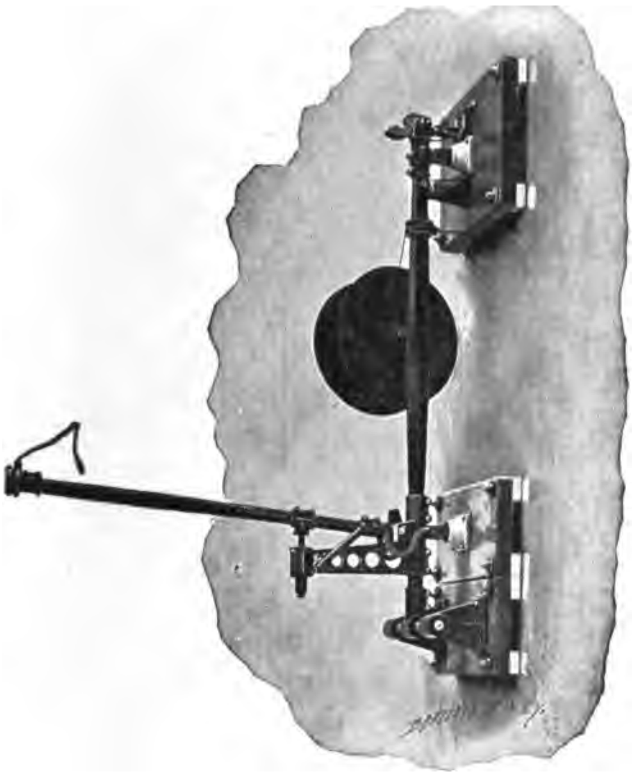


Fig. 121. High-Tension Circuit-Breaker. 2d Position.

termed a *plug* switch. It consists of a long wood handle, on the upper end of which is a contact-piece made to fit over a tapered terminal mounted near the top of the board, out of reach of the attendant. The contact-piece has a flexible cable, the other end of which connects with the lower terminal, inserted into the marble. The connection between cable and lower terminal is through the medium of a removable plug

surrounded by a hard-rubber handle protected by a rubber guard. A locking device prevents the circuit being broken at this point. The contact-piece at the top has a projection forming a slide between

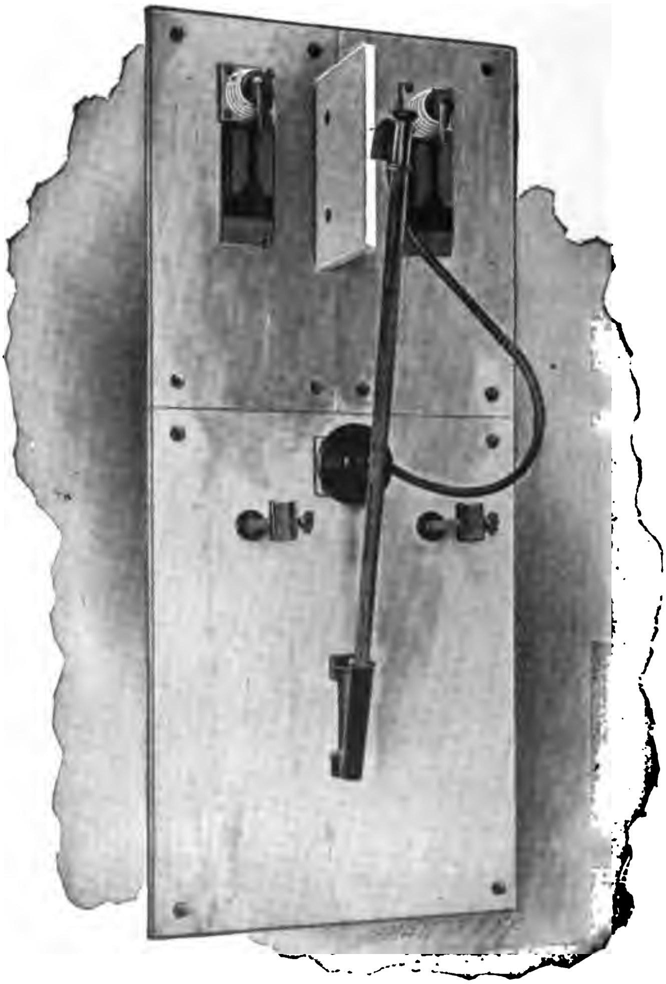


Fig. 122. High-Tension Circuit-Breaker.

the hard-rubber guide. A straight-line motion is thus assured and danger of breaking or bending the terminal removed.

The circuit is opened by pulling the handle down, thus releasing the contact-piece from the taper plug and then swinging it back over the shoulder, moving away from the board until the arc is ruptured.

An arc of considerable length may be formed without damaging the switch, as the breaking-points are zinc-tipped.

111. The earliest form of lightning-protector, or what is now sometimes (though erroneously) called a static protector, consisted in a small air-gap between either one or both sides of the circuit and the ground. Usually such air-gaps took the form of one or more serrated plates, and so long as the problem was merely the protection of telephone and telegraph circuits from atmospheric electricity this device answered after a fashion, though it was by no means satisfactory. Protectors of this kind were based upon a theory that the high potential of a discharge would jump the air-gap of the plates rather than traverse other apparatus possessing considerable impedance. It is now believed that the phenomena sometimes ascribed to the action of atmospheric electricity, sometimes to the resonant action of the lines, and sometimes to so-called "static" effect consist of a great number of very rapid electrical oscillations, and that the impedance of even a straight rod is sufficient to cause a discharge to spit off in all directions. So the first step in improvement of protectors of this nature was to increase the number of spark-gaps.

112. High-Resistance Arresters have been made, consisting of a number of thin metal plates, each separated from the other by means of a sheet of mica or other infusible insulating substance. The line is connected to the top plate, while the bottom plate is in electrical communication with the ground. On the occurrence of a lightning flash the enormous electromotive force of the discharge is supposed to enable the current to find its way over all of the insulating gaps and to ground, while the gaps introduce sufficient resistance to prevent the continuance of an arc after the atmospheric discharge has ceased. This form of lightning-arrester is exemplified in Fig. 123. From its simplicity this invention has received quite wide introduction.

Fig. 123. High-Resistance Arrester.

The life of this arrester, however, is short, owing to the fact that at each discharge globules of molten metal are likely to form on the edges of the disks that bridge across the insulating

gaps, and in a short time completely eliminate them, thus short-circuiting the line to earth.

An improved form of this protector is shown in Fig. 124, in which *A* is an elevation and *B* a section. There are two nests of concentric

Fig. 124 A. Improved Spark-Gap Protector.

cylinders secured by an insulating porcelain base. The line is connected to the inside cylinder, while the outer ones are grounded. Each gap is $\frac{1}{16}$ inch wide; the end of each cylinder flares; and each is perforated with a number of holes to allow free access of air.

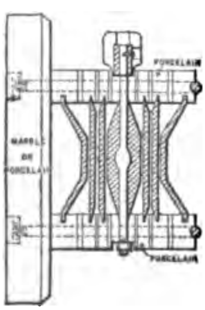


Fig. 124 B.

In the operation of the arrester the lightning enters from the line to the central cylinder, jumping the gaps in the narrower portions of the arrester and passing from the outer cylinder to the ground connection. If the dynamo current follows the lightning, a current of air is at once established through the perforations of the porcelain caps and between the cylinders. This air-current blows the arc upward into the spaces between the flaring ends of the cylinders, where the arc is instantly ruptured on account of the increased distance between the

conical tops and on account of the increased surface of metal exposed.

113. The Magnetic Blow-Out Arrester. — This apparatus, invented by the Thomson Houston Company, is represented in Fig. 125. It consists of a coil of wire, in series with the line and apparatus to be protected, forming the helix of an electro-magnet, between the poles of which are placed two cam-shaped pieces of metal, one being connected to the line and the other to earth, and which are separated by a small air-gap. Under an atmospheric discharge the electro-magnet presents sufficient impedance to divert the flash, and make it cross the air-gap between the cam-shaped pieces of metal, while the arc which is thus formed, being in a strong magnetic field, is extinguished by the action of the electro-magnet. For central-station work this has proved an efficient and valuable instrument.

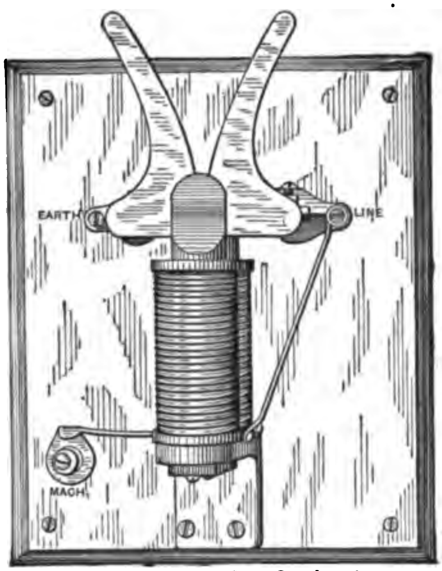


Fig. 125. Magnetic Blow-Out Arrester.

114. There seem to be three ways in which a thunder-cloud may affect a transmission line. First, by direct discharge from the cloud to the line. This is probably of very rare occurrence and would usually be of sufficient severity to render protection impracticable. Second, by electro-magnetic induction; that is, a flash in the vicinity of a line might create a sufficient electro-magnetic field to produce a disturbance upon a neighboring wire, but it is very doubtful if this ever occurs. Lastly, there is the effect of electro-static induction, which probably takes place frequently. The whole of a transmission line and its accompanying apparatus may be regarded as a conductor insulated from the earth.

Suppose a heavily charged cloud to move toward the transmission line; by the laws of electro-static induction there will be a charge, say positive, set free upon the transmission system. If the advent of the

cloud be gradual, this charge will leak to earth through the insulation without inflicting any injury; if not, it will either spit off, or will find some weak point in the insulation to puncture with a corresponding

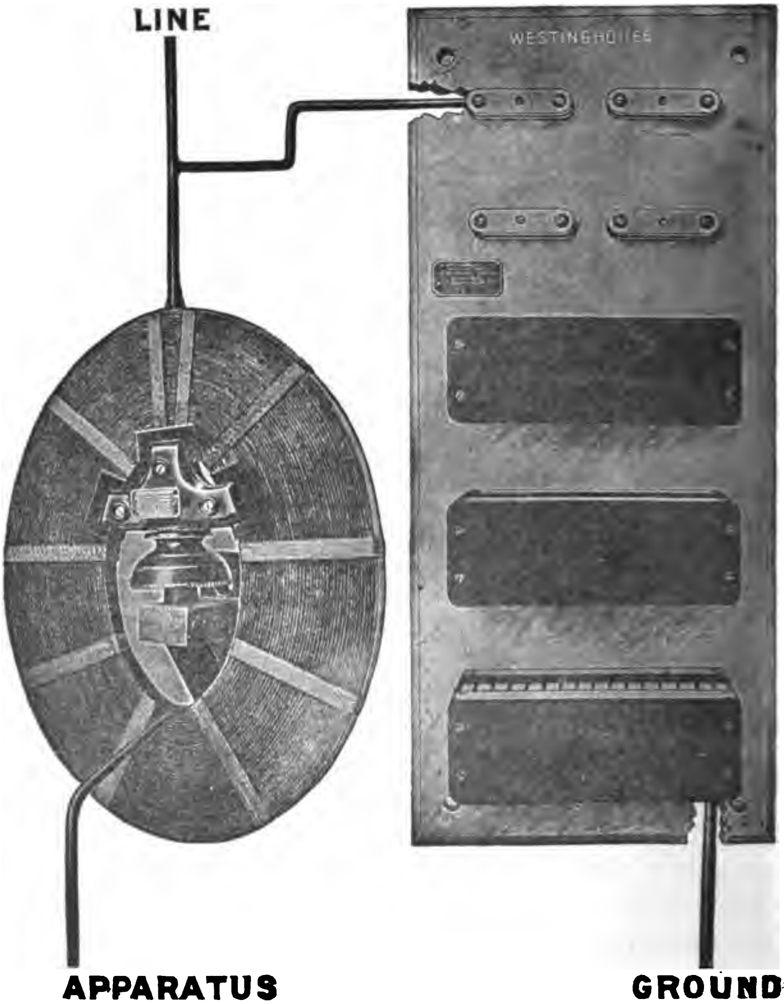


Fig. 126. Westinghouse Lightning-Arrester.

injury. Assume now that the cloud suddenly discharges itself by a flash; the negative static charge which was previously induced on the transmission line was bound by the presence of electricity in the cloud, but is now set free and surges to and fro on the line in an endeavor to seek

earth. Either the insulation of the line must be strong enough to withstand these pulsations until they die away, or a harmless path must be provided for their discharge, or the insulation of the line or some of its apparatus will be punctured and destroyed. It is the object of the modern lightning-arrester to provide for such contingencies, and the proper function of the lightning-arrester is to obviate such an abnormal rise of potential in any part of the circuit or translating devices as will exceed the proper factor of safety of the insulation. The modern arrester essentially consists of a number of specially designed spark-

Fig. 127. Spark-Gap for Lightning-Arrester.

gaps between the line and the ground, while all apparatus is protected by choking-coils. In this highly developed form the protector is shown in Fig. 126. It consists of a marble panel, carrying the spark-gaps to which the line is attached by a short straight wire. On the apparatus side a choking-coil is placed as shown. The apparatus upon the marble slab consists of two sets of air-gaps. These gaps are made of non-arcing metal (whose peculiarities will be presently explained), mounted as shown in Fig. 127; while the circuit is shown in Fig. 128, from which it is seen that the line is connected to a set of series gaps. Succeeding this is another set of gaps which is shunted

by a resistance R, and lastly there is a non-inductive series resistance to ground. The operation is as follows: When the potential at A rises sufficiently the series gaps are broken down and the discharge takes place. If this is heavy, the shunt resistance R forces it to cross the shunted gaps and thence through the series resistance to ground. The arc which follows is then withdrawn from the shunted gaps by the shunt resistance, and is gradually suppressed in the series gaps by both resistances. By properly proportioning the number of gaps it is easy to construct such an arrester to discharge at almost any predetermined excess of potential over that normally designed for the

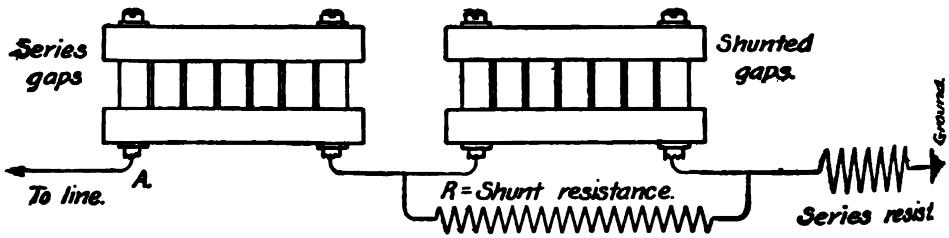


Fig. 128. Circuit for Lightning-Arrester.

line. Arresters of this type may be designed for any voltage from 1000 to 50,000 and have been highly successful.

Attention has been called to the fact that the sudden opening or closing of a switch or a short circuit or other resonant action may set up electrical surges in a transmission line that are likely to be highly injurious. Now these surges differ in nowise from the electrostatic phenomena produced by a lightning flash, and therefore the same apparatus which protects from atmospheric disturbances will also operate to guard the line from injurious electrical waves incident to its own operation.

115. The so-called tank lightning-arrester has been found particularly applicable to the direct-current system of electric railways. This arrester is illustrated in Fig. 129. Between the line and the generators a marble base is placed, containing a number of choke-coils. The coils are divided into several sections, and from each section a conductor passes to the tank. The tank is provided with an outlet so arranged that a stream of water may be kept running through it. A static discharge passing along the line encounters the impe-

dance of the choking-coils and follows the conductors to the water of the tank and thence to ground. With normally pure water the resistance of the liquid in the tank is so great that the leakage from the dynamos is reduced to a very few amperes, but this resistance is

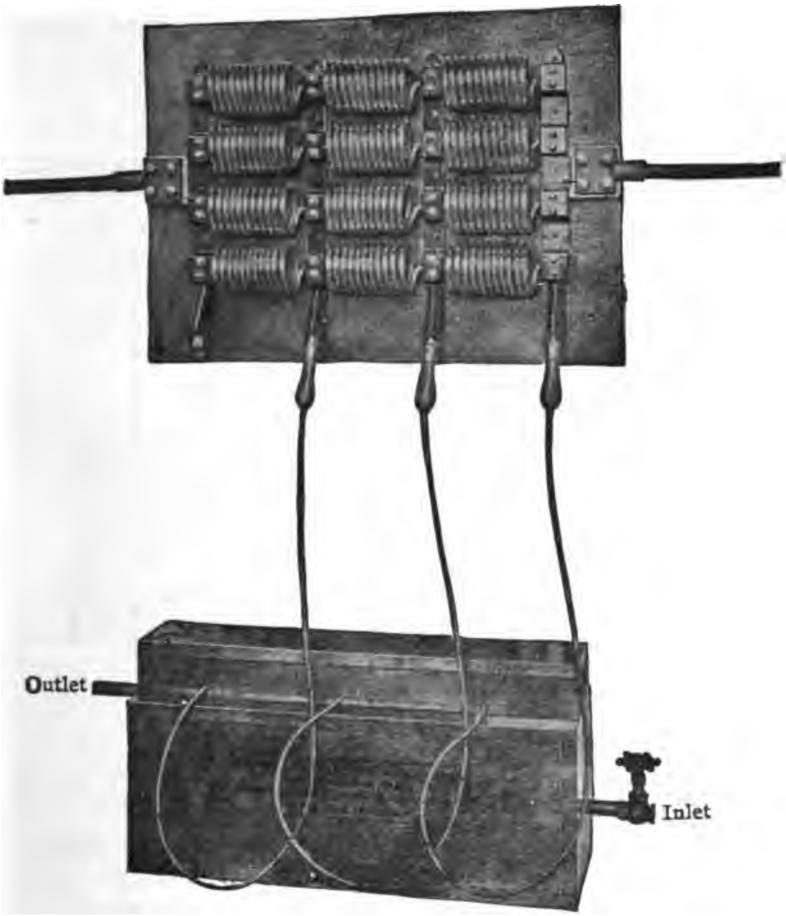


Fig. 129. Westinghouse Tank-Arrester.

entirely non-inductive, and to the static discharge means but little opposition. A single tank lightning-arrester of this description can carry about 1200 amperes, line current, but it is easy to use a number of such instruments in parallel and build up an almost indefinite capacity. Obviously such arresters need only be used when a thunder-

storm is anticipated, and by turning all the water off all leakage may be avoided during fair weather.

116. Non-Arcing Metal Arresters.— While experimenting on the subject of lightning-arresters, Mr. Wurtz made the singular discovery that metals of a certain chemical group would not allow of the formation or continuation of an arc. This singular and fortunate discovery has led to the invention of a lightning-arrester made of non-arcing metals, as shown in Fig. 130.

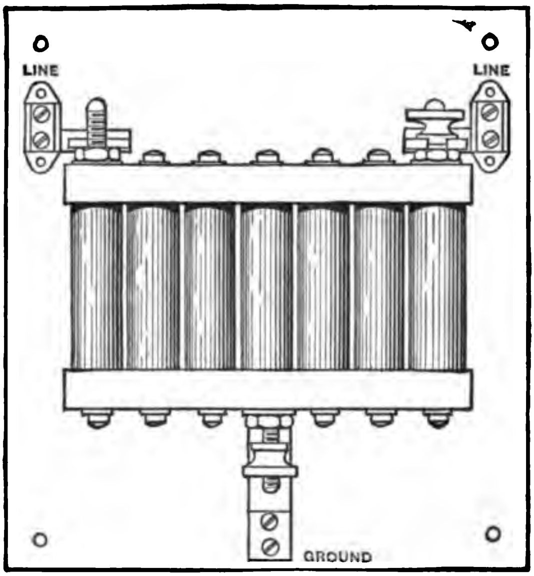


Fig. 130. Non-Arcing Arresters.

From the illustration it will be seen that upon a slate base there are placed seven metallic cylinders, the exterior cylinders on each side of the center being connected with the line, while the central cylinder is grounded, the intermediate cylinders forming a series of air-gaps. The cylinders are separated by about the thirty-second of an inch. When a flash takes place, the discharge crosses the air-gaps, seeking the center cylinder, and thence to earth. Mr. Wurtz's discovery lies in the fact that certain metals of the cadmium group do not permit the continuance of an arc, probably due to the fact that when the spark first crosses the gaps it volatilizes and oxidizes a certain amount of the metal, and extinguishes itself by interposing a

non-conducting medium between the two surfaces. The lightning-arrester here described is chiefly valuable in alternating-current work, as it is found that with continuous-current machines the arc is not entirely extinguished, though it is rendered comparatively harmless.

117. Discriminating Arresters. — In a paper before the American Institute of Electrical Engineers, Mr. A. J. Wurtz illustrates two forms of what he terms "discriminating lightning-arresters."[1]

The invention of a non-arcing lightning-arrester, by Mr. Wurtz, seemed to present an exceedingly satisfactory method of protection for alternating-current circuits. On continuous-current circuits, however, this lightning-arrester does not entirely extinguish the arc.

A continued study of the subject led to the invention of a still more simple non-arcing constant-current arrester, which for simplicity and

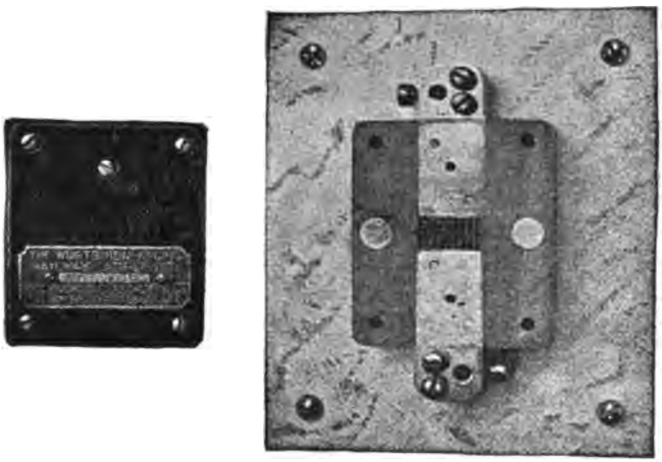


Fig. 131. The Wurtz Non-Arcing Continuous-Current Arrester.

cheapness, as well as for effectiveness in action, seems to be almost unsurpassable. This arrester is illustrated in Fig. 131, and consists of two blocks of marble about 3″ wide, $3\frac{1}{2}$″ long, and 1″ thick. In the lower block two brass electrodes 1″ wide are laid and finished flush with the surface, the distance between the ends of the electrodes being about $\frac{1}{2}$″. This space is occupied by a series of blocks of lignum vitæ, which are thoroughly charred, or contain a series of charred grooves about $^1/_{16}$″ wide and $^1/_{32}$″ deep. The outer marble block is

[1] See Transactions of American Institute of Electrical Engineers, May, 1894.

intended simply to act as a cover, and to protect the apparatus from injury. The charred wood seems to form a conducting path of so high resistance as to prevent the dynamo voltage from crossing it, and yet forms a conductor of sensibly low resistance to the high potential of the lightning flash. Furthermore, the carbon being essentially non-volatile, even under the temperature of the electric spark, no adequate metallic vapors are formed to sustain the arc that is initiated by the flash. As a result, the electrical oscillations set up in the conducting circuit by an electric storm pass with comparative readiness across the carbon blocks, which are of sufficiently high resistance to extinguish and prevent any dynamo arc.

118. Telephone Cable and Switchboard Protectors.—Underground cables have been severe sufferers from foreign currents of sufficient

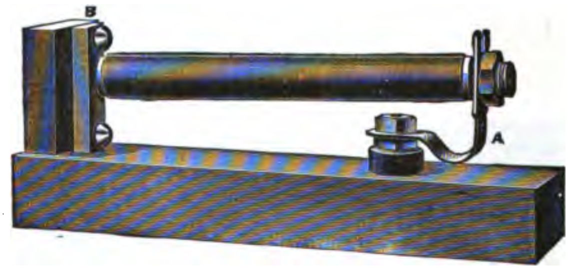


Fig. 132 A. Cable Protector.

magnitude to fuse the lead covering of the cable and entirely destroy them. To afford a protection, a special fuse device, indicated in Fig. 132 A, has been put in service upon cable-heads forming the junction between aerial lines and underground systems. The illustration shows a single protector, the entire cable being secured by arranging a sufficient number of these protectors, one after another, to correspond to the number of wires. The protector consists of a tube of vulcanized fiber or india-rubber about 6″ in length; the ends of the tube carry metallic bushings, one end being attached to a spring A, to which the aerial line is connected, while the other end is secured to a metallic block B, in electrical connection with the wire in the cable. Through the insulating-tube extends a lead fuse arranged to blow at any desired current, forming the only connection between the cable and aerial line. As the end B is sealed, while the end A is open to the atmosphere, the blowing of the fuse

causes a disruptive charge, forcing the products of the fusion of the lead wire violently out into the air, thus extinguishing the arc. Experience with this protector has been gratifying, and such apparatus appears in many different forms.

119. Switchboard Arresters. — At the switchboard end of Telegraph and Telephone lines, an endeavor has been made to combine a lightning arrester with a sneak current protector, by means of the device indicated in Fig. 132 B.

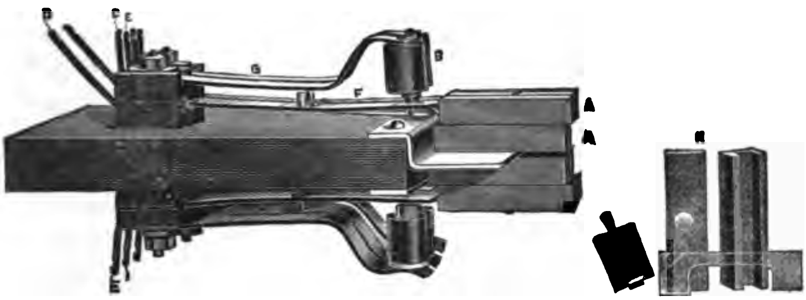


Fig. 132 B. Switchboard Protector.

The contrivance consists of two parts, — a lightning arrester made of two carbon plates, seen at AA, and a sneak current arrester in the form of a fusible heat-coil at B. One wire of each pair of cable wires enters the terminal C, the other the terminal D. The wire D passes through spring F to spring G, and thence to the switchboard through E. Spring F, however, is in contact with two peculiarly shaped carbon plates. These carbon plates are shown in detail at H. The plates consist of two little blocks of carbon about $\frac{1}{8}''$ thick, one having a groove in the top into which the spring fits. The lower carbon rests upon a metal plate that is thoroughly grounded, and the carbon in addition has a small central concavity filled with a drop of fusible metal. The two carbons are separated by a sheet of mica about $\frac{1}{200}''$ in thickness, cut out in such a manner as to bring the drop of fusible metal in the lower carbon almost in contact with the upper carbon plate. A flash of lightning entering D is supposed to follow through spring F to the carbon plate, and then to jump the small air-gap presented by the film of mica, and go to ground through the lower carbon plate. In case the flash should be of sufficient intensity to cause a sensible

amount of heat, the drop of fusible metal is melted, and flows across the gap formed by the mica, thus shorting the carbon plates and dead-grounding the entire combination, and affording protection to the switchboard.

120. In the case of a sneak current, there might be insufficient voltage to cause the discharge to leap the $\frac{1}{100}''$ of the mica gap, and yet a sufficient quantity of current might be presented to injure the switchboard. To avoid this, the heat-coil B is introduced in the circuit between the springs G and F. This heat-coil consists of a fine German silver wire wrapped around a small metal plug, which is held in its place by a drop of solder having an exceedingly low melting-point. The heat-coil is individually shown at H, where the projecting metal point may be readily distinguished. A close inspection of the cut indicates that the heat-coil rests between the springs F and G, while the point projects through spring F, and rests upon a thin spring directly underneath spring F, over the ground plate, and yet not in contact with the same. Upon the passage through the heat, coil of a current of sensible magnitude, in the neighborhood of .5 amperes, there is sufficient heat evolved in the fine German silver wire to melt the drop of solder, thus allowing the tension of spring G to force the pin through the coil, down upon the auxiliary spring, pressing this spring to metallic contact with the ground plate, thus dead-grounding the system, short-circuiting the switchboard, and preventing the sneak current from injuring the apparatus. Inasmuch as the heat-coils are found to ground the line with the passage of .5 amperes, and as most telephonic and telegraphic apparatus can, for some time, resist a current of 1 ampere, it seems that this device furnishes fairly reasonable protection to switchboard apparatus. The other half of the cable pair, entering by the terminal C, passes, by means of an insulated bolt, through the iron frame on which the apparatus is placed, to a second set of springs, heat-coils, and carbon plates, arranged to duplicate the first set, and passes to the switchboard by E'. Thus both wires of a metallic line are protected.

APPENDIX TO CHAPTER V.

INSURANCE REGULATIONS FOR THE INSTALLATION OF CIRCUITS.

THE following extracts from the National Electrical Code give a consensus of the best expert opinion as to precautions desirable in the construction of electric circuits.

121. GENERAL ARRANGEMENT OF RULES.

Class A. — CENTRAL STATIONS, dynamo, motor, storage-battery rooms, and transformer sub-stations, Rules 1 to 11.

Class B. — OUTSIDE WORK, Rules 12 to 13.

Class C. — INSIDE WORK, Rules 14 to 39.

a. GENERAL INSTRUCTIONS, all systems and voltages, Rules 14 to 17.

b. CONSTANT-CURRENT SYSTEM, Rules 18 to 20.

c. CONSTANT-POTENTIAL SYSTEMS.

1. All voltages, Rules 21 to 23.
2. Voltages not over 300, Rules 24 to 31.
3. Voltages between 300 and 3,000, Rules 32 to 37.
4. Voltages over 3,000, Rules 38 and 39.

Class D. — SPECIFICATIONS FOR WIRES AND FITTINGS, Rules 40 to 55.

Class E. — MISCELLANEOUS, Rules 56 to 59.

Class F. — MARINE WIRING, Rules 60 to 72.

122. Class A. 1. GENERATORS MUST BE —

a. Located in a dry place.

b. Never placed where hazardous processes are carried on or where exposed to inflammable gases or flyings of combustible material.

c. Insulated upon floors or base-frames kept filled to prevent the absorption of moisture, and must be kept clean and dry. If base-frame insulation is impractical, the metal frame should be permanently and thoroughly grounded. If frame insulation is impractical, the machine should be surrounded with insulated platforms to prevent injury to operators.

d. Protected by approved safety fuses.

e. Supplied with waterproof covers when not in use.

f. Supplied with plate, stating maker, capacity in volts, amperes, and normal speed, R. P. M.

123. 2. CONDUCTORS, EXTENDING FROM GENERATORS TO SWITCHBOARDS OR OTHER INSTRUMENTS AND THENCE TO OUTSIDE LINES, MUST —

a. Be in plain sight or readily accessible.

b. Have approved insulation as called for by rules in Class C. In central stations exposed circuits must have a heavy braided non-combustible outside covering. Bus bars may be made of bare metal.

c. Be rigidly placed.

d. In all other respects be installed as required by rules in Class C.

124. 3. SWITCHBOARDS.

a. Must be so placed as to prevent danger of communicating fire to adjacent material.

b. Must be made of non-combustible material or of hard wood in skeleton form, filled to prevent absorption of moisture.

c. Must be accessible from all sides when wired from the back, but may be placed against a non-combustible wall when wired on the face.

d. Must be kept free from moisture.

e. Bus bars must be equipped in accordance with Rules for conductors.

4. Resistance Boxes and Equalizers.

a. Should be placed upon switchboard, or, if not, placed at least one foot from all combustible material, and protected by non-inflammable, non-absorptive, insulating material.

5. Lightning-Arresters Must Be —

a. Attached to each side of every overhead circuit.

b. Located in accessible places, away from combustible materials, and as near as possible to the point where the wires enter the building. Station-arresters should be placed upon the switchboard. Kinks, coils, sharp bends in wires, between the arresters and outside lines, must be avoided.

c. Provided with a good, permanent ground by conductors equal in conductivity to a No. 6 B. & S. copper wire. Ground conductors should be run in straight lines from the arresters, and never attached to gas-pipes. Choke-coils can be introduced between the arresters and the dynamos. Ground wires should not be inclosed in iron pipes.

125. 6. Care and Attendance.

a. Competent employees must be provided for all dynamo machinery.

b. Oily waste must be kept in metal waste-cans, with legs raising the bottoms at least 3 inches from the floor, and with self-closing covers; remove waste daily.

7. Testing.

a. All circuits must be provided with reliable, preferably automatic ground detectors, never grounded to gas-pipes within buildings, and —

b. Where automatic detectors are not practicable, tested daily.

c. Data from tests must be preserved for Inspection Department.

126. 8. Motors.

a. The general provisions for generators, *a* to *f* inclusive, apply to motors.

b. Must be wired with the same precautions required by rules in Class C.

c. The motor and its resistance-box must be protected by cut-out and controlled by switch, plainly indicating whether current is on or off. For one-quarter horse-power, or smaller motors on low-tension circuits, a single-pole switch will suffice. The switch and rheostat must be located within sight of the motor.

d. Rheostat or starting boxes must conform to Rule 4.

e. Must not be run in series-multiple or multiple-series.

f. Must be provided with waterproof cover when not in use.

g. Electric ceiling-fans must be hung from insulated supports.

127. 9. Railway Power Plants.

a. Each feed-wire before it leaves the station must be equipped with an automatic circuit-breaker, mounted upon fireproof base in full view and easy reach of the attendants.

128. 10. Storage or Primary Battery Installations.

a. When current for light or power is taken from primary or secondary batteries, the same general regulations must be observed as applied to

similar apparatus fed from generators developing the same difference of potential.

b. Storage-battery rooms must be thoroughly ventilated.

c. Secondary batteries must be insulated.

d. Metal connections liable to corrosion must be avoided.

129. **11.** TRANSFORMERS —

Must be so placed that the burning of the coils or boiling over of the insulating-oil can do no harm.

130. **Class B.** OUTSIDE WORK. **12. Wires.**

a. Service-wires must have approved rubber insulation. Line-wires must have approved weatherproof or rubber insulation. Tie-wires must have an insulation equal to that of the conductors they confine.

b. Must be so placed that moisture cannot form a cross between them, not less than one foot apart, and not in contact with any substance other than their insulating supports. Service-blocks must be covered with two coats of waterproof paint.

c. Must be at least seven feet above the highest point of flat roofs, and at least one foot above the ridge of pitched roofs.

d. Must be protected by dead insulated guard iron or wires from possibility of contact with other conducting wires or substances to which current may leak.

e. Must be provided with petticoat insulators only, glass or porcelain.

f. Must be so joined as to be both mechanically and electrically secure without solder; joints must then be soldered, and covered with an insulation equal to that of the conductors.

g. Must, where they enter buildings, have drip-loops, and the holes bushed with non-combustible, non-absorptive tubes slanting upward toward the inside.

h. Telegraph, telephone, or similar wires must not be placed on the same cross-arm with electric light or power circuits.

i. The metal sheaths of cables must be permanently grounded.

Trolley Wires.

j. Must not be smaller than No. 0 B. & S. copper or No. 4 B. & S. silicon bronze, and must readily stand the strain put upon them when in use.

k. Must have a double insulation from ground. In wooden-pole construction, the pole will be considered as one insulation.

l. Must be capable of being disconnected at the power-plant or of being subdivided into sections. This rule also applies to feeders.

m. Must be protected against accidental contact where crossed by other conductors.

13. Transformers.

a. Must not be placed inside of any building excepting central stations.

b. Must not be attached to the outside wall of buildings unless separated by substantial supports.

131. **Class C.** INSIDE WORK. **a.** *General Instructions.* **14. Wires.**

a. Must not be smaller than No. 14 B. & S., except as specified under 24-u and 40-c.

b. Tie-wires must have an insulation equal to that of the conductors.

c. Must be joined so as to be mechanically and electrically secure without solder, and then soldered, and covered with an insulation equal to that of

the conductors. Stranded wires must be soldered before being fastened under clamps or binding-screws; and, when they have a greater conductivity than No. 10 B. & S. copper wire, they must be soldered into lugs.

d. Must be separated from contact with all portions of the building through which they pass by non-combustible, non-absorptive, insulating-tubes.

e. Must be kept free from contact with all other conducting material by a continuous, firmly fixed non-conductor, providing a separation of at least one inch.

f. Must be so placed in damp places that moisture cannot form crosses between conductors or between the conductors and other things.

15. Underground Conductors.

a. Must be protected, when entering buildings, against moisture and mechanical injury, and all combustible material kept removed from the immediate vicinity.

b. Must not be so arranged as to shunt the current around any protective device.

132. **16.** TABLE NO. 40 OF CARRYING CAPACITY OF WIRES.

B. & S. G.	Table A. Rubber-Covered Wires. *See No. 40, a.* Amperes.	Table B. Weather-proof Wires. *See No. 40, b.* Amperes.	Circular Mills.	Table A. Rubber-Covered Wires. *See No. 40, a.* Amperes.	Table B. Weather-proof Wires. *See No. 40, b.* Amperes.
18	3	5	200,000	200	300
16	6	8	300,000	270	400
14	12	16	400,000	330	500
12	17	23	500,000	390	590
10	24	32	600,000	450	680
8	33	46	700,000	500	760
6	46	65	800,000	550	840
5	54	77	900,000	600	920
4	65	92	1,000,000	650	1,000
3	76	110	1,100,000	690	1,080
2	90	131	1,200,000	730	1,150
1	107	156	1,300,000	770	1,220
0	127	185	1,400,000	810	1,290
00	150	220	1,500,000	850	1,360
000	177	262	1,600,000	890	1,430
0000	210	312	1,700,000	930	1,490
			1,800,000	970	1,550
			1,900,000	1,010	1,610
			2,000,000	1,050	1,670

133. **17.** SWITCHES, CUT-OUTS, AND CIRCUIT-BREAKERS.

a. Must be so arranged that each cut-out switch or circuit-breaker will disconnect all the wires of the circuit to which it is attached.

b. Must not be placed in the immediate vicinity of inflammable material.

c. Must, when exposed to dampness, be inclosed in waterproof box or mounted on non-absorptive material.

134. (*b*) CONSTANT-CURRENT SYSTEMS. **18. Wires.**

a. Must have an approved rubber insulation.

b. Must be arranged to enter and leave buildings through an approved double-contact service switch kept free from moisture and of easy access. Snap-switches must not be used.

c. **Must always be arranged in plain sight and not incased.**

d. Must be supported upon glass or porcelain insulators, which separate the wire at least one inch from surface, wired over, and be kept rigidly eight inches from each other, except within the structure of lamps or on hanger-boards.

e. Must, on side walls, be protected from mechanical injury by boxing inclosing an air-space of at least one inch all around the conductors. The boxing must be closed at the top, the wires passing through bushed holes, and must extend not less than seven feet from the floor. When crossing floor-timbers where the conductors might be exposed to injury, wires must be attached by their insulating supports to the under side of a wooden strip not less than one-half an inch in thickness.

19. Arc Lamps.

a. Must be carefully isolated from inflammable material.

b. Must always be provided with whole-glass globes inclosing the arc, securely fastened on a closed base.

c. Must be provided with a wire netting of not over 1¼ inch mesh and an approved spark-arrester, when inflammable material is in the vicinity of the lamps.

d. Where hanger-boards are not used, lamps must be hung from insulating supports other than their conductors.

20. Incandescent Lamps in Series Circuits.

a. Must have the conductors installed as provided for in Rule 18. Each lamp is to be provided with an automatic cut-out, and

b. Suspended from a hanger-board by means of a rigid tube.

c. Electro-magnetic devices for switches and systems of multiple-series or series-multiple lighting must not be used.

d. Must not be attached to gas-fixtures.

135. (*c*) CONSTANT-POTENTIAL SYSTEMS, ALL VOLTAGES.

21. Automatic Cut-outs.

a. Must be placed on all service-wires as near as possible to the point where they enter the building, inside the walls, and arranged to cut off the entire current.

b. Must be placed at every point where change is made in the size of the wire, unless the cut-out in larger wire will protect the smaller.

c. Must be in plain sight, or inclosed in an approved box readily accessible. Must not be placed in the canopies or shells of fixtures.

d. Must be so placed that no set of incandescent lamps requiring more than 6 amperes shall be dependent upon one cut-out.

e. Must be provided with fuses, the capacity of which does not exceed the allowable carrying capacity of the wire. Circuit-breakers must not be set more than 30 per cent above the allowable carrying capacity of the conductors they protect.

22. Switches.

a. Must be placed on all service-wires, in a readily accessible place, as near as possible to the point where the wires enter the building, and arranged to cut off the entire current.

b. Must always be placed in dry, accessible places, and grouped as far apart as possible. Knife-switches must be so placed that gravity will tend to open the switch.

c. Must be double-pole except when the circuits they control supply 3 amperes or less.

d. Gangs of flush-switches must be inclosed in boxes of fire-resisting material. Where two or more switches are placed under one plate, the box must have a separate compartment for each. No push-buttons for bells, gas-lighting circuits, etc., shall be placed in the same wall-plate with switches controlling lighting or power circuits.

23. Electric Heaters.

a. Must be placed at a safe distance from inflammable material, and be treated as sources of heat.

b. Each must have a cut-out and indicating-switch.

c. Must have the attachment of feed-wires in plain sight, easily accessible, and protected from interference.

d. Flexible conductors for portable apparatus must have an approved insulating covering.

e. Each must be provided with a name-plate giving maker and normal capacity in volts and amperes.

136. 2. LOW-POTENTIAL CIRCUITS.

Voltages not over 300. (Any circuit which develops a difference of potential between 10 and 300 volts shall be considered a low-potential circuit.)

24. Wires.

a. Must not be laid in cement, plaster, or similar finish.

b. Nor fastened with staples.

c. Nor be fished for any great distance, and only where inspectors can be satisfied that rules are complied with.

d. Twin wires must not be used excepting in conduits or where flexible conductors are necessary.

e. Must be protected on side walls from mechanical injury, and protected when crossing floor-timbers by attaching the wires by their insulating supports to the under side of a wooden strip, not less than one-half inch thick and not less than three inches wide.

f. When run immediately under roofs, or near water pipes or tanks, they will be considered as exposed to moisture.

Special Rules for Dry Places.

g. Must have an approved rubber or weatherproof insulation.

h. Must be rigidly supported on non-combustible, non-absorptive insulators, which separate the wires at least one-half inch from the surface wired over, and two and one-half inches from each other.

Special Rules for Damp Places.

i. Must have an approved rubber insulation.

j. Must be rigidly supported on insulators which separate the wires at least one inch from the surface wired over, and two and one-half inches from each other.

k. Must have no joints or splices.

For Molding Work.

l. Must have an approved insulation.

m. Must never be placed in molding in concealed or damp places.

For Conduit Work.

n. Must have approved insulation.

o. Must not be drawn in until all mechanical work on the building is completed.

p. Must not have wires of different circuits drawn in the same conduit.

q. Must, for alternating systems, have all wires of the same circuit drawn in the same conduit. This is advised for direct-current systems.

For Concealed Work.

r. Must have an approved rubber insulation.

s. Must be rigidly supported upon non-combustible, non-absorptive insulators, which separate the wires at least one inch from the surface wired over, and ten inches from each other.

t. When it is impossible to place concealed wiring on non-combustible supports, the wires, if not exposed to moisture, may be fished on the loop-system, if inclosed in approved continuous flexible tubing or conduit.

For Fixture Work.

u. Must have approved rubber insulation, and shall not be less than No. 18 B. & S. in size.

v. Supply conductors must be kept clear of the grounded parts of fixtures. Shells must be constructed in a manner to permit of this requirement.

w. Must, when fixtures are wired outside, be so secured as not to be cut or abraded by the motion of the fixtures.

25. Interior Conduits.

a. Must be continuous from one junction to another or to fixtures, and the conduit tube must properly enter all fittings.

b. Must be first installed as a complete conduit system without conductors.

c. Conduits must extend at least one-half inch beyond the finished surface of walls or ceilings, except that, if the end is threaded and a coupling screwed on, the conduit may be left flush.

d. Must, after conductors are introduced, have all outlets plugged with special wood or fibrous plugs, made in parts, and the outlet then sealed with approved compound. Joints must be air-tight and moisture-proof.

e. Must have the metal of the conduit permanently and effectually grounded.

26. Fixtures.

a. Must, when supported from the gas-piping, be insulated from the gas-pipe system by means of approved insulating joints.

b. Must have all burs or fins removed before the conductors are drawn into the fixture.

c. The upper end of all fixtures must be sealed moisture-proof.

d. Combination fixtures must not conceal the conductors in a space less than one-fourth inch.

e. Must test free from contacts between conductors and fixtures, from short circuits, and from grounds.

f. Ceiling-blocks should be made of insulating material; or if not, the wires passing through the plates must be surrounded with non-combustible, non-absorptive, insulation.

27. Sockets.

a. Where inflammable gases exist, the lamp and socket must be inclosed in a vapor-tight globe, wired with approved rubber-covered wire soldered directly to the circuit.

b. In damp places or over specially inflammable stuff, waterproof sockets must be used.

28. Flexible Cord.

a. Must have approved insulation and covering.

b. Must not be used as a support for clusters.

c. Must not be used except for pendants, wiring of fixtures, or portable lamps or motors.

d. Nor in show-windows.

e. Must be protected by insulating bushings where the cord enters the socket.

f. Must be so suspended that the entire weight of the sockets will be borne by knots under the bushing in the socket, and above the point where the cord comes through the ceiling-block or rosette.

29. Arc Lights on Low-Potential Circuits.

a. Must have a cut-out for each lamp or series of lamps.

b. Must be furnished with such resistances or regulators as are inclosed in non-combustible material. Incandescent lamps must not be used for resistances.

c. Must be supplied with globes, and protected by spark-arresters.

30. Economy Coils.

a. Economy and compensator coils must be mounted on non-combustible, non-absorptive insulating supports, allowing an air-space of at least one inch between the frame and support, and in general treated as sources of heat.

31. Decorative Series Lamps.

a. Incandescent lamps in series shall not be used excepting by special permission.

137. 3. HIGH-POTENTIAL CIRCUITS. *Voltages between 300 and 3,000.*

32. Wires.

a. Must have approved rubber insulation.

b. Must always be in plain sight, and never incased except where required by the inspection department.

c. Must be rigidly supported on glass or porcelain insulators carrying the wires at least one inch from the surface wired over, and at least four inches apart for voltages up to 750 and at least eight inches apart for voltages over 750.

d. Must be protected on side walls from mechanical injury by substantial boxing, retaining an air-space of one inch around the conductors, closed at the top, the wires passing through bushed holes, and extending not less than seven feet from the floor. When crossing floor-timbers wires must be attached by their insulating supports to the under side of a wooden strip not less than one-half inch in thickness.

33. TRANSFORMERS MUST BE PLACED —

a. At a point as near as possible to that at which the primary wires enter the building.

b. In an inclosure constructed or lined with fire-resisting material, this inclosure to be securely locked, and access allowed only to responsible persons.

c. Effectually insulated from the ground and in a practically air-tight inclosure, except that it shall be thoroughly ventilated to the outdoor air, and there must be a six-inch air-space on all sides of the transformer.

34. CAR-WIRING.

a. Must always be run out of reach of passengers, and have an approved rubber insulation.

138. 35. CAR-HOUSES.

a. Must have the trolley-wires securely supported on insulating hangers.

b. Must have the trolley-hangers placed at such a distance apart that in case of a break in the trolley-wire, contact could not be made with the floor.

c. Must have cut-out switch located at proper place outside the building, so that all trolley circuits in the building can be cut out at one point, and line circuit-breakers must be installed, so that when this cut-out switch is open the trolley-wire will be dead at all points within 100 feet of the building. The current must be cut out of the building whenever not in use.

d. Must have all lamps and stationary motors installed so that one main switch can control the whole of each installation, independently of main feeder-switch. No portable incandescent lamps or twin wires allowed except that portable incandescent lamps may be used in the pits, connections to be made by two approved flexible rubber-covered wires properly connected, and controlled by a switch placed outside the pit.

e. Must have all wiring and apparatus installed in accordance with rules under Class C.

f. Must not have any system of feeder distribution centering in the building.

g. Must have the rails bonded at each joint with not less than No. 2 B. & S. annealed copper wire; also a supplementary wire to be run for each track.

h. Must not have cars left with trolley in electrical connection with the trolley-wire.

36. LIGHTING AND POWER FROM RAILWAY WIRES.

a. Must not be permitted under any pretense in the same circuit with trolley-wires with a ground return except in electric railway cars, electric car-houses and their power-stations, nor shall the same dynamos be used for both purposes.

37. SERIES LAMPS.

a. No system of multiple-series or series-multiple for light or power shall be used.

b. Under no circumstances can lamps be attached to gas-fixtures.

4. *Voltages over 3,000.* 38. PRIMARY WIRES.

Must not be brought into or over buildings except power- and sub-stations.

39. SECONDARY WIRES.

a. Must be installed under Rules for high-potential systems when their immediate primary wires carry a current at a potential of over 3,000 volts.

139. **Class D.** FITTINGS, MATERIALS, AND DETAILS OF CONSTRUCTION.

All Systems and Voltages.

40. **Wire Insulation.**

a. **Rubber-Covered.** — The insulating covering must be solid, at least $\frac{3}{64}$ of an inch thick, and covered with a substantial braid. Must not readily carry fire, must show an insulation of one megohm per mile after two weeks' submersion in water at seventy degrees Fahrenheit, and three days' submersion in lime-water, and after three minutes' electrification with 550 volts.

b. **Weatherproof.** — The insulating covering must not support combustion, must resist abrasion, must be at least one-sixteenth of an inch thick, and thoroughly impregnated with a moisture-repellent.

c. **Flexible Cord.** — Must be of two-stranded conductors, each having a carrying capacity of not less than a No. 16 B. & S. wire, and each covered by an approved insulation, and protected by a slow-burning, tough-braided outer covering.

1. Insulation for pendants must be moisture and flame proof.
2. Insulation for all other purposes must be solid, at least $\frac{1}{32}$ of an inch thick, and must show an insulation resistance between conductors and between either conductor and the ground of at least one megohm per mile after one week's submersion in water at 70° Fahrenheit, and after three minutes' electrification with 550 volts.
3. The flexible conductors for portable heating-apparatus must have an insulation that will not be injured by heat, which must be protected from mechanical injury by an outer substantial braided covering, and so arranged that mechanical strain will not be borne by the electrical connection.

d. **Fixture Wire.** — Must have a solid insulation, with a slow-burning, tough outer covering, the whole to be at least $\frac{1}{32}$ of an inch thick, and show an insulation between conductors and between either conductor and ground of at least one megohm per mile, after one week's submersion in water at 70° Fahrenheit, and after three minutes' electrification with 550 volts.

e. **Conduit Wire.** — 1. For insulated metal conduits, single wires and twin conductors must comply with Section *a* of these Rules. Concentric wire must have a braided covering between the outer conductor and the insulation of the inner conductor, and must comply with Section *a*.

2. For non-insulated metal conduits, single wires and twin conductors must comply with section *a*, and in addition have a second outer fibrous covering at least $\frac{1}{32}$ of an inch thick, and sufficiently tenacious to withstand the abrasion of being hauled through the metal conduit. Concentric conductors must have a braided covering between the outer conductor and the insulation of the inner conductor, and comply with section *a* of this rule, and must have a fibrous covering $\frac{1}{32}$ of an inch thick, and sufficiently tenacious to withstand the abrasion of being hauled into the conduit.

140. 41. Interior Conduits.

a. Each length of conduit must have the maker's name or initials stamped in the metal so that inspectors can see the same.

Insulated Metal Conduits.

b. The metal covering must have an equal resistance to penetration by nails as the ordinary commercial form of gas-pipe of same size.

c. Must not be seriously affected by burning out a wire inside the tube when the metal is connected to one side of the circuit.

d. Must have the insulating lining firmly secured.

e. The insulating lining must not crack or break when a length of conduit is uniformly bent, at temperature of 212° Fahrenheit, to an angle of 90° with a curve of fifteen inches radius, for pipes of one inch and less, and fifteen times the diameter of pipe for larger pipes.

f. The insulating lining must not soften injuriously at a temperature below 212° Fahrenheit, and must leave the water in which it is boiled practically neutral.

g. The insulating lining must be at least $\frac{1}{32}$ of an inch thick, and the materials composing it must be of such a nature as will not have a deteriorating effect on the insulation of the conductors, and must be sufficiently tough to withstand the abrasion of drawing in and out of long lengths of conductors.

h. The insulating lining must not be mechanically weak after three days' submersion in water, and when removed from the pipe entire must not absorb more than ten per cent of its weight of water during 100 hours of submersion.

i. All elbows must be made for the purpose, and not bent from lengths of pipe. The inner radius of any elbow of pipe must not be less than 3½ inches. Must not have more than the equivalent of four quarter bends from outlet to outlet, the bends at the outlets not being counted.

Uninsulated Metal Conduits.

j. Plain iron or steel pipes of equal strength to resist penetration by nails as ordinary gas-pipe of the same size, provided the interior surfaces are smooth and free from burs, may be used. Pipe to be galvanized, or the interior surfaces coated, to prevent oxidization, with some substance which will not become sticky, and prevent wire from being withdrawn.

k. All elbows must be made, and not bent from lengths of pipe. The inner radius of any elbow must not be less than 3½ inches. Must have not more than the equivalent of four quarter bends from outlet to outlet, the bends at the outlets not being counted.

141. 42. Wooden Moldings.

a. Must have, both inside and outside, at least two coats of waterproof paint, or be impregnated with a moisture-repellent.

b. Must be made of two pieces, backing and capping constructed to thoroughly incase the wire, and provide a one-half inch tongue between conductors and a solid backing, which under-grooves shall not be less than three-eighths of an inch thick, and must afford suitable protection from abrasion.

142. 43. Switches.

a. Must be mounted on non-combustible, non-absorptive bases.

b. Must have carrying capacity sufficient to prevent undue heating.

c. Must, when used for service-switches, indicate whether the current is on or off.

d. Must be plainly marked with the name of the maker and the current and voltage for which the switch is designed.

e. Must, for constant-potential systems, operate successfully at 50 per cent overload, in amperes, with 25 per cent excess voltage under the most severe conditions to be met with in practice.

f. Must, for constant-potential systems, have a firm and secure contact; must make and break readily, and must not stop when motion has once been imparted by the handle.

g. Must, for constant-current systems, close the main circuit and disconnect the branch wires when turned "off"; must be constructed to be automatic in action, not stopping between points when started, and must prevent an arc under all circumstances. Must indicate whether the current is on or off.

143. 44. Cut-outs and Circuit-Breakers.

a. Must be supported on bases of non-combustible, non-absorptive insulating material.

b. Cut-outs must be provided with covers when not arranged in approved cabinets.

c. Cut-outs must operate successfully with fuses rated at 50 per cent above and voltage 25 per cent above that for which they are designed, under the most severe conditions of practice.

d. Circuit-breakers must operate successfully with a current 50 per cent above and a voltage 25 per cent above that for which they are designed, under the most severe conditions of practice.

e. Must be plainly marked with the name of the maker and current and voltage for which designed.

45. Fuses.

a. Must have contact surfaces or tips of harder metal having perfect electrical connection with the fusible part.

b. Must be stamped with about 80 per cent of the maximum current they can carry indefinitely.

c. Fuse terminals must be stamped with the maker's name, initials, or known trade-mark.

46. Cut-out Cabinets.

a. Must be so arranged as to obviate any danger from melted fuses.

144. 47. Sockets.

a. No portion of the lamp-socket exposed to contact with outside objects must be allowed to come into electrical contact with either conductor.

b. Must, when provided with keys, comply with the requirements for switches.

145. 48. Hanger-Boards.

a. Hanger-boards must be so constructed that all wires and current-carrying devices thereon shall be exposed to view, and thoroughly insulated by being mounted on a non-combustible, non-absorptive insulating substance.

All switches shall be double-pole automatic in action and strictly non-arcing.

146. 49. Arc Lamps.

a. Must be provided with reliable stops to prevent carbons from falling out, in case clamps become loose.

b. Must be carefully insulated from the circuit in all exposed parts.

c. Must, for constant-current systems, be provided with an approved hand-switch, also an automatic switch that will shunt the current if they fail to feed properly. The hand-switch, if placed anywhere except on the lamp itself, must comply with Rule 48.

147. 50. Spark-Arresters.

a. Spark-arresters must so close the globe that it will be impossible for any sparks to escape.

148. 51. Insulating-Joints.

a. Must be entirely made of material that will resist the action of illuminating gases, and will not soften under the heat of an ordinary gas-flame, or leak under moderate pressure, and arranged so that a deposit of moisture will not destroy the insulation, and shall have a resistance of not less than 250,000 ohms, and be sufficiently strong to resist the strain to which they will be subjected.

b. Insulating-joints employing soft rubber will not be allowed.

149. 52. Resistance Boxes and Equalizers.

a. Must be equipped with metal or with other non-combustible frames.

150. 53. Reactive Coils and Condensers.

a. Reactive coils must be made of non-combustible material, and treated as sources of heat.

151. 54. Transformers.

a. Must not be placed in any but metallic or non-combustible cases.

152. 55. Lighting-Arresters.

a. Must be mounted on non-combustible bases, and must be so constructed as not to maintain an arc after the discharge has passed, and must have no moving parts.

153. Class E. MISCELLANEOUS. 56. Insulation Resistances.

All wiring in buildings must test free from grounds, and must have an insulation between conductors and between all conductors and ground as follows. Circuits carrying current up to — TABLE No. 41.

5 amperes	4,000,000 ohms.	200 amperes	100,000 ohms.
10 amperes	2,000,000 ohms.	400 amperes	50,000 ohms.
25 amperes	800,000 ohms.	800 amperes	25,000 ohms.
50 amperes	400,000 ohms.	1,600 amperes, and over,	
100 amperes	200,000 ohms.		12,500 ohms.

All cut-outs and safety devices must be in place when test is made. When lamp-sockets, receptacles, and electroliers, etc., are connected, one-half of the above insulation will be required.

154. 57. Protection Against Foreign Currents.

a. Where telephone, telegraph, or other wires connected with outside circuits are bunched together within a building, and where inside wires are laid in ducts with lighting or power wires, covering of such wires must be fire-resisting or they must be inclosed in air-tight ducts.

b. All conductors under (*a*) which run to aerial lines must be provided with approved protective devices which will shunt the instruments in case of a dangerous rise of potential, and will open the circuit and arrest abnormal current. Protectors must have non-combustible insulating bases, and covers provided with a lock, and must be installed under the following requirements: —

1. Protectors must be located at the point where the wires enter the building, either immediately inside or outside the same. If outside, the protector must be inclosed in a metallic waterproof case.
2. If protectors are placed inside the building, the wires from the support outside to the binding-post of the protector shall be of a grade of insulation equal to that of electric light or power wires, and the holes through the outer wall must be bushed as for high-tension service.
3. The wires from the point of entrance to the protector must be run in accordance with rules for high-potential wires.
4. Ground wires shall be insulated, not smaller than No. 16 B. & S. Ground wires shall be kept at least three inches from all conductors, and run in as straight a line as possible to the ground.
5. Ground wires shall be attached to a water-pipe if possible, and shall be carried to and attached to the pipe outside the first joint inside of the foundation walls, and connection be made by soldering. In the

absence of other good ground, the ground shall be made by means of a metallic plate buried permanently in moist earth.

58. Electric Gas-Lighting.

Where electric gas-lighting is to be used on the same fixture with the electric light —

a. No part of the gas-piping or fixture shall be in electric connection with the gas-lighting circuit.

b. The wires shall have non-inflammable or, when concealed, such insulation as required for fixture-wiring for electric lights.

c. The whole insulation must test free from grounds.

d. The two insulations must test perfectly free from connection with each other.

155. Class F. MARINE WORK. **60. Generators must be —**

a. Located in a dry place.

b. Insulated from their bed-plates.

c. Provided with waterproof cover, and —

d. With name-plate, giving the maker, voltage, amperes, and normal speed R. P. M.

156. 61. Wires.

a. Must have an approved insulation, not less than $\frac{1}{8}$ inch thick for all conductors except portables, and covered with substantial water- and flame-proof braid. The physical characteristics shall not be affected by any change in temperature up to 200° Fahrenheit. After two weeks' submersion in salt water at 70° Fahrenheit, it must show an insulation resistance of one megohm per mile after three minutes' electrification with 550 volts.

b. Must have no single wire larger than No. 12 B. & S. Stranded conductors must be used when greater carrying capacity is required. No single solid wires smaller than No. 14 B. & S., excepting in fixture-wiring. Stranded wires must be soldered before being fastened under binding-screws. When they have a greater conductivity than No. 10 B. & S., they must be soldered into lugs.

c. Must be supported in approved molding except at switches and portables.

d. Must be bushed with hard-rubber tubing one-eighth of an inch thick when passing through beams and non-watertight bulkheads.

e. Must have, when passing through watertight bulk-heads and through all decks, a metallic-box tube lined with hard rubber. In case of deck-tubes they shall be boxed to prevent mechanical injury.

f. Necessary splices or taps must be made both electrically and mechanically secure without solder. They must then be soldered, and covered with an insulating compound equal to that of the wire, and protected by waterproof tape, and then painted with waterproof paint.

157. 62. Portable Conductors.

a. Must be made of two-stranded conductors, each having a carrying-capacity equivalent to No. 14 B. & S. wire, and covered with an approved insulation. When not exposed to moisture or severe mechanical injury, each conductor must have a solid insulation, at least $\frac{1}{32}$ of an inch thick, and must show an insulation between conductors and between each conductor and ground of at least one megohm per mile after one week's submersion in water at seventy degrees and after three minutes' electrification at 550 volts, and be protected by slow-burning, tough-braided

covering. Where exposed to moisture or mechanical injury, each conductor shall have at least $\frac{1}{32}$ of an inch solid insulation protected by tough braid. The two conductors shall then be stranded together, using a jute filling. The whole shall be covered with a layer of flax, at least $\frac{1}{32}$ of an inch thick, and treated with a non-inflammable, waterproof compound. After one week's submersion in water at seventy degrees, with 550 volts and a three minutes' electrification, must show an insulation between the two conductors, or between each conductor and the ground, of one megohm per mile.

63. Bell or Other Wires.

a. Shall never be run in the same duct with lighting or power circuits.

158. 64. Table of Capacity of Wires.

B. & S. G.	Area. Actual C. M.	No. of Strands.	Size of Strands. B. & S. G.	Amperes.	B. & S. G.	Area. Actual C. M.	No. of Strands.	Size of Strands. B. & S. G.	Amperes.
19	1,288	. .	. .	. .	. .	38,912	19	17	60
18	1,624	. .	. .	3	. .	49,077	19	16	70
17	2,048	. .	. .	. .	. .	60,088	37	18	85
16	2,583	. .	. .	6	. .	75,776	37	17	100
15	3,257	. .	. .	. .	. .	99,064	61	18	120
14	4,107	. .	. .	12	. .	124,928	61	17	145
12	6,530	. .	. .	17	. .	157,563	61	16	170
. .	9,016	7	19	21	. .	198,677	61	15	200
. .	11,368	7	18	25	. .	250,527	61	14	235
. .	14,336	7	17	30	. .	296,387	91	15	270
. .	18,081	7	16	35	. .	373,737	91	14	320
. .	23,709	7	15	40	. .	413,639	127	15	340
. .	30,856	19	18	50					

When greater conducting area than that of 12 B. & S. G. is required, the conductor shall be stranded in a series of 7, 19, 37, 61, 91, or 127 wires, as may be required; the strand consisting of one central wire, the remaining laid around it concentrically, each layer to be twisted in the opposite direction from the preceding.

159. 65. Switchboards.

a. Must be of non-combustible, non-absorptive insulating material.
b. Must be kept free from moisture, and accessible from all sides.
c. Must have a main switch, cut-out, and ammeter for each generator, a voltmeter, and ground conductor.
d. Must have a cut-out and switch for each side of each circuit.

160. 66. Resistance Boxes.

a. Must be non-combustible material.
b. Must be mounted on non-inflammable, non-combustible material, preferably on the switchboard.
c. Must be constructed to allow sufficient ventilation.

161. 67. Switches.

a. Must have non-combustible, non-absorptive bases.
b. Must operate successfully at 50 per cent overload in amperes, with 25 per cent excess voltage under the most severe conditions, and must be plainly marked with the name of the maker, current, and voltage.

c. Must be double-pole when circuits which they control supply more than six 16-candle-power lamps or their equivalent.

d. Must, when exposed to dampness, be inclosed in a watertight case.

162. 68. Cut-outs.

a. Must have non-combustible, non-absorptive insulating bases.

b. Must operate successfully on short circuits, with fuses rated at 50 per cent above and with a voltage 25 per cent above the current and voltage for which they were designed, and must be plainly marked with the name of maker, current, and voltage.

c. Must be placed at every point where a change is made in the size of the wire, unless the cut-out in the larger wire will protect the smaller.

d. In places such as upper decks, holds, cargo spaces, and fire-rooms, a watertight and fireproof cut-out may be used, connecting directly to mains when such cut-out supplies not more than six 16-candle-power lamps or their equivalent.

e. When placed anywhere except on switchboards and places as specified in *d*, they shall be in a cabinet lined with fire-resisting material.

f. Shall be so placed, except for motors, search-lights, and diving-lamps, that no group of lamps requiring more than six amperes shall depend upon one cut-out.

163. 69. Fixtures.

a. Shall be mounted on blocks of well-seasoned lumber, treated with two coats of white lead or shellac.

b. Where exposed to dampness, the lamp must be surrounded by a vapor-proof globe, and where exposed to mechanical injury, surrounded by a globe protected by a stout wire guard.

c. Shall be wired with the same grade of insulation as portable conductors which are not exposed to moisture or mechanical injury.

164. 70. Sockets.

a. No portion of the lamp-socket which is exposed to contact with outside objects shall come into electrical contact with either of the conductors.

165. 71. Wooden Moldings.

a. Must be of well-seasoned lumber, and treated inside and out with two coats of white lead or shellac.

b. Must be made in two pieces, so constructed as to thoroughly incase the wire, and provide a one-half inch tongue between conductors and a solid backing under grooves not less than three-eighths of an inch thick.

c. Where molding is run over rivets, beams, etc., it must be secured to a backing-strip.

d. Capping must be secured by brass screws.

166 72. Motors.

a. Must be wired under the same rules as apply to circuits of the same current and potential for lighting. The motor and resistance-box must be protected by a double-pole cut-out, and controlled by a double-pole switch, unless of one-quarter horse-power or less.

b. Must be thoroughly insulated.

c. Shall be covered with waterproof covers when not in use.

d. Must be provided with a name-plate, with maker's name, capacity in volts and amperes, and the normal speed in revolutions per minute.

CHAPTER VI.

THE CONSTRUCTION OF UNDERGROUND CIRCUITS.

CONDUITS.

167. The rapid multiplication of electrical circuits, particularly in business centers of the large towns, has increased the number of aerial wires to such an extent as to become an unbearable street obstruction. To obviate this difficulty the practice has arisen of constructing underground subways, or conduits, into which circuits may be placed.

168. Classification. — Conduits can be divided into two classes, that may, respectively, be termed flexible and inflexible systems, depending upon the possible mutability of the circuits *after* the structure is completed.

In the flexible system, a structure is designed and built under the pavement of the street in such a manner that the electrical circuits which it is to contain may be introduced at any time after the completion of the subway; and from time to time the circuits may be extended, or rearranged or replaced, as the business of the territory shall indicate to be advisable.

Under the inflexible system, as the conduit is built, *all* of the wires which it can ever contain are introduced at the time of construction, the design being such as to preclude any modification of the circuits after the completion of the work. In thickly settled districts, where the amount of electrical business can be fairly accurately gauged, and in which the changes or extensions in the business, beyond that of the original estimate, are small from year to year, the inflexible system presents the advantage of cheapness in initial construction. The design of the structure must contemplate, however, sufficient capacity to embrace all of the probable business which is ever likely to be done, for increased capacity can only be secured by constructing an entirely new conduit. The inflexible system being more economically constructed, and of more economical

maintenance, presents the attractiveness of cheapness, though, unless the amount of business can be accurately gauged and located, this very quality is apt to prove deceptive, and the subsequent cost of extension and rearrangement may greatly exceed the initial expense of a flexible system.

With the flexible system, the subways are designed with sufficient room to be capable of receiving all of the circuits which the most sanguine estimate of the business of the future can call for. The conduits are planned with a number of separate chambers, or ducts, into which the circuits may be placed. The expense, therefore, of the conductors can be reserved until business shall demand their introduction.

Every form of conduit should embrace the following conditions: —

The conduit should be reasonably economical in cost of construction.

It should afford a thorough protection to the inclosed circuits, securing them from the effects of street excavations, and from the incursion of gas, water, or organic acids from the streets, and should protect the insulation of the circuits, and maintain it at a high working average.

It must be rapid and easy of construction, so as not to present undue obstruction to street traffic.

It must be sufficiently flexible to accommodate itself to existing street structures.

It must have sufficient mechanical strength to successfully resist the ordinary destructive influences to which street structures are exposed.

It must present a minimum annual maintenance expense.

169. The Valentine Conduit. — One of the earliest underground distributions was the Valentine system, consisting of a rectangular wooden box some ten to fifteen feet in length, subdivided by vertical and horizontal partitions, into ducts about three inches square, for the reception of the circuits. These boxes were constructed of creosoted yellow pine, and were buried in the earth at a safe distance below the street pavement. At the joints the boxes were spliced by wooden battens, covered with felt and thoroughly pitched, to exclude

moisture. After the wooden box was laid, the whole structure was thoroughly tarred as an additional precaution against decay.

Experience shows that conduits of wood are the cheapest and most easily and quickly installed of any form. If thoroughly creosoted with pure dead oil of tar lead-covered cables can be safely installed, but with all precautions the life of a wooden conduit is short; in comparison with that of other forms, possibly a life of twenty-five years may be expected, while under unfavorable circumstances ten or fifteen years is all that can be counted on.

170. The Wyckoff or MacDonald Conduit. —This is an attempt at a structure slightly more substantial than that of Valentine's. It

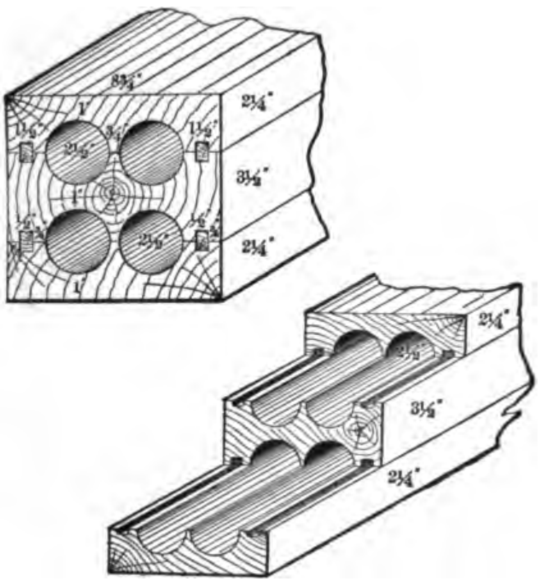


Fig. 133. The Wyckoff or MacDonald Conduit.

consists of a number of circular ducts, as represented in Fig. 133, bored in blocks of creosoted wood, the blocks being tongued and grooved together in a substantial manner. This conduit ought be so built that the different pieces should always break joint, and therefore the difficulties of unequal settlement of successive lengths is avoided. The Wyckoff is laid precisely the same way as the Valentine.

171. The Paper Conduit. — A conduit involving the use of paper tubes has been proposed, consisting of a rectangular wooden box in which a number of tubes of pasteboard of the requisite size are laid, and the interstices between the tubes filled with asphalt. As a result, the pasteboard forms a mold, around which the asphalt may be poured, thus forming a block filled with smooth cylindrical holes for the reception of the circuits. It is stated that this device gives good results. However, there is as yet insufficient experience with it upon which to base a conclusive verdict.

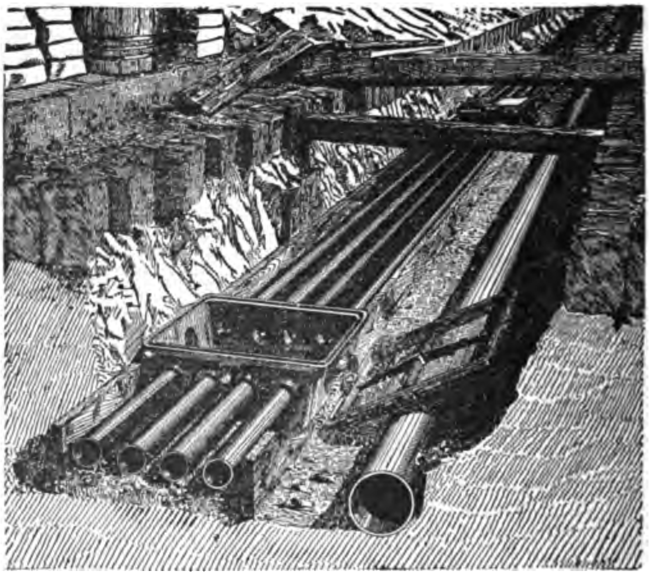


Fig. 134. Iron Pipe in Conduit.

172. Pipe Conduits. — Four very successful forms of conduit depend upon the use of metal pipe, surrounded either on the interior or the exterior with a cementing compound.

FIRST. — *Wrought-Iron Pipe in Hydraulic Cement.* — This conduit is constructed by opening an appropriate trench in the street, the bottom of which is covered with a layer of 6″ or 7″ of good concrete. A suitable mixture for this purpose may be made of two parts Rosendale cement, three parts sand, and five parts broken stone. After the bottom is thoroughly leveled, and the concrete rammed

into place, being carefully graded, a layer of wrought-iron pipe of appropriate diameter to receive the circuits is laid upon the concrete. These pipes are jointed by means of vanishing screw thread couplings, making a joint which is absolutely tight. An entire section of conduit, embracing the space between two adjacent manholes, is laid at one time. As soon as the first layer of pipe is in its place, the spaces between and around the tubes are carefully filled with concrete, thoroughly rammed into place. Upon the setting of this concrete, a second layer of pipe is introduced, and this process repeated until a sufficient number of iron pipes are laid to give the necessary capacity to the subway. See Fig. 134.

After the last row is in place, a top coating of concrete, 3″ to 6″ in thickness, is spread over the pipe, a layer of plank placed upon the top of the concrete to secure the structure against damage from the tools of workmen opening the streets, and the pavement replaced. This structure makes an excellent conduit in every respect, being probably the best one now known. It is water and gas tight. It may be built to accommodate at pleasure any number of circuits, and is sufficiently flexible to enable reasonable bends between manholes to be successfully made; and in cases of streets crowded with underground structures, the iron pipe may thread through or around other obstructions in a way impracticable in any other form of conduit. Experience with this form of subway has shown that in process of time the iron pipe may rust away; but in this event a smooth cylindrical hole is left, extending through a solid block of concrete, which during the time required for the destruction of the iron pipe has become as hard as stone, thus leaving ample protection for the inclosed circuits. While from a constructive and maintenance standpoint, this device presents all the advantages to be desired for a conduit, it is quite expensive to build.

SECOND. — *Wrought-Iron Pipe in Asphaltic Concrete.* — A similar conduit has been proposed, by imbedding wrought-iron pipe in asphaltic concrete instead of cement. The substitution, however, of the asphalt for the cement concrete possesses no particular advantage, and is still more expensive to build.

THIRD. — *Zinc Tubing in Hydraulic Cement.* — To cheapen the iron-pipe conduit, it has been proposed to bed zinc tubing in hydraulic cement; the idea being that economy would be affected by the use

of very thin and light zinc tubing, which would be much cheaper than the previously proposed iron pipe. The zinc tube was simply to serve as a mold around which concrete would be placed; the expectation being that the zinc tube, in any event, would certainly disappear, leaving the desired hole in the cement. This device, however, has not met with success, as the zinc tube, when made

Fig. 135. Cement-lined Iron Pipe.

sufficiently heavy to stand the ramming of the concrete, proved more expensive than the corresponding iron pipe.

173. FOURTH. — *Cement-Lined Iron Pipes.* — Another effort to cheapen the iron-pipe conduit has resulted in the construction of a very thin pipe made of sheet iron into which a layer of cement is introduced, surrounding a mandrel, that is subsequently withdrawn,

thus leaving a continuous tube of cement protected by a thin shell of sheet iron, the expectation being that pipe of this description would be sufficiently strong to stand laying in the street, and that after the pipe was once in place it would be protected from further injury by the surrounding concrete and soil. This conduit has met with much deserved success, and so far as first cost is concerned, is decidedly cheaper than that involving the iron pipe. The appearance and method of constructing a conduit of cement-lined iron pipe are shown in Fig. 135.

174. The Dorset or Callender-Webber Conduit. — This conduit consists of tubular blocks some 4 ft. in length, made of asphaltic or pitch concrete molded around mandrels of the required size to

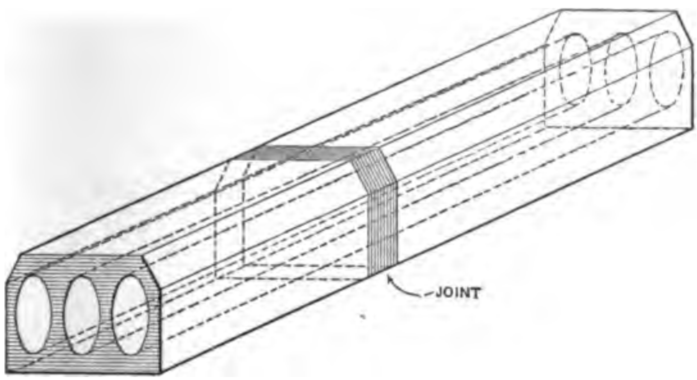


Fig. 136. The Dorset Duct.

give a number of 3″ holes extending entirely through the block. See Fig. 136.

The conduit is constructed by laying, at the bottom of an appropriate excavation, a series of the perforated blocks; the joints being made by carefully abutting the ends of successive blocks, and uniting them with a mixture of hot asphalt, pitch, or tar. It was found, however, exceedingly difficult to align the blocks sufficiently accurately to make the ducts exactly continuous; and any subsequent settlement of the soil caused the conduit to open at the respective joints.

175. The Chenowith Conduit. — The Chenowith conduit was an attempt to build between manholes continuous tubes of cement. This was accomplished by making a series of mandrels split lon-

gitudinally into three parts, held in place during construction by a spiral ribbon of sheet iron. A number of these mandrels were placed at the bottom of the street excavation, and concrete tamped solidly around them. The metal ribbon was lubricated with soapstone to facilitate the extraction of the mandrel, and on the withdrawal of the mandrel remained in place in the concrete. After the concrete is set, the metal ribbon can be pulled out and used again. By this means, between successive manholes, a continuous block could be

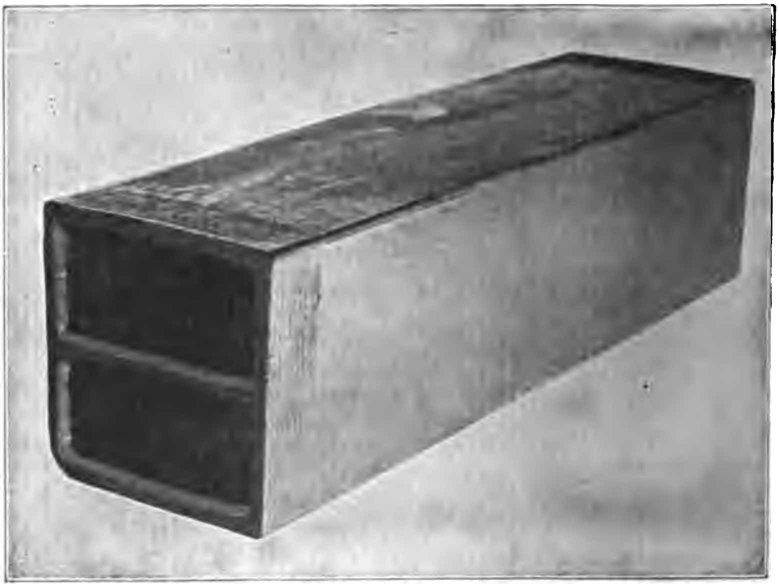


Fig. 137. 10″ × 10″ Terra Cotta Duct.

constructed having the appropriate ducts to receive the necessary circuits.

176. The Terra-Cotta Conduit. — An exceedingly valuable form of conduit, embracing nearly all of the points required for the successful protection of electrical circuits, being withal economical to construct, has been found in the use of terra-cotta blocks for the purpose of forming the subway. A rectangular pipe is made of terra-cotta ware about 3 ft. long, having a partition in the center. See Fig. 137.

Successive lengths of this pipe are joined by wrapping the suc-

ceeding sections with heavy jute dipped in asphalt. The jute wrapping makes a joint which successfully holds the lengths of pipe correctly in place, and a thorough application of asphalt ensures a joint which is water and gas tight, and does not decay. Care must be observed not to apply the asphalt too hot, or the jute will be injured. The conduit is formed by placing at the bottom of the

Fig. 138. Laying 10″ × 10″ Ducts.

street excavation the requisite number of earthenware pipe to give the desired capacity. It is usual to lay the pipe upon a bed of concrete, and to protect it on either side and on the top by a concrete wall 4″ to 6″ thick. Conduits of this class have been constructed to a large extent, and so far have proved eminently successful. The chief objection to this style of construction is found in the fact that

the earthenware pipes are so designed as to accommodate several cables in one duct. While there is little or no difficulty in introducing several cables into one division, it is often exceedingly difficult to withdraw them after they have been in place any length of time without destroying the sheath. The operation of introducing the 10″ × 10″ terra-cotta pipe is shown in Fig. 138.

177. Terra-Cotta Separate Duct System. — To overcome the difficulty in withdrawing cables, which is experienced in 10″ × 10″ ducts, an earthenware conduit has been devised, which consists of a number of blocks of earthenware pipe, each having a separate

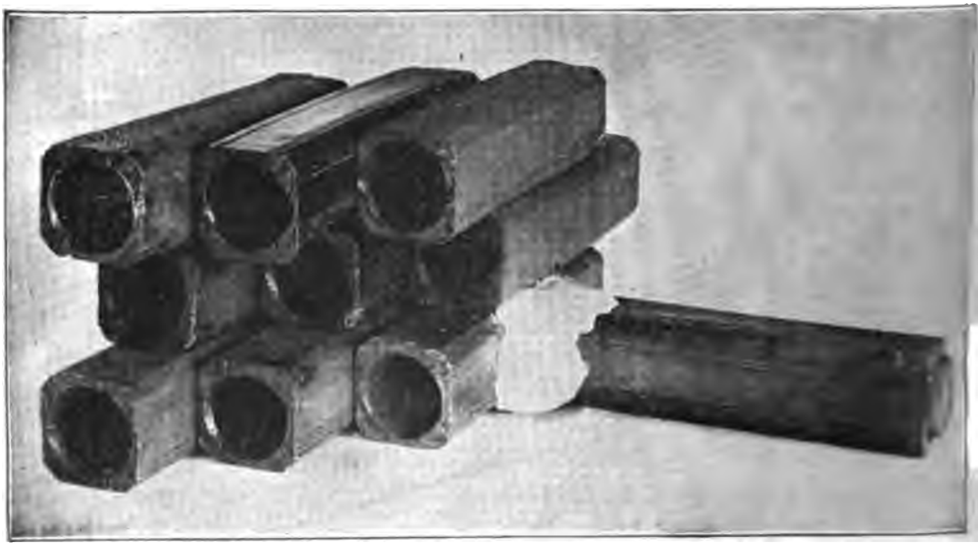


Fig. 139. The Terra-Cotta Separate Duct System. Pipe Sections.

duct. These blocks are 5″ square, and from 18″ to 2 ft. in length. They are made of earthen pipe, having the general appearance indicated in Fig. 139.

To construct a subway out of this material, an appropriate street excavation is made, the bottom of which, after having been carefully graded, is lined with a layer of 6″ of concrete. Upon this concrete the earthenware ducts are built, in precisely the same fashion as a brick wall is laid up. To secure proper alignment, it is customary to lay one line of duct through the center of the trench, to guide the alignment of all the succeeding layers of pipe. As the work

progresses, mandrels some 6 ft. or 8 ft. in length, which closely fit the ducts, are placed in each row of pipe, thus ensuring correct alignment, until the cement in which the pipes are laid has had a chance to set. As fast as the subway is built, the mandrels are pulled along, thus keeping the pipe constantly in true line.

Conduits of this description should be constructed by laying the pipe up in a strong mixture of either Rosendale or Portland cement. The joints in the pipe should be hammered down so as to be as close as possible, and not to exceed $\frac{1}{4}''$. As all the pipe in burning is slightly concave, care should be taken to lay the subway with the convex sides upward in every instance, so that no obstacles may be experienced in the subsequent introduction of the cables. As the ducts are laid, all the joints between the successive blocks should be thoroughly grouted with cement. This form of conduit presents the advantage of great flexibility; as a subway of any number of ducts can be formed, and, in order to accommodate street obstructions, the geometrical cross-section of the conduit can be varied at pleasure. After completion, a scraper, similar in shape to a boiler-tube cleaner, should be drawn through each of the ducts, which serves to cut away and clean out all gravel and cement which has found its way accidentally into the ducts, and then by washing the ducts with a stream of water from a hose, a clean, polished hole is obtained, extending between the adjacent manholes. While this form of conduit is slightly more expensive than rectangular earthen pipe, it presents the inestimable advantage of giving a separate duct for each cable. The method observed in building this form of conduit is seen in Fig. 140.

178. While the Camp duct presented many improvements over the older $10'' \times 10''$, it was open to some objections. The multiplicity of separate pieces entailed considerable expense in laying, and as there was no means of securing the ends of the separate tiles to each other, there was always the possibility that subsequent settlement or misalignment might occur and produce obstacles to the introduction of cable. To obviate these objections conduit-makers have studied to devise means whereby several ducts could be combined in a single tile in order that installation cost might be minimized, and to produce some method whereby the end of each piece could be secured to its neighbor and thus prevent any possibility of misalignment. These efforts have resulted in the production of

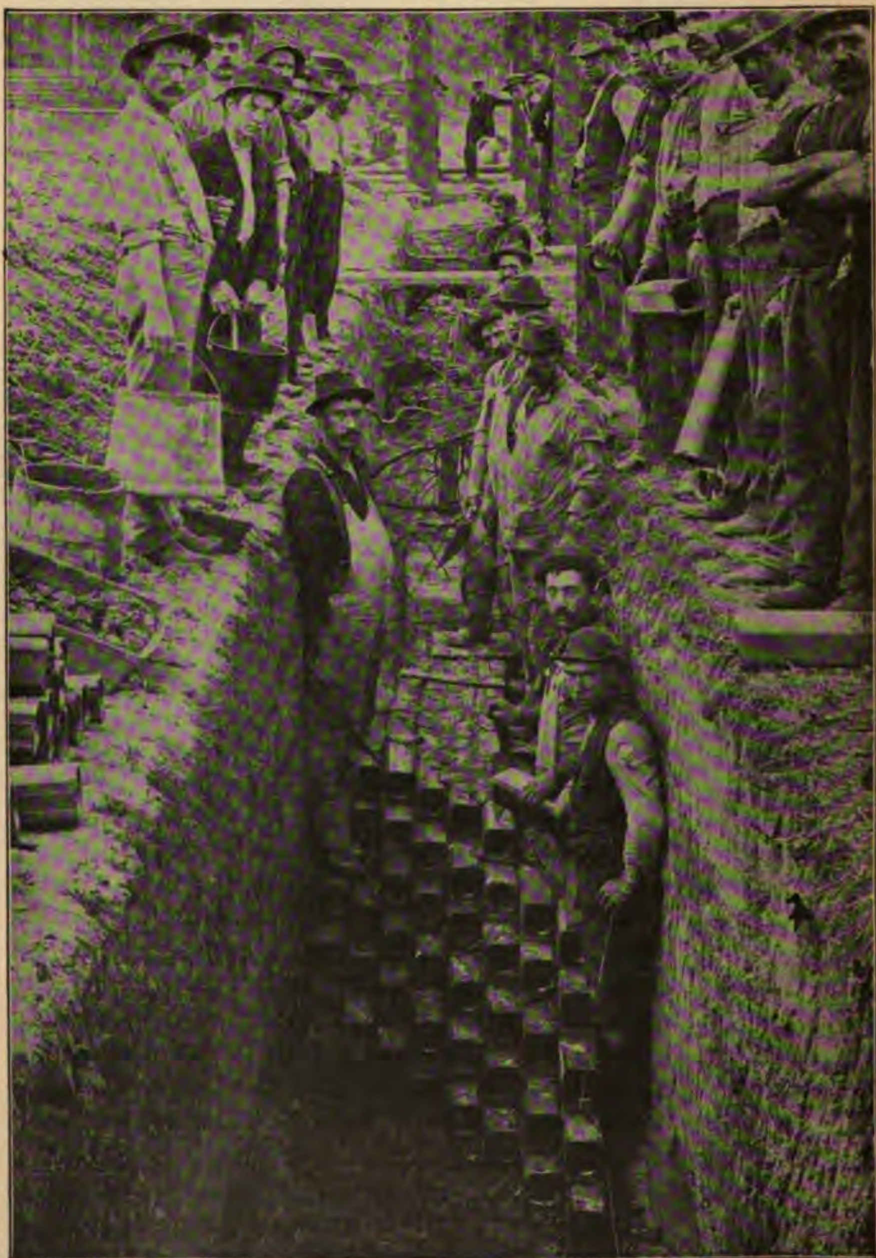


Fig. 140. The Separate Duct Terra-Cotta Conduit.

what is known as the "Multiple-Duct Conduit," of which there are several forms on the market, the McRoy being the first to appear, while that manufactured by the American Vitrified Conduit Co. is a recent exponent. The various makers adopt methods of construction which differ slightly in details, but the result in all cases is essentially similar. Fig. 141 is a representation of the various forms of ducts produced by the American Vitrified Conduit Co. The single-duct tile, as it is seen from the illustration,

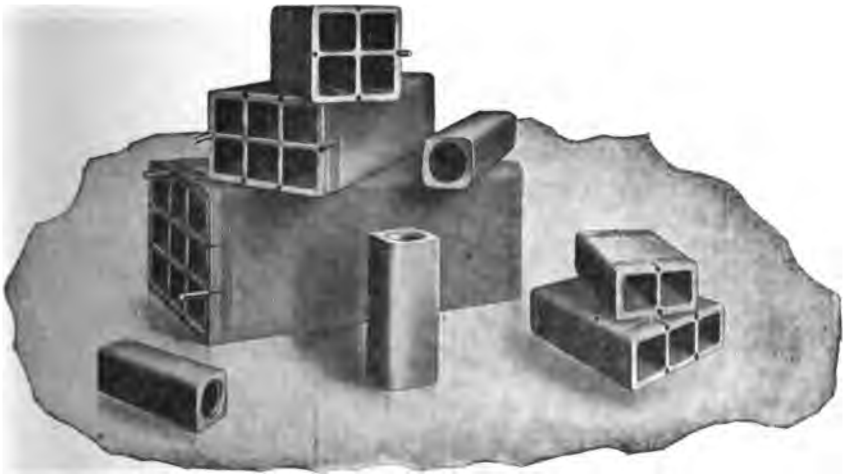


Fig. 141. Multiple-Duct Conduit.

closely resembles the original Camp tile, excepting that each piece is provided with a molded male and female end, something after the fashion of drain-pipe. When this conduit is installed the male end of each piece is thinly coated with cement mortar and inserted in the female end of the preceding one, the projection upon the male end being centered by the concavity in the preceding piece, while the addition of the mortar is assumed to make the joint reasonably gas- and water-tight. In other respects the installation of the *single-duct self-centering conduit,* as this type is termed, is the same as that already described for the Camp tile.

179. The multiple duct is produced by forcing the properly prepared clay through a die so constructed as to produce a tile which contains several compartments each of which is intended as a single

Fig. 142. Laying Multiple Ducts.

PLATE III.

Installation of Multiple Duct Conduit.

To face page 203.

cable-space. As is shown in Fig. 141, the multiple duct contains two, three, four, six, and nine cable-spaces in each piece. This variety enables the engineer greatly to economize time and labor in street work. It is often possible to secure the desired number of cable-spaces in a single piece of tile, while almost any combination can be obtained by the employment of three or four pieces. Along the sides of each piece a quadrilateral groove or channel is molded, and into this groove a similar-shaped piece of steel from four to six inches long is inserted. This steel bar acts as a dowel, and in this way the alignment of each piece with its preceding and succeeding neighbors is secured.

180. The general method of constructing multiple-duct conduits is shown in Fig. 142 and in Plate III. A trench of sufficient width and to contain the appropriate number of ducts is excavated in the street. After being carefully leveled to proper line and grade the bottom of the trench is paved with concrete about 3″ thick, and upon this flooring the tile is laid. In case it is necessary to use more than one layer of ducts a thin coating of mortar is spread upon the top and sides of each layer and the succeeding ones bedded in the mortar in the same manner as adopted for the single-duct tile. When all of the tile is in place the top and sides are covered with two or three inches of concrete to preserve the conduit from the tools of street excavators. In some cases the joints between successive pieces of ducts are protected only by the concrete casement which surrounds the conduit. The preferable plan is to make the joint as shown in Fig. 143. This artifice consists in wrapping each joint with two or three layers of strong burlap previously dipped in melted asphalt and then mopping the entire joint with a coating of the same substance exactly as has been explained for the 10″ × 10″.

The junction between the conduit and the necessary manholes is made as exemplified in Fig. 144, which also represents a very advantageous method of constructing manholes. As is indicated in the illustration, the manhole consists of a monolithic structure of concrete, which is made by building a mold of the appropriate size and shape to form the manhole, and placing it upon the top of a concrete floor laid upon the bottom of the proper excavation in the street. The sides and top of the mold are then covered with a layer of from six to eight inches of concrete, that is

PLATE III.

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allowed to set, after which the mold is taken to pieces inside of the manhole and passed out in the opening left in the top, that is subsequently furnished with the proper manhole cover. This method gives the strongest, most durable, and cheapest construction, and is always advisable where street obstacles will permit of its adoption. Otherwise manholes are constructed of brick, as shown in Fig. 145, the end of the conduit being built into the manhole wall. In such cases it is customary to form the roof of the manhole by covering the top of the manhole with old rails or any form of struc-

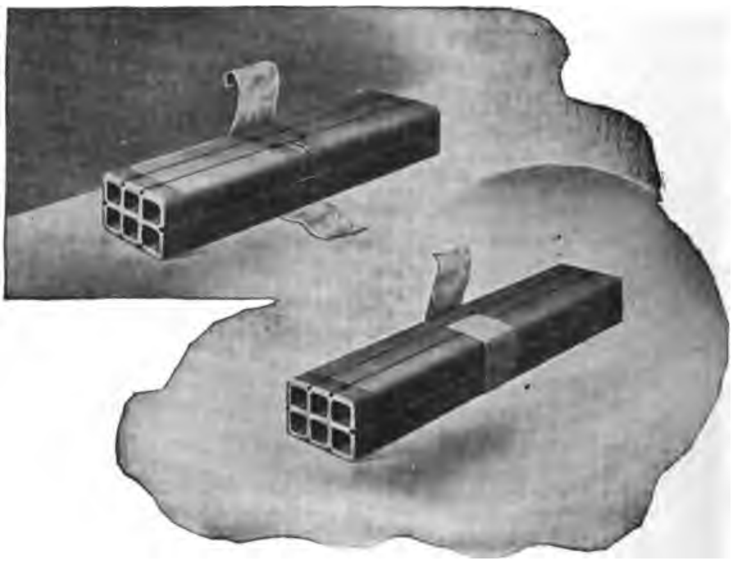


Fig. 143. Joining Multiple Duct.

tural steel pieces of the necessary strength, upon which a course of brick is laid, and the completed structure provided with the proper cover. Fig. 145 contains four illustrations. The upper one shows an elongated vault built about a water-pipe; the second one shows a manhole ready to receive the ironwork for the roof; in the lower one the vault is shown ready to receive the manhole cover; while the third illustration from the top indicates the finished manhole ready for the reception of paving. The mechanical features of single- and multiple-duct tile are shown in Table No. 43.

Fig. 144. Entrance to Concrete Manhole.

Fig. 145. Brick Manhole Building.

181. The Crompton System. — The subways so far considered have been only adapted to the use of highly insulated cables, as the designs have been such as to afford no insulation to the circuits.

TABLE NO. 43.

Standard Vitrified Conduit.

Style of Conduit.	Dimension of Square Duct in inches.	Dimension of Round Duct in inches.	Outside Dimensions of End Section in inches.	Reg. Stock Lengths in ins.	Short Lengths in inches.	Approx. Weight per duct ft.
2-duct multiple	$3\frac{3}{8}$ sq.	$3\frac{1}{4}$	5 × 9	24	6, 9, and 12	8 lbs.
3-duct multiple	$3\frac{3}{8}$ sq.	$3\frac{1}{4}$	5 × 13	24	6, 9, and 12	8 lbs.
4-duct multiple	$3\frac{3}{8}$ sq.	$3\frac{1}{4}$	9 × 9	36	6, 9, and 12	8 lbs.
6-duct multiple	$3\frac{3}{8}$ sq.	$3\frac{1}{4}$	9 × 13	36	6, 9, and 12	8 lbs.
9-duct multiple	$3\frac{3}{8}$ sq.	$3\frac{1}{4}$	13 × 13	36	6, 9, and 12	8 lbs.
Common single duct		$3\frac{3}{8}$	5 × 5	18	6, 9, and 12	8 lbs.
Single duct self-centering		$3\frac{3}{8}$	5 × 5	18	6, 9, and 12	10 lbs.
Round single duct self-centering		$3\frac{1}{4}$	5 in. round.	18	6, 9, and 12	10 lbs.

Steel connection keys $\frac{3}{8}$ in. square × $4\frac{1}{8}$ in.

Several European attempts have been made to construct a subway in which bare copper conductors could be used, thus avoiding the expense of insulated conductors. Notable among these systems is that of Crompton, which has received quite an extended development in London, Nottingham, and Birmingham. In the Crompton system the conduit is usually laid under the foot-walks, and not under the street proper, as is customary in this country. The construction involves the excavation of a trench, which, for the ordinary-sized

conduit is about 3 ft. 5″ wide, by 1 ft. 9″ deep. This trench is then supplied with a bottom and side walls of concrete, the bottom being about 3″ in thickness, while the sides are from 6″ to 8″. At intervals of about 50 ft., so arranged as to be opposite every alternate house, a handhole cover is introduced, to give access to the conduit for the purpose of taking out service wires. Directly under this handhole a heavy rectangular piece of oak timber is set in the concrete sides of the subway. Upon this timber the requisite number of porcelain insulators are placed to carry the circuits for which the conduit is intended. These insulators have a simple slot on

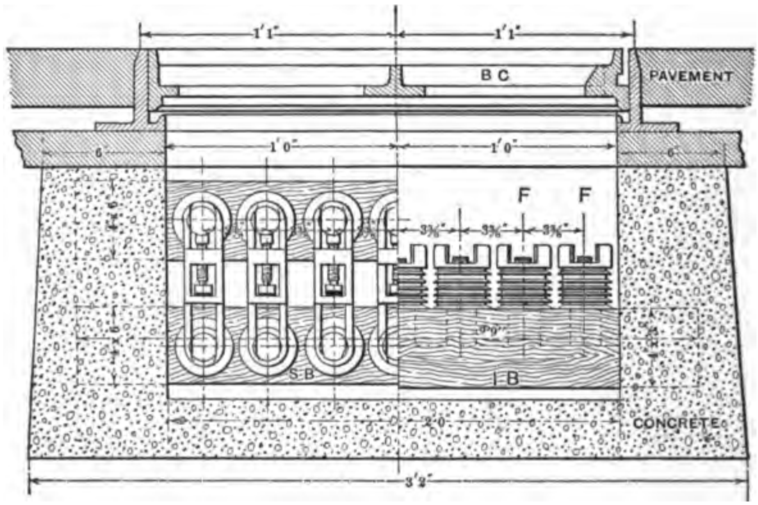


Fig. 146. Crompton Conduit, Half Plan and Section at Handhole.

their tops in order to receive and carry the lines, that are merely naked strips of copper. Fig. 146 indicates a half-plan and half-section of the conduit at a handhole. At intervals of about 300 ft. the handholes are made quite large in order to afford the necessary room for stretching the copper strips. The subway is completed by placing the requisite iron casting at each of the handholes, and by covering the entire top of the trench with a layer of Yorkshire flagging, after which the paving of the foot-walk is replaced, the only indication remaining being the handhole covers at the houses. The circuits consist of copper strips about ¼″ thick, and from 1″ to 1½″ wide. The conductors are introduced by running a cord between

the successive handholes, and then joining the copper strips in continuous lengths and hauling them in. The hauling cord is introduced by attaching it to the collar of a small dog, who is trained to run along the bottom of the subway from handhole to handhole. At each handhole cover an inspector is stationed who sees the copper ribbon is placed in the slot of the insulator as it makes its appearance. When a length of 300 ft. of copper ribbon is introduced, one end of it is made fast in the end insulator, and then by means of a hydraulic jack, the ribbon is pulled up to the appropriate tension,

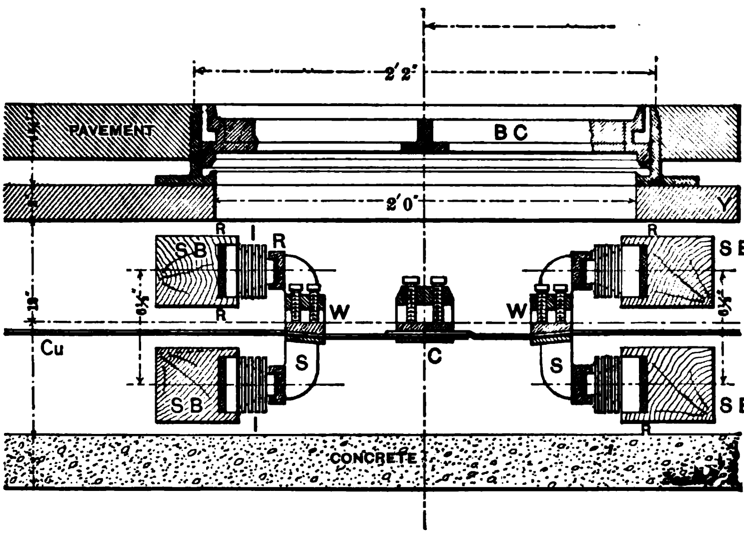


Fig. 147. Crompton Conduit, Longitudinal Section.

and secured at the other end in a similar insulator. The method of joining and securing the strips is indicated in Fig. 147.

Here it will be seen that there are two sets of heavy oak blocks (SB) set in the concrete. On these blocks the insulators (I) rest, and carry clamp (S), that by means of set screws (W) tightly pinch and hold the copper strips in place. A pad of rubber (R) distributes the pressure equally over the insulator and prevents cracking the porcelain. This structure certainly seems to present a maximum of advantage in the way of small street space occupied, ease and economy of construction, and flexibility and convenience for rearrangement or extension of circuits. The English reports are so satisfactory that it seems strange that similar devices are not tried in this country.

182. The Brooks System. — This system, the invention of David Brooks of Philadelphia, embodies the use of a heavy mineral oil, one of the best insulators known. Mr. Brooks's invention consisted in placing in a trench, excavated under the pavement, an iron pipe of

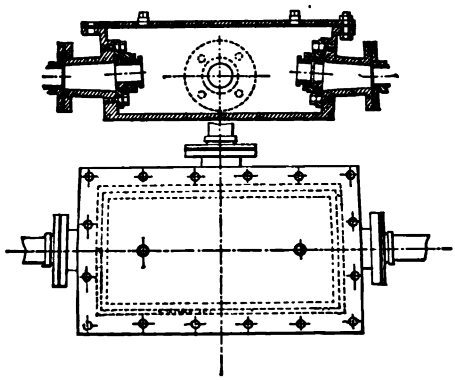


Fig. 148. Brooks System Junction-Box.

sufficient size to carry the necessary conductors for the system. Occasionally a rectangular box is introduced, as shown in Fig. 148, into which each end of the pipe opens by means of a flanged joint. This box serves the purpose of affording an opening into the pipe

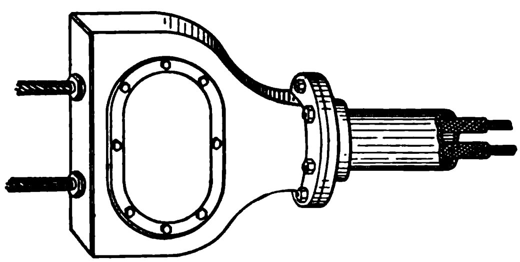


Fig. 149. Brooks System Service Box.

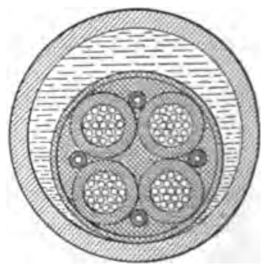


Fig. 150. Brooks System Cross-section of Cable and Oil Pipe.

through which the conductors could be drawn in. At various intervals a service-box, as shown in Fig. 149, is introduced, by means of which distribution could be accomplished. The cables used in the Brooks system consisted of solid or stranded copper conductors, shown in section in Fig. 150, covered with a layer of raw jute or hemp, to prevent contact between the conductors, or grounding

on the iron pipe. This covering was placed on the outside of the conductors by a braiding-machine, and then the cable drawn into the pipe through the service-boxes. As soon as the conductor was in position the pipe was filled with boiling resin oil, which formed the insulating material, and preserved the electrical qualities of the cable. While in some instances the Brooks system has been found to stand up admirably under severe tests, considerable difficulty has been experienced in keeping the pipe sufficiently tight to retain the fluid insulator. For electric lighting service the Brooks system has in some instances given fair satisfaction ; yet its largest use has probably

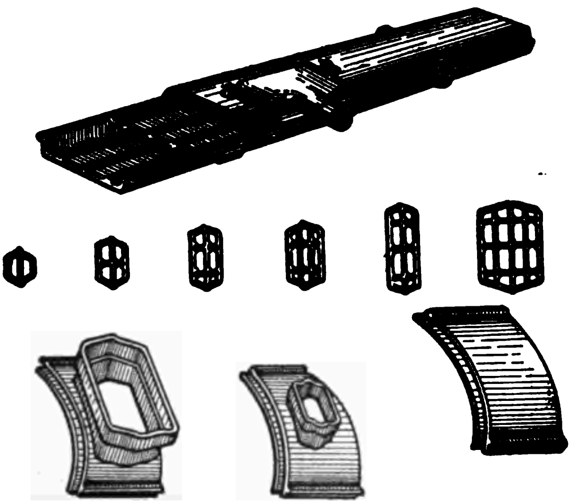


Fig. 151. Johnstone System.

been found in telephone and telegraph work, and for this purpose it has not fulfilled expectations, as much of this conduit which has been introduced has, after a time, been found gradually to fail in insulation.

183. The Johnstone System. — The Johnstone conduit system bears evidence of exceedingly careful design, the structure having been planned to meet all exigencies to which subways are called to respond. The arrangement consists of a series of cast-iron troughs made in lengths of about 6 ft., so designed in sections that a conduit of any desired capacity can be built. The sections, as shown in the illustration, Fig. 151, are arranged to comprise a series of rectangular ducts, into which the circuits may be at any time placed by

drawing in cables in the ordinary way. At frequent intervals a service-box is arranged upon the top of the upper row of ducts, out of which appropriate leads can be taken to serve the desired installation. At the street corners iron manholes (Fig. 152) are introduced, into which the ducts end, and from which all rearrangements or connections can be made. While this conduit is admirable in every

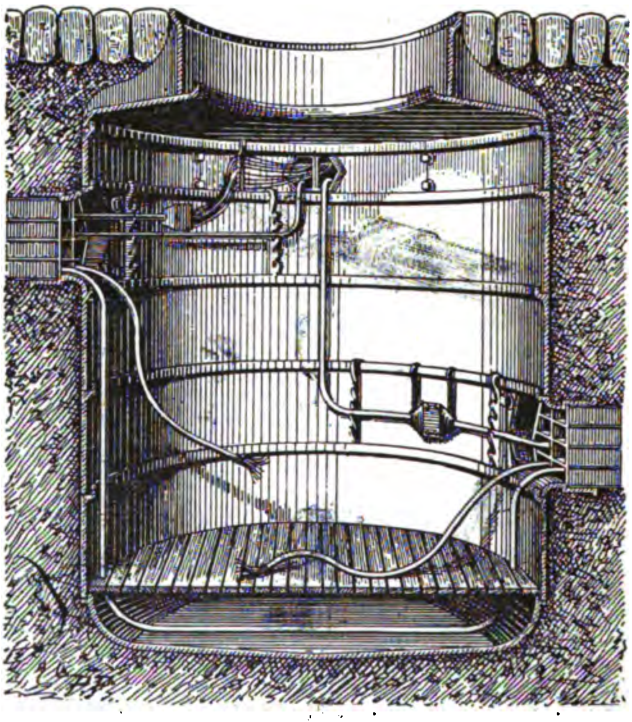


Fig. 152. Manhole of Johnstone System.

respect, it is one of the most expensive forms of subway constructed; its cost being so great as to almost prohibit its use.

184. The Kennedy System. — The Westminster Company of London have employed a modification of the Compton conduit, designed by Professor Kennedy, that has given good satisfaction. A general idea of this method may be obtained from the accompanying illustration, Fig. 153, showing a cross-section of this conduit designed to carry two lines of main feeds, and a three-wire distributing system. A trench is excavated in the street, which, in a manner

similar to that of the Compton construction, is lined on the bottom and sides with concrete. The conductors, as in the Compton system, are bare copper strips; but instead of being supported upon insulators set in oak blocks, the insulators are solid porcelain supports, which

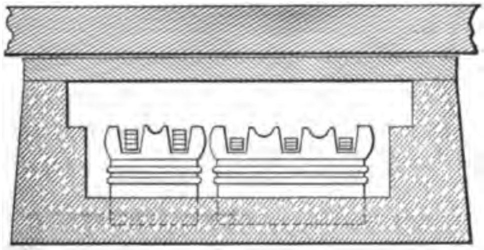


Fig. 153. Section of Kennedy System.

are set directly in the concrete bottom. The top of the conduit is covered with flagging or iron casting. The circuits, formed of bare strips of copper wire, are drawn through the conduit, being placed in the insulators by means of handholes in a manner similar to that adopted by the Compton system.

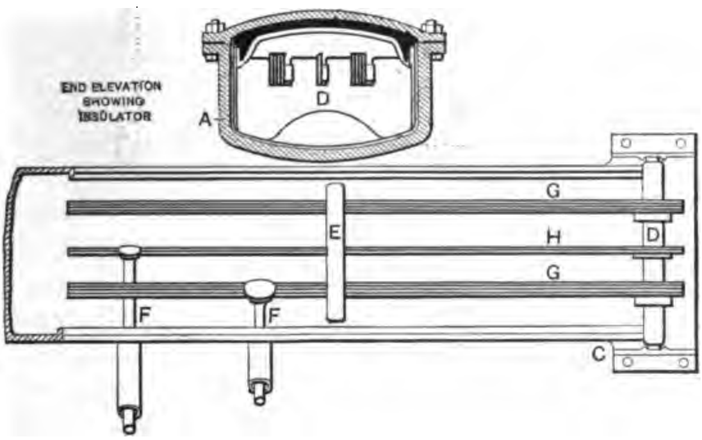


Fig. 154. St. James System.

185. St. James System, London. — This arrangement is very similar to that of the Compton and Kennedy systems. The conduit, however, instead of being formed of concrete, is made of an iron trough, Fig. 154, set at the bottom of the street excavation, thus avoiding the

use of cement, and greatly expediting the street work. This trough is provided with a water-tight cover, secured by means of bolts and packing. At frequent intervals throughout the trench, a porcelain bridge is placed for supporting the circuits, which consist of bare copper strips set on edge, or stranded cable, and strained to be sufficiently taut to remain in a straight line. It is obvious that in all of the systems where bare wire mains are employed, special precaution must be taken to insure careful drainage, so that all incursions of moisture from the street may be readily and quickly removed in order not to flood the mains.

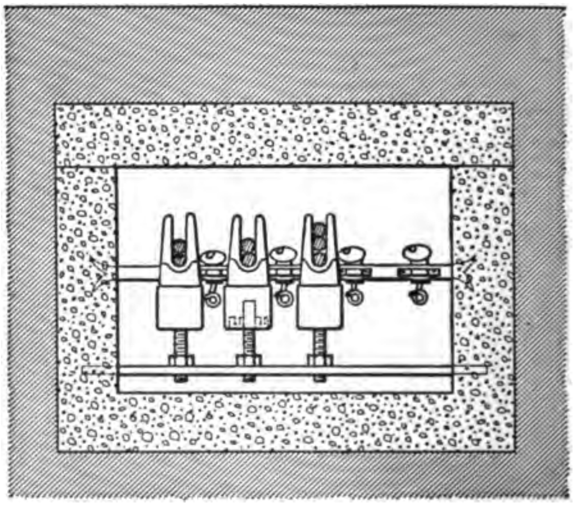


Fig. 155. Parisian System.

186. The Parisian Systems. — A large part of the underground distribution of Paris has been accomplished by the use of bare conductors extended through concrete trenches in a manner similar to the London system. The subway is formed by excavating a trench under the sidewalk, which is lined with concrete on the sides and bottom, having a flagstone, or similar covering, placed over the top (Fig. 155). The cables are almost universally bare stranded copper wire. They are carried through the trench, being supported upon porcelain insulators, carried upon iron pins set and secured in the concrete forming the bottom of the trench. In many of the German and Italian cities similar methods of

distribution have a widespread introduction, and are successfully operated.

187. Inflexible Systems. — The examples of subway systems so far cited appertain exclusively to the flexible system, that, owing to its greater adaptability to service fluctuations, has deservedly obtained a more widespread development. In the succeeding illustrations the inflexible system is presented. Numerous modifications of the methods here given will readily occur to the facile designer, in order to adapt this principle to the varying circumstances of particular localities.

188. The Callander Solid System. — By the Callander solid system a series of cast-iron troughs are arranged along the bottom of a trench excavated in the street. In the troughs the requisite number of cables are extended, supported from time to time upon insulating pieces fixed in the troughs. This protection is found to be necessary, from the fact that the insulating compound, with which the trough is to be filled, is never absolutely hard, but behaves like a very viscous fluid; and if the cables were unsupported they would gradually settle, and ultimately lie upon the cast iron forming the exterior of the subway, thus short-circuiting, and spoiling the entire structure. The cables are usually stranded copper rope of the appropriate size, covered with an additional insulating compound. In view of the melted asphalt, or insulating compound, which is subsequently poured in, this would seem unnecessarily expensive, as bare copper conductors thus arranged would answer equally as well. After the cables are in place, the entire trough is filled with Trinidad asphalt, thus completing the structure, and presenting an appearance as indicated in Fig. 156, in cross-section.

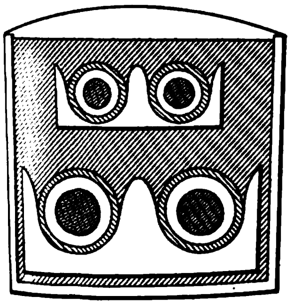


Fig. 156. Cross-section of the Callander Solid System.

The troughs are laid in lengths of 6 ft., and are of about $\frac{5}{16}''$ in thickness, a cast-iron cover being placed over the top to protect the conduit from injury. At appropriate intervals manholes are introduced (see Fig. 157).

The design of manhole adopted by the Callander system is one to

afford the greatest possible protection to the circuits. As indicated in Fig. 157, the manhole consists of an excavation below the street level, into which is set an iron chamber surmounted with a watertight cover, which is screwed down and rendered moisture-proof by being bolted to a rubber gasket that sets upon the top of the manhole casting. The mains contained in their trough of asphalt are carried through the iron walls of the manhole and carefully

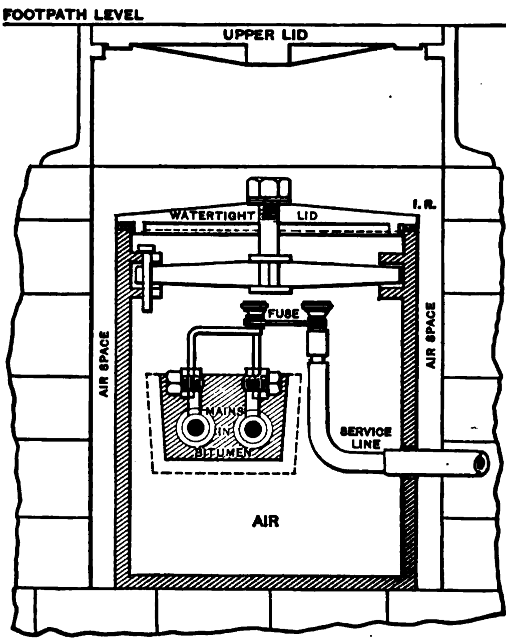


Fig. 157. Manhole, Callander Solid System.

cemented into place. The service mains are in a similar manner carried out through a hole drilled into the manhole wall, and through a packed joint that is moisture-proof. The cast-iron box forming the manhole is set inside a cemented chamber that is so arranged as to be entirely surrounded by an air-space, into which drainage may accumulate, and be conducted away by appropriate connection to the sewer. While the Callander system presents a very perfect form of underground service, the iron trough carrying the cable seems an unnecessarily expensive precaution.

189. Two forms of cheaper construction are indicated in Figs. 158 and 159. The arrangement shown in Fig. 158 is that which is adopted in the distributing systems in Cologne and in two or three other European cities. The arrangement consists of a wooden box, shown in cross-section, into which a carefully insulated concentric cable is placed, being suspended at intervals of every few feet by means of an iron strap. After the cable is laid in place, the box is poured full of asphalt, or concrete, thus entirely surrounding the cable with an insulating material. The asphalt and the box serve to protect the cable from injury in the street. Installations of this description have given good service, though they have, as yet, not been in operation sufficiently long to determine their probable life. It is necessary to use a very carefully prepared cable, as is indicated in the illustration. The cable is concentric, the outer conductor of which is protected by two lead sheaths and an iron armor. The exterior layer is of tarred hemp. It will also seem that an installation of this kind, made and protected by only a wooden box, would be subject, sooner or later, to the decay of the woodwork.

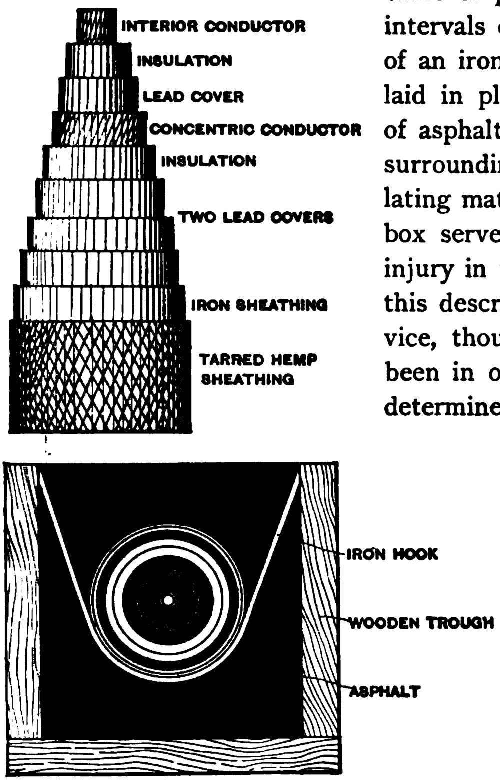


Fig. 158. Cologne Conduit.

As an improvement both in durability and economy, the construction indicated in Fig. 159 has been adopted in Zurich. The cable is a concentric conductor, insulated with paper, and having a lead sheath separating the inner and outer mains. The conduit consists of an earthenware trough placed just beneath the pavement level. It is strong, simple, cheap, can be rapidly laid, and affords an

excellent mechanical protection to the cable. It is made in sections of 3 ft. in length, and, if carefully placed in the street excavation, requires only a little grouting at the joints. The cables are simply laid at the bottom of the earthenware trough, which is then filled with sand, and completed with an earthenware cover. Some 18 miles of this conduit, containing 60 miles of cable, are in active operation.

190. Manholes. — It is essential, at frequent intervals, to provide means of access to underground conduits for the introduction or rearrangement of circuits, for distribution, and for such changes in direction of the subway as the location of streets renders essential. Such opportunities for access consist in chambers, constructed under the pavement of the streets, of sufficient size to allow reasonable room for two or three men to work. Usually these chambers are

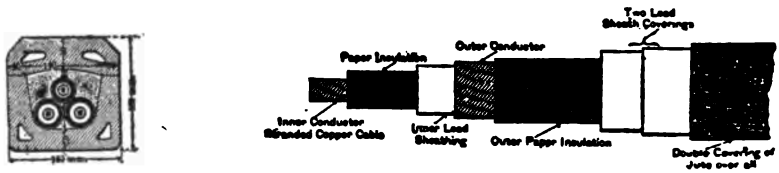


Fig. 159. Zurich Conduit.

rectangular vaults, built of either concrete or brick. They are roofed over, either with arches or structural iron carrying an arch brick, and provided with an iron frame supporting the manhole cover. The various branches of the conduit are arranged to extend through the walls of the chamber, giving free access to all of the ducts converging at the particular manhole. It is advisable to construct the manholes of ample dimensions; for while, by increasing the size of the chamber, the initial cost of the underground system is slightly augmented, yet the future expense entailed in introducing the circuits, and the labor constantly necessitated by the rearrangement and maintenance, is so much decreased by affording to the workmen a reasonable amount of space in which to perform their avocations, that the extra capital invested is usually found to be well expended.

A vault 5 ft. wide by 7 ft. long, and from 4 ft. 6″ to 6 ft. high, is as small as should be designed for large underground systems,

where considerable splicing and rearrangement of circuit is to be expected.

Inasmuch as the manholes are the lowest points of the subway, it is essential to provide for drainage, by connecting the bottoms of each of the vaults with the sewers, by the means of an ample drain-pipe provided with a catch-basin and trap. If

Fig. 160. A Terminal Manhole.

precautions of this kind are omitted, it frequently happens that in heavy rainfalls the conduit becomes flooded, and the circuits much injured. This provision is of paramount importance in subways containing uninsulated circuits. If lighting circuits are reasonably convenient of access, it is well to arrange in the vaults provision for incandescent lamps, as by this means workmen are afforded reasonable illumination for the prosecution of their work,

and all of the dangers attendant upon lanterns, or other methods of lighting, are avoided. While the manholes are in use by the workmen, it is necessary to provide some method of protection to prevent travel in the street from being injured by falling into the manhole, and also to prevent injury occurring to the workmen from causes of this kind. It is common to arrange a circular iron pipe guard, so designed as to be readily folded up. The guard can be unfolded and secured by setting inside the iron ring forming the manhole; and then a suitable red flag, lantern, or other signal can be attached in order to call the attention of the passer-by to the fact that the street is open at these points.

As an example of manhole construction, Fig. 160 is from a

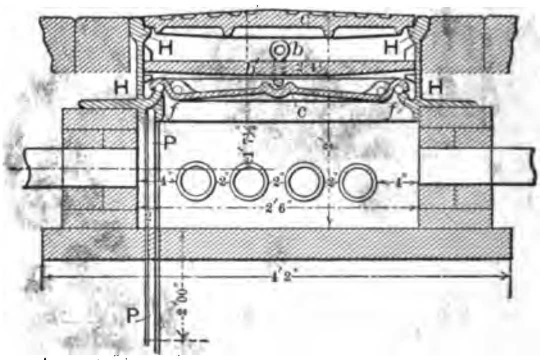


Fig. 161. New York Subway Manhole.

photograph of the terminal manhole of a large underground system. The subway may be seen entering the wall of the manhole, the lower rows of ducts being filled with cables that in the foreground turn and run upward into the station above.

As additional illustration, the vault of the Johnstone, Fig. 152, a splendid but expensive device, and that of the Callender Solid System, Fig. 157, may be consulted.

The typical manhole adopted for New York subways, arranged for distributing circuits of all kinds, is shown in Figs. 134 and 161. In Fig. 161 the cross-section of the construction is shown, indicating a brick or iron chamber, into which the iron-pipe ducts open. Special attention is given to the method of securing the cover in order to hermetically seal the chamber. The device con-

sists in a heavy iron frame carrying two covers — an inner and an outer cover. The inner cover rests upon an elevated ring; tightness being secured by means of a circular cylindrical gasket; the cover being forced into place by means of the screw *b*, and cross-piece *b'*. Drainage is secured by connecting the gutter formed by the elevated ring to the sewer by means of the pipe P. The structure is completed by the addition of the street cover. The perspective view of this method of construction is shown in Fig. 134, wherein the arrangement for aiding the ventilation of the subways by means of a large air-pipe placed parallel to, and in the same trench with, the ducts, will be noted. The air-pipes extend from the blower stations and connect with the various manholes; maintaining throughout the subway system a pressure slightly in excess of that of the atmosphere, thus preventing the ingress of gas.

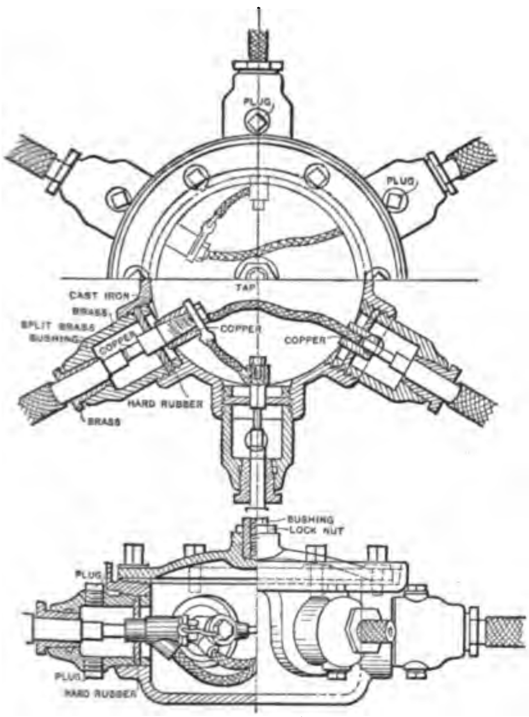


Fig. 162. Street Railway Junction-Box.

191. Junction-Box for Underground Railway Feeds. — A large system of underground distribution has recently been installed in Philadelphia, in connection with the substitution of electricity for animal power upon the street railway lines.

The conduits consist of cement-lined iron pipes, set in concrete. The main subway from the station consists of eight pipes, three inches in internal diameter, laid two feet deep, and intended each to carry two No. 0000 lead-covered cables. The manholes are placed at suitable distances, to enable careful and efficient handling of the wires, and to allow the railway company to make any combination

or rearrangement of the circuits of the different streets, as may be found advisable. The accompanying illustration, Fig. 162, includes the design of a manhole which has been worked out in admirable fashion for this purpose.

A circular iron chamber is arranged, carrying six inlets, something in the shape of a six-point star. Each of the inlets consists of a stuffing-box, through which the cable is introduced into the interior of the manhole by means of a water-tight joint, to which the duct of the subway is hermetically attached. All of the cables thus open into the box, and may obviously be arranged or changed in any desired manner. The cover of the box is firmly locked in place by means of bolts and a water-tight gasket.

192. Introduction of Circuits. — After the completion of the underground structure, the introduction of the circuits becomes a matter for consideration. This is accomplished by the process of "rodding." The workmen are supplied with a number of light-jointed pine sticks, about $\frac{3}{4}''$ in diameter, and from 3 to 4 ft. in length. These rods are equipped with a screw or bayonet joint at either end, that they may be successively jointed together in series. Upon entering the manholes, the operator proceeds to connect up one or two lengths of these sticks or rods, and pushes them through the duct of the subway, into which it is proposed to introduce the circuit. By successive additions to the rod, it may be thus shoved entirely through the duct into the next adjacent manhole. Connection is thus obtained between the two manholes, whereby a rope or wire may be extended through the duct. As soon as this is accomplished, the reel upon which the conductor is wound is placed over the opening into one of the manholes, and cables lowered into the vault around a large-sized sheave or pulley, and introduced into the duct. The cable is then attached to the rope, and hauled through the duct from one manhole into the next one; the necessary traction being supplied by means of a fall and windlass located at the farther vault. Thus length after length of circuit may be introduced, until the entire quantity is laid. The different lengths may be then spliced so as to form a continuous circuit.

Where it is desired to rod a number of ducts extending between two adjacent manholes, the process may be very greatly facilitated by the recently devised method of "Pneumatic Rodding." The

workmen in one manhole are furnished with a "dart," which is a spool-shaped piece of wood about six inches long, having a leather washer on each end, arranged to make the dart fit tightly into the ducts. The rear end of the dart carries a ring to which a light cord is attached, loosely coiled on the bottom of the manhole. At the more remote manhole, the workmen are provided with an air-pump capable of being fitted over the mouth of the duct. The dart being placed on the mouth of the duct, a few strokes of the air-pump produce sufficient vacuum to cause the dart to fly swiftly through the duct, dragging the cord after it, a few seconds of time being all that is required to perform the operation.

193. Gas. — Perhaps the greatest danger to which underground subways are exposed is the accumulation of illuminating gas, either in the ducts or in the vaults. The pipes forming the gas-plants, ramifying through the streets of all the larger towns, are usually constructed of cast iron, which permits a considerable leakage of gas directly through the material of the pipe itself. In addition to this, the leaky joints and service-pipes are sufficient to completely impregnate the soil of the streets with illuminating gas. In fact, the statistics of some cities show that the gas companies are unable to account for some 15 to 20 per cent of the gas manufactured by them, this loss being almost entirely ascribed to street leakage. The vaults and ducts of the subways form convenient places for the accumulation of the gas, which collects in them by percolating through the soil. In the early days of subway construction, many serious accidents happened, either from asphyxiation of the workmen entering the vaults for the purpose of drawing in, or making changes in, the circuits, or from veritable explosions in the subways, due to the gas forming explosive mixtures with the atmosphere, and chancing to ignite from some accidental spark. To obviate casualties from this cause, it is now customary to provide all of the important subways with means of ventilation, two plans having been adopted which have proved fairly successful.

First. — The least expensive method consists in grading each subway between adjacent manholes, so that the ducts shall have a slight fall from one manhole to the next. In order to provide for a reasonably uniform grade throughout the entire subway, sections between the manholes can be arranged so that two adjacent sections

grade *into* the manhole, and the next two grade *away* from the manhole. Then, if the cover of the vault is arranged with suitable openings, so as to provide a chance for the entrance of air, it is found that the subways will keep fairly pure, in cases where the gas leakage into the soil is not too rapid or excessive.

SECOND. — The method of ventilation which is perhaps the surest, although the most expensive, requires the installation of one or more ventilating plants at various points along the subway. Under these circumstances no particular attention need be paid to grading the conduit, excepting for purposes of drainage; and the vault covers, instead of being rendered open to the atmosphere, are made as nearly air-tight as is practicable. At the ends of the subway, or if the length of the underground construction requires it, at several intermediate points, ventilating plants are arranged, consisting of air-blowers driven by some form of prime mover, which constantly forces into the subway quantities of fresh air. At first sight it would seem as if it were essential to keep a steady flow of air through the subway, in order to clear the ducts of the accumulation of gas; but, on the contrary, the attempt of the ventilating plant is to produce a pressure in the subway which is a little greater than that of the atmosphere. Under these circumstances the incursion of gas into the subway is prevented from the mere fact that the pressure there being greater than that of the exterior atmosphere, causes the gas to flow *away from* and not *toward* the subway. Under these circumstances, the air which is forced in finds a natural outlet through the porosity of the soil itself.

194. Metallic Conduits and Cable Sheaths for Alternating Currents. — With the extending tendency to transmit energy by means of alternating currents, the question of the effect of metallic sheaths for conductors, or metallic structures for conduits, assumes considerable importance. Lead and iron are the only metals that have so far been used for this purpose. The impedance to which any alternating current is subjected, is greatly affected by the magnetic permeability of the medium surrounding the conductors; and also depends upon the mechanical disposition of the media. Conductors which are sheathed with metal so disposed as to form a closed circuit have the impedance factor largely increased. Iron sheaths still further augment it. If all the constants of a circuit are known, it is

possible to calculate the impedance; but the value of many of the factors, however, is still quite uncertain. Some French experiments upon lead-covered cables and currents at a frequency of 100 per second show the conductors to be subjected to a loss of energy varying from 1 to 2 per cent. With iron sheaths, from 5 to 10 per cent loss is reported, with one extraordinary instance of a 35 per cent loss. While this subject needs much more investigation, it is certainly safe from present appearances to relieve alternating circuits from the presence of metallic surroundings.

CHAPTER VII.

THE CONSTRUCTION OF UNDERGROUND CIRCUITS.

CABLES AND CONDUIT CONDUCTORS.

Art. 195. Conduit Conductors. — For all underground circuits, excepting such as are designed to go into conduits arranged for bare conductors, some special form of insulation is essential in order to maintain the electrical integrity of the circuits. To accomplish this end, various designs, leading toward the formation of the conductor into cables, have been invented.

196. Armored Cables. — THE SIEMENS CABLES. — The earliest attempts toward the construction of underground circuits consisted in the mere excavation of a trench through the street, into which the insulated cable, carrying the distributing circuits, was placed. Experience demonstrated that it was difficult to build a cable with sufficient mechanical strength to be self-protective against destructive influences constantly at work to cause the deterioration of street structures. Even the best armored cables are liable to be ruined by a single stroke of the pickax; so additional precaution was needed, thus causing the development of the more modern types of subway structures. The armored cable, however, is by no means to be despised as a method of underground distribution. On the contrary, the entire system adopted by the Siemens Bros. is based upon a superior construction of the cable (provided with ample protection against damage), laid directly in trenches excavated under the pavement in the street. The Siemens cable is of the concentric type; that is, the two conductors forming the circuit are not laid side by side, but are arranged one inside the other, separated by the appropriate insulating material. Two advantages accrue from this method of construction, one being a notable saving of space, insulating material, and cost of manufacture, and the other the practical impossibility of forming a short circuit with any exterior object, thus affording an immunity against fire risks or injuries to workmen. Three of

the most valuable forms of concentric cable are shown in Figs. 163, 164, and 165.

Cable No. 1, Fig. 163, is used by the electric light installations

Fig. 163. Siemens Incandescent Light Cable, Paris. No. 1.

in the Théâter du Chatelet, and the Opéra Comique, Paris. This cable has a central conductor of 61 wires, each of three millimeters

Fig. 164. Edison Cable, Paris. No. 2.

in diameter, separated by a layer of rubber five millimeters ($\frac{3}{16}$ of an inch) in thickness, outside of which is placed a surrounding conductor composed of 22 strands, each of 7 wires, $1\frac{7}{10}$ millimeters in

diameter. A second coating of india-rubber, covered with a lead sheath, completes the cable.

Cable No. 2, Fig. 164, is the cable used by the Secteur Edison in Paris, that, in addition to the concentric conductors and lead sheath, is provided with a steel wire armor.

Cable No. 3, Fig. 165, is a special cable used by the Siemens Bros. on the five-wire system that they have established in Paris.

1 and 2 are the concentric conductors; 3 the balancing main, to which allusion will be made in the chapter on parallel distri-

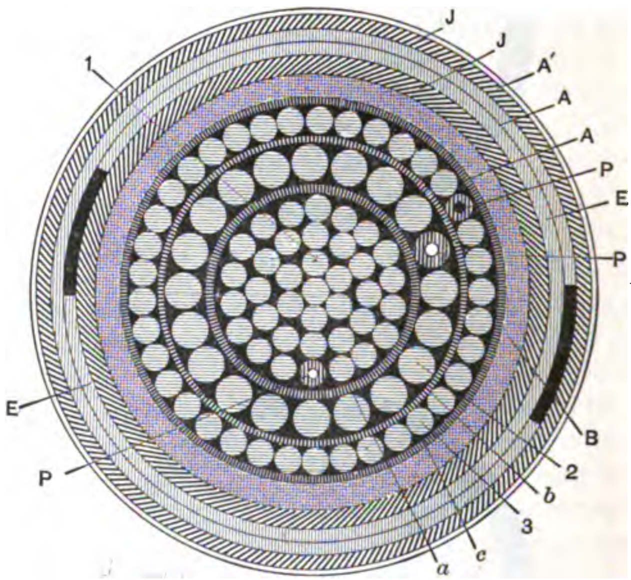


Fig. 165. Siemens Concentric Cable. No. 3.

bution; *a*, *b*, and *c* insulation; B lead covering; J jute; A asphaltum insulation; and E iron armor.

The mains are laid by merely excavating a trench in which a bed of sand is placed for the reception of the cable.

The methods of making service connections are indicated in Fig. 166, being very similar to the Edison system adopted in this country. In order to protect the cable from excavators, it is customary to lay directly over the mains a layer of plank or a length of iron-wire netting. While the iron-wire netting may call the attention of the excavator to the presence of the structure beneath him, the

plank is found to be by far the most sufficient protection, inasmuch as it actually prevents pick or shovel from cutting through and coming in contact with the cable.

197. The Edison System. — Under this system, bare conductors are inclosed in an iron pipe, protected by an insulating compound, the iron pipe having ample strength to protect the circuits from

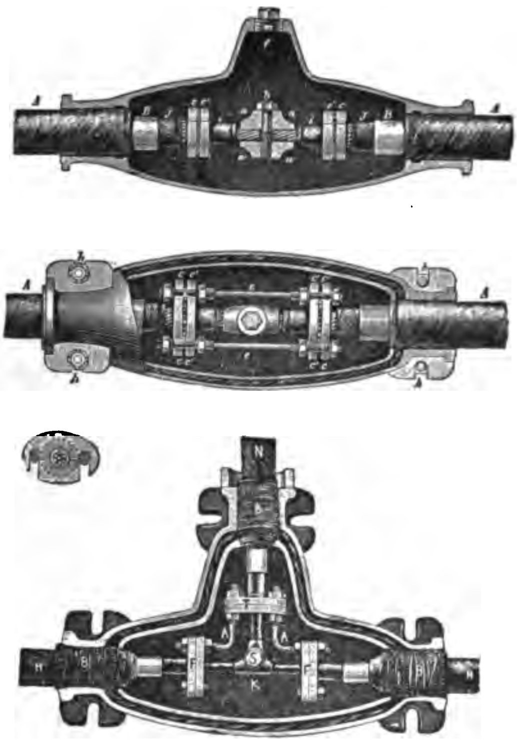


Fig. 166. Junction-Box for Siemens Cable.

external injury. The Edison circuit is formed by inclosing in a 16-ft. length of iron pipe, about 3″ in diameter, three copper conductors of appropriate size. These conductors are separated from each other by winding each one with a loose spiral of jute or cotton yarn of sufficient thickness to insure the separation of each individual rod from any of its neighbors, and from the exterior pipe. The three copper rods, after having been wound in this fashion, are bundled together, and slipped inside of a length of iron pipe. When

the rods are in position the pipe is poured full of a melted special insulating compound, that, on cooling and hardening, holds the conductors firmly in their place. As soon as the insulating material has hardened, the completed section of pipe, with its three conductors, is carefully tested, and, if found satisfactory, becomes a complete section. All of the Edison underground plant is planned upon the three-wire system. Four different sizes of pipe, carrying correspondingly different sized conductors, are in common use, and are as follows:—

1½″ pipe containing 80 M to 120 M circular mils of conductor.
2″ pipe containing 150 M to 300 M circular mils of conductor.
2½″ pipe containing 350 M to 600 M circular mils of conductor.
3″ pipe containing 700 M to one million circular mils of conductor.

The cross-sections of the various sizes of electrical tube are shown in Fig. 167. To lay the mains, excavation is made in the street just under the surface of the pavement. Successive lengths of appropriate size electrical tubes are then laid loosely along the bottom of the trench; each successive length is connected to its neighbor by means of a junction-box shown in Figs. 167 and 168.

In the illustrations it will be seen that the two ends of the pipe enter an egg-shaped casting through two water-tight sleeves at either end of the oval. Inside of the casting, the separate conductors are joined by connectors formed of heavy copper rope. The connectors are screwed to the conductors by means of set screws running through copper castings on the ends of the connecting rope. After the connectors are in place, they are thoroughly soldered to the ends of the mains, thus making the electrical joint. The covering of the egg-shaped casting is screwed down upon the lower half; and by means of a small hole in the top of the casting, the whole of the box is filled full of melted insulating compound, thus forming an absolutely water-tight joint. To connect the consumer to the main line, a junction-box is provided which takes the place of the ordinary connecting-box joining the ends of the two successive pipe lengths, Fig. 168. The service-box is essentially the same as the junction-box, with the exception that it has three outlets instead of two, the third outlet forming a means whereby a third electrical tube may be carried from the street

main to the inside of the wall of the premises of the consumer. At various points along the underground net-work, large distributing-boxes are placed, into which all of the mains from several

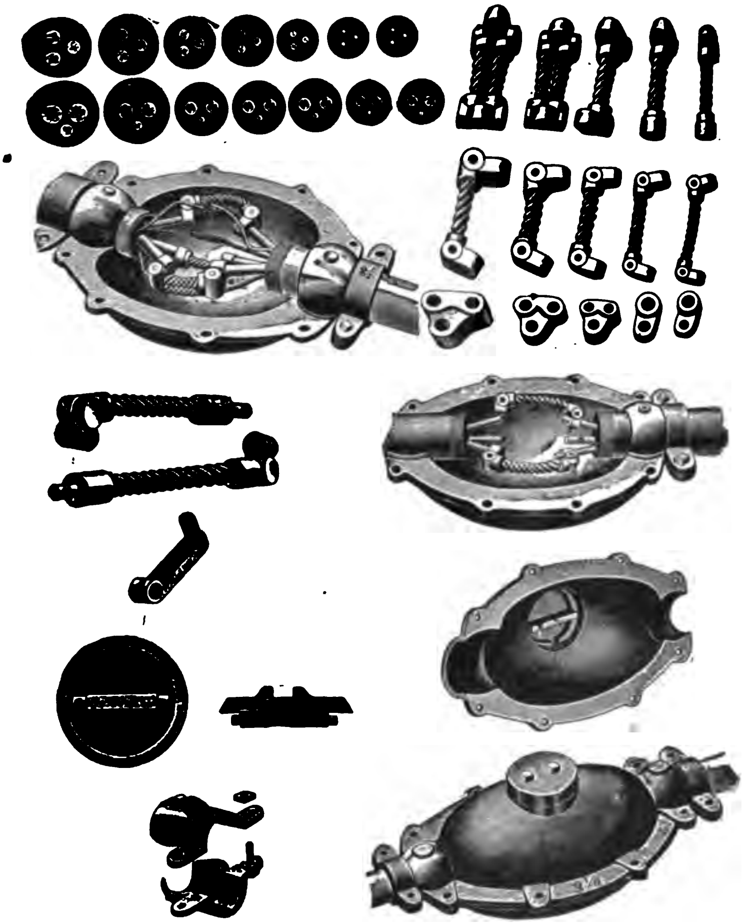


Fig. 167. Edison Tubes and Junction-Box.

adjacent streets extend. By means of the flexible connections shown in Figs. 167 and 168, any desired combination or rearrangement of the circuits may be effected. The service-box also forms a ready means of testing and inspecting all of the circuits so as to insure their adequate and proper maintenance. The Edison

system of conduit has in this country received very large development, a great proportion of our cities being supplied with incandescent lights by means of this system. The Edison system presents

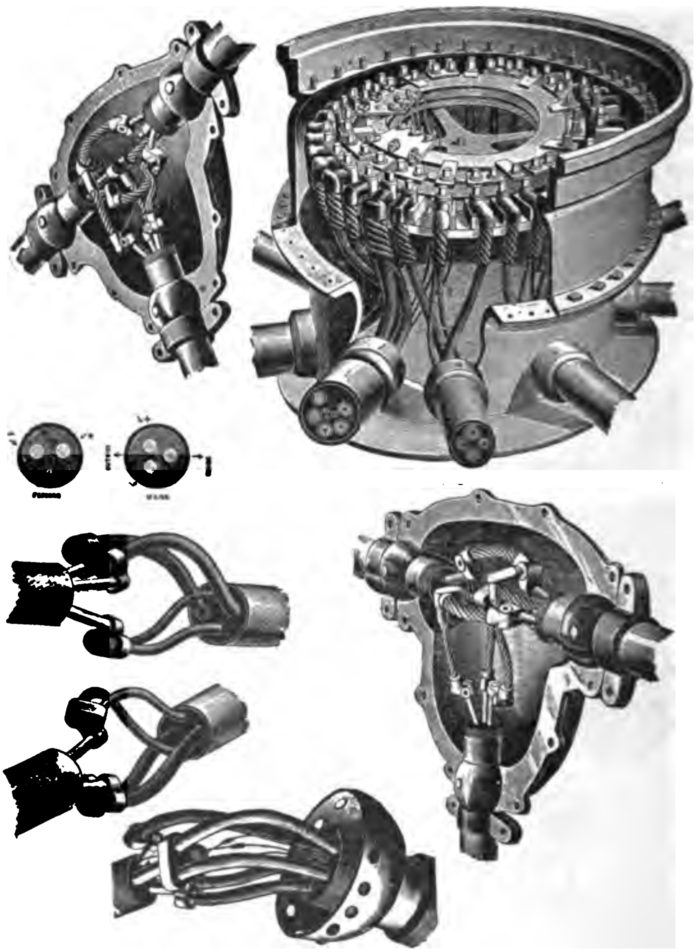


Fig. 168. Edison Distributing and Service Boxes.

the advantage that all the work of manufacture can be done in the factory by machinery, by skilled labor, and under the supervision of thoroughly competent inspectors. The street work simply consists in excavating an exceedingly shallow trench, and laying the mains loosely along the bottom of it, and in suitably connecting

the ends of the adjacent tubes. While the Edison system is one of the most admirable that has been devised, it is obvious that it entails considerable expense on account of the necessity of providing each group of mains with a separate iron pipe.

198. The Ferranti Mains. — A description of cable systems would not be complete without reference to the method successfully put into practice in London by Mr. Farranti, involving transmission of alternating currents at a potential of 10,000 volts. Present practice would but rarely justify such pressures, but the time may not be far distant when this amount will be frequently exceeded. The Ferranti mains consist of two concentric copper tubes, A and E, Fig. 169, separated by half an inch of paper, C, saturated with black mineral wax, and protected from injury by a solid iron sheath, D. About thirty miles of these conductors have

Fig. 169. Edison Distributing-Box.

been made, and are working from the Deptford Station in London. The greater proportion of these conductors are designed to carry 250 amperes, and consist of an inner tube $\frac{9}{16}''$ in diameter, giving a cross-section of one-fourth of a square inch. This is separated from the outer tube by the requisite paper insulation, while in turn the outer tube is similarly separated from the iron sheath. A longitudinal and cross-section of the main is indicated in Fig. 170.

The chief point of interest lies in the construction of the joints in order that both the requisite insulation and conductivity should be secured. For convenience in handling, the mains are made in 20-ft. lengths; and the ends of each piece are turned to form long conical male and female sockets, as exhibited at A in the illustration. When the mains are laid, the successive lengths are forced into accurate

contact by a hydraulic jack. The inner conductors are joined by a copper plug *a*, while the outer ones are connected by a copper sleeve G, that is secured by corrugating the sleeve onto the mains.

199. Telegraph Cables. — For underground telegraph work, the cable is usually made by twisting together a sufficient number of carefully insulated wires to form the desired number of circuits. The wire chosen for the purpose is usually some one of the better forms of rubber-covered wire, to attain the requisite insulation. By means of twisting-machines, the conductors are laid up into a flex-

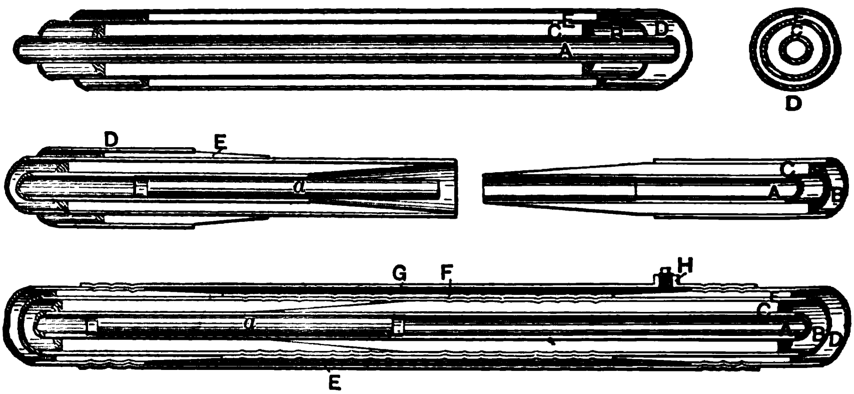


Fig. 170. The Ferranti Mains.

ible rope, which is afterwards covered with one or more layers of heavy braid, treated with an insulating compound, in order to retain the circuits in their appropriate positions. Cables of this description may be made of any desired size, and supplied with circuits of either large or small wire, at the pleasure of the designer. For an additional mechanical protection, they may be subsequently supplied with either a lead sheath or an armor of iron or steel wire. This, however, is rarely necessary, excepting where specially severe service is to be expected.

200. Subaqueous Cables. — Cases frequently arise where it is necessary to cross, with an electrical circuit, a stream or other body of water. Under these circumstances, a cable specially prepared is necessary, to resist the greater severity of the service. The circuits may be arranged as already described for ordinary telegraph

cables, due care being observed to proportion the copper cross-section for the work which the cable is called upon to perform. After the circuits are laid up, it is necessary to afford the cable a much greater protection than is essential for ordinary underground lines. To this end subaqueous cables are frequently supplied with two or more sheaths, in order to make them absolutely waterproof, and then are supplied with an additional armor of iron or steel wire, which is

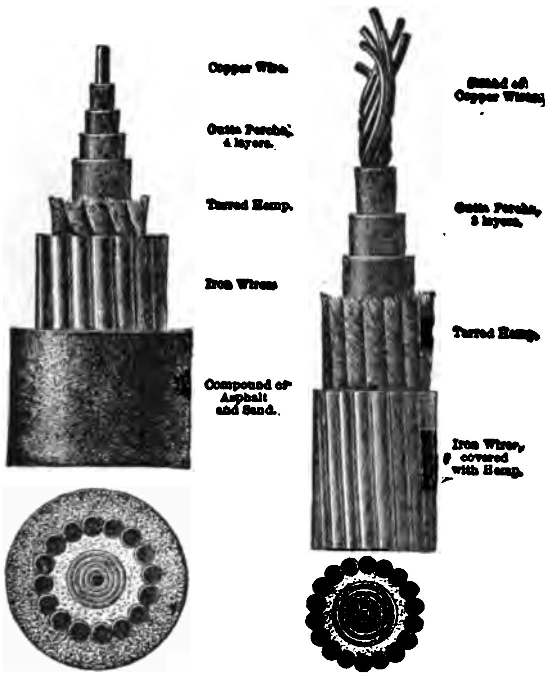


Fig. 171. Submarine Telegraph Cable.

braided over the surface of the sheaths. For submarine telegraph work, the lead sheaths often are omitted, as sufficient insulation can be obtained by covering the wire with a number of layers of insulating compound. The steel armor, however, is an absolute necessity, in order to protect the cable from injury during the period of laying, and to protect it from such destructive influences as chafing against rocks and other obstructions which may be found in the bed prepared for it, and to enable it to resist all injury which may be caused by the keels or anchors of passing vessels. In espe-

cially shallow water, extra precautions must be taken, as here the cable is much more liable to injury. Examples of submarine telegraph cable construction are exemplified in Fig. 171.

201. Power Circuits. — Cables for power circuits may be manufactured of any desired capacity, and especially designed and adapted for particular cases of transmission. For copper cross-sections which are less than 100,000 circular mils, it is customary to use a solid conductor, which is overlaid with several layers of insulating material. For sizes larger than this, the stranded conductor becomes impera-

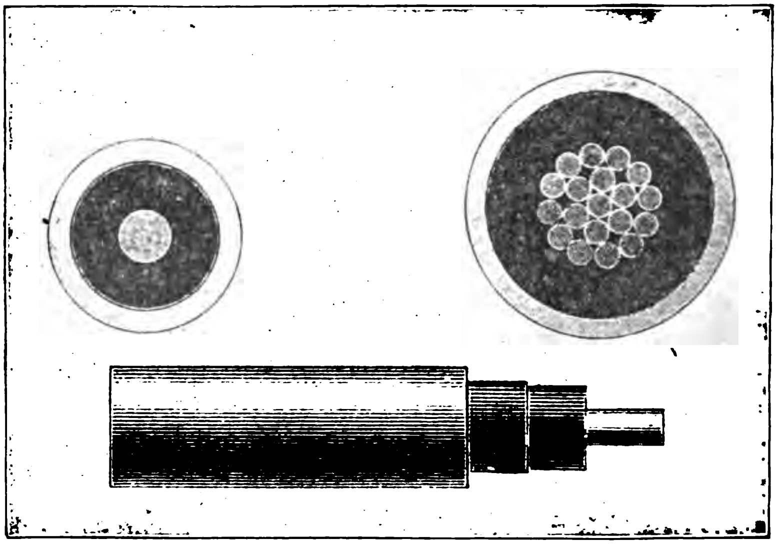


Fig. 172. Power Cables.

tive, as the solid rod is too stiff to permit of the necessary mechanical manipulations that are required for installation. The general appearance of such cables is indicated in Fig. 172. The practice of drawing a lead sheath over cables of all descriptions is rapidly increasing; as it is found that the continuous film of lead affords an almost perfect protection to the cable, and guarantees to the insulating material a much longer life than can be obtained in any other way.

Experience has also shown that paper thoroughly impregnated with insulating compound, such as the various tars or resins, forms

one of the best insulating materials, provided paper can be kept reasonably dry, as is insured by the use of the lead sheath. A very large class of distributing cables are now made with paper insulation, which give the highest satisfaction in actual service. Some of the varieties of paper insulated cables are shown in Fig. 173.

202. The possible variety in design that could be attained for transmission cables is without limit. In Fig. 174, from Nos. 1 to 22, a variety of cable cross-sections are shown which experience has indicated to be serviceable in various forms of transmission, and which may be obtained in the market without the necessity for special manufacture.

No. 1, No. 3, and No. 15 are examples of feeder cable intended for three-wire distributing systems. Nos. 1 and 3 contain stranded conductors. In No. 1 each con-

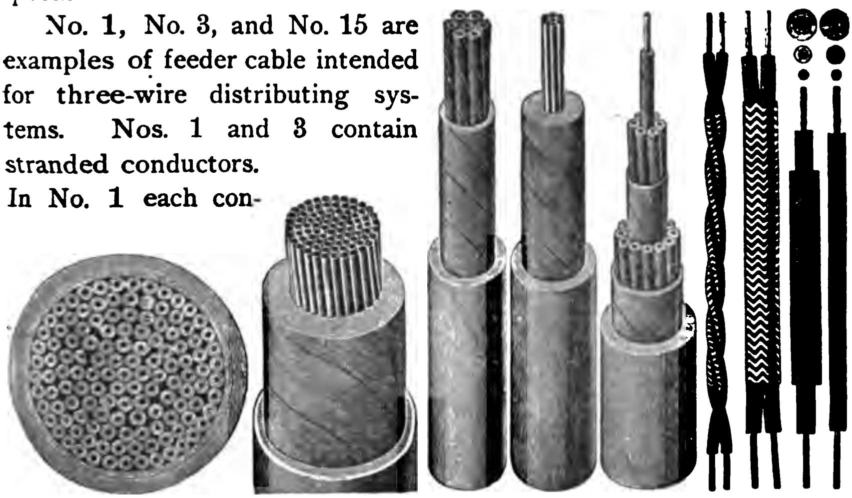


Fig. 173. Paper Cables.

ductor is surrounded by an independent lead sheath, while in No. 3 the lead sheath embraces all three of the mains. Nos. 1 and 3 may be commonly obtained, having from 20,000 to 250,000 circular mils. No. 15 contains no lead sheath, and solid conductors are used, as the cable is rarely called for excepting in small sizes. Nos. 2, 4, 6, 7, and 21 are examples of conductors with lead sheaths and exterior and interior insulation. They are stranded for the sake of greater flexibility, and may be obtained in all the commercial wire sizes. The finer strands, as in No. 2, No. 6, and No. 7, are much more flexible than the coarse wire of No. 4. No. 5 is an unarmored, unsheathed submarine cable designed for

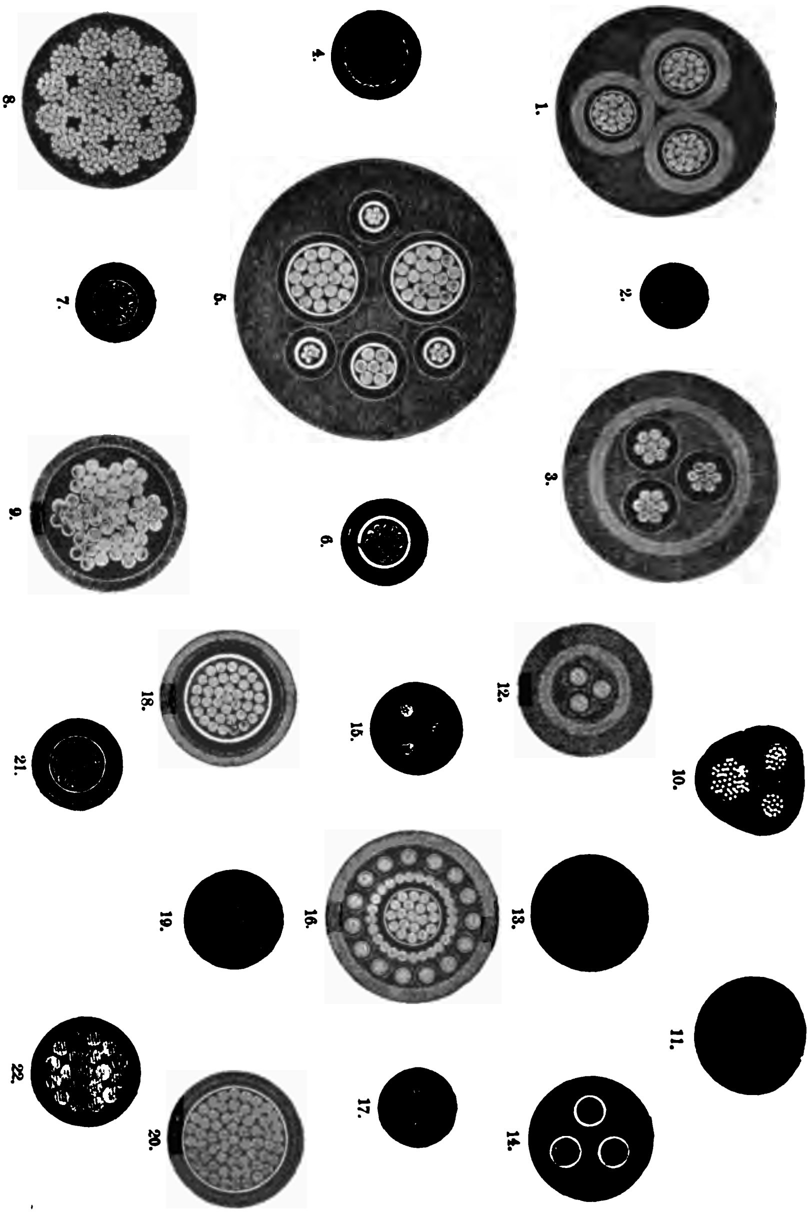


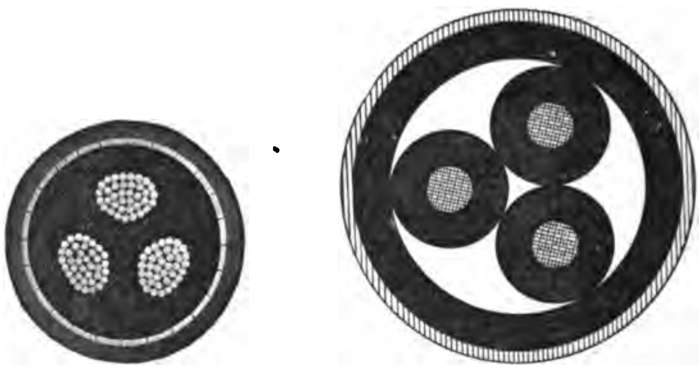
Fig. 174. Sections of Transmission Cables.

transmission on the three-wire system. The two large conductors are intended for the outside mains, copper cross-section, 450,000 mils., while the smaller one fills the office of the third wire. The three small conductors are intended to serve as pilot-wires. Nos. 8, 9, and 11 are feeder cables, designed for underground power distribution, and may be obtained up to 1,000,000 circular mils. No. 18 is also a feeder cable, with an extra protection of the lead sheath. Nos. 20 and 22 are feeder cables with light insulation, but are only intended for interior work in dry locations, and not for underground service. Copper section up to 1,000,000 circular mils. Nos. 13 and 19 are examples of solid lead-covered and plain insulated underground cable, suitable for arc-light service. Nos. 12 and 15 are similar triple-conductor cables of solid conductors. No. 16 is a triple-conductor concentric cable, with lead sheath, especially adapted to triphase transmission or other high-potential work.

203. Increasing experience in the manufacture and operation of underground cables carrying alternating currents at comparatively high potentials tends to confirm the belief that lead-covered cables insulated with paper saturated with some oily or resinous compound are at once the best and cheapest design. It was formerly considered that paper insulation should be limited to low voltages and rubber compounds used for high tension, but it is beginning to be believed that under long-continued severe electric stresses india-rubber deteriorates, either by a kind of electrolytic action or by the purely physical stress to which the electric tension subjects it, while treated paper seems to be exempt from such action. There are now a number of large distribution systems operating cables at from 6,000 to 12,000 volts, a few working from 15,000 to 20,000, and one at 25,000 vol.s. While there is as yet insufficient experience upon which to base an exact rule for the thickness of insulation required, there are many successful plants the insulation of whose cables is about $^1/_{32}$ of an inch for every 1000 volts of pressure. Possibly future experience may show that this thickness can be decreased, but present success makes the cautious reluctant to incur the risk that any reduction might entail.

The safe carrying capacity of underground cables is another point upon which there is a paucity of information. Doubtless carrying capacity depends much upon environment. Thus in electric-railway work, where a number of cables are installed in adjacent

ducts the heat developed by the I^2R losses will increase with the number of cables, and as the number of cables increases there will be both more heat and less opportunity for radiation and conduction through the ground. In such cases the safe carrying capacity will be much lower than for a cable placed in a single duct uninfluenced by companion lines. Conversely, a submarine cable laid on the bed of a running stream would have a very large carrying capacity, as the cooling action of the water would preserve the insulation in safety, even if great volumes of current were forced through the cable. There appears to be a general opinion that about 1000 circular mils should be allowed for every ampere transmitted. In some cases this amount



A B

Fig. 175. Types of High Tension Cables.

can be safely exceeded, but from present experience it is a safe and conservative rule.

Fig. 175 represents some modern designs. At A an English triphase cable is shown. The three conductors are flattened to afford maximum insulation in a minimum of space, and the whole volume of the sheath filled with insulation. At B is an American cable designed for 10,000 to 12,000 volts. Each of the conductors is 500,000 C.M., covered with $^{12}/_{32}$ in. of insulation. The conductors are then cabled and surrounded with $^{12}/_{32}$ in. more of insulation and enclosed in a lead sheath $\frac{1}{8}$ in. thick.

204. Telephone Cables. — For telephone service cables are required which possess slightly different characteristics from those which would meet the specifications for telegraphic or for power distribution. In

the telephone service it is found essential to reduce the electrostatic capacity to the lowest possible figure, in order to produce a conductor which shall have the requisite talking ability, and also to so arrange the conductors as to neutralize all the effects of either self-induction between the talking circuits, or induction produced by the presence of neighboring currents. To successfully accomplish these requisites many experiments have been tried, producing a variety of cables that have been more or less successful.

205. The British Post-Office Cable. — A cable largely in use by the English postal service is composed of tinned copper conductors, each of three strands, aggregating a weight of about 20 lbs. per mile, with a resistance of 45 ohms per mile. Each conductor is covered with two coatings of india-rubber, and then taped with thin india-rubber-coated cotton, and finally with ozokerite. The conductors are then twisted together in pairs, and laid up in cables of the required number, served with jute, and wrapped with stout asphalted tape. After the core is thus completed it receives an additional coating of hemp and another layer of asphalted tape.

206. The Patterson Cables. — The Patterson cables, made by the Western Electric Company, are composed of a number of copper conductors, usually of No. 18 or 19 gauge, which are insulated from each other by being loosely wrapped with a spiral layer of paper, and are then protected by means of a lead sheath. In the earlier cables it was thought necessary to secure the insulation by forcing liquefied paraffine into the cable, the paraffine being aerated with carbonic acid gas. By this means a very high insulation resistance was obtained, with a notable reduction in electrostatic capacity compared with the rubber cable. All cables of this description are made in twisted pairs; the respective conductors, after being insulated, and before being laid up in the core, are twisted together to give a lay of one turn in some 5″ to 7″. The entire core is then formed by laying up successive pairs of twisted conductors in a similar spiral manner. While the use of the aerated paraffine was found to be a marked improvement, so far as the electrostatic capacity was concerned, over the former methods, the cable was yet found to present an objectionable amount. To still further reduce this feature, recourse was had to the *dry-core cable*, which simply consists in paper-

covered copper conductors laid up and covered with a lead sheath, with no other form of insulation. By this means a cable is obtained in which the dielectric consists largely of air. Cables of this description are made as low as .04 microfarad per mile. The objection to this style of cable lies in its liability to injury, in case the lead sheath is ruptured, and the cable subjected to moisture. When the cables are manufactured, it is customary to seal each end of the cable by the introduction of paraffine or similar insulation, for a space of a few feet, in order to prevent the incursion of moisture when the cable shall be spliced. So long as the lead cover remains intact, no difficulty is experienced; but a rupture of the lead is sufficient to admit moisture to the paper core, when by the capillary attraction of the paper, the moisture is liable to become distributed throughout the entire length of the cable, thus utterly ruining it.

207. The W. T. Glover Cables. — The earlier cables manufactured by this firm were designed for grounded circuits, and were constructed in a manner to lessen and avoid cross-talk, and termed "Anti-Induction Cables." The cable was formed of the requisite number of insulated wires, usually of No. 18 gauge. The wire was of tinned copper, insulated with several thicknesses of pure rubber strip, and served with prepared tape. A number of the wires in the cables were then coated with a continuous layer of tin-foil, and placed in definite positions in the cable, with regard to the remainder of the conductors. Inasmuch as the location of the wires covered with lead foil was accurately known, they served as a means of locating the positions of the other circuits. The arrangement of the wires in the cables also was such that the lead foils were all in electrical contact. The core thus formed was further protected by means of a lead sheath arranged to come in contact with the previously sheathed conductors. As a result, all of the sheaths were grounded by being connected to the exterior coating. This arrangement of sheathing was designed to intercept induced currents, and protect the cable from cross-talk and the other effects of induction. To design a cable for metallic circuits, a new form was arranged that has been termed the "Magpie Cable." This consists of a number of wires arranged in double pairs. The wires are insulated in the manner already described. The arrangement of the conductors is such that

the wires are laid up in strands of four, one of each pair in each strand being covered with white tape and the other pair with black, thus serving as a means of distinguishment, in order that the appropriate conductors may be readily picked out and assigned in arranging the circuits. While these cables have found a wide introduction, the lead sheath and tin-foil wrapping cause them to have a very high electrostatic capacity, something like .27 of a microfarad per mile.

208. The Fowler-Waring Cable. — Two different forms of cable are manufactured under this trade name.

The first class, the Waring cable, is arranged by twisting together a sufficient number of copper conductors and incasing them in a leaden sheath. When this is accomplished, the whole cable is forced full of heavy petroleum oil, something in the fashion of the Brooks system. It is claimed for this cable that it has the ability to resist very high temperatures without serious injury to the insulation. Experiments have been made which show that individual conductors may be heated nearly red-hot, or the exterior of the cable heated to the fusing-point of the leaden sheath, without serious injury.

The second class of cables, known as the Dry Core, is made by wrapping the conductors with a specially prepared vegetable fiber, arranged to be non-hydroscopic. The wires are then twisted together and lead-covered. This arrangement attains practically the same result as is secured by the paper cables, with the supposed advantage that the prepared fiber does not render the cable so likely to absorb moisture. The Waring cable has an electrostatic capacity of about .16 of a microfarad, and the Dry Core about .07 of a microfarad per mile.

209. The Felten-Guilleaume Cables. — The cables manufactured by this firm are similar, so far as their styles of rubber insulation are concerned, to those of other manufacturers; but they offer a very ingenious and exceedingly valuable form of paper cable. The design of these cables is shown in the accompanying illustrations, Figs. 176 and 177, giving a cross-section and the method of making a single "twisted pair." The conductors are arranged either in pairs or in fours, and are separated by a single strip or two strips

of paper. From the illustration, it will be observed that each pair is made by laying between the copper conductors a strip of paper,

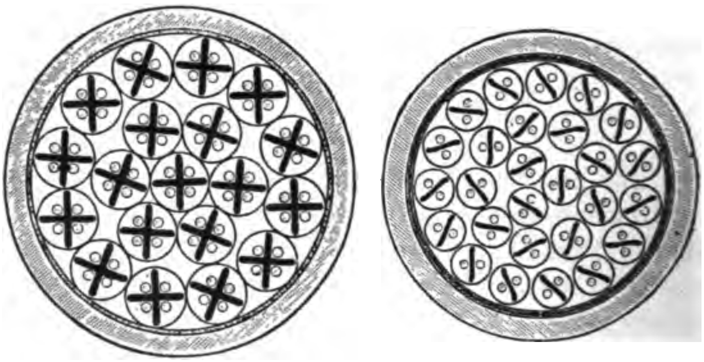


Fig. 176. Sections of Felten-Guilleaume Cable.

which is then twisted, and subsequently surrounded with an additional layer of paper. Each pair of conductors, or each set of four,

Fig. 177. A Twisted Pair.

is thus inclosed in a little paper tube separated transversely by one or two diaphragms of paper. After the completion of the core, the whole cable is incased in a leaden sheath, and then may be further protected by an additional layer of tape or of iron armor. The paper for these cables is either ordinary dried paper, or it may be impregnated with an oil or resin to prevent the incursion of moisture. It is asserted that specimens of these cables have been shown with a capacity as low as .05 microfarad per mile. The same firm also manufactures at present the most successful telephone cables for submarine work. The same system is used, but the construction for marine work is necessarily somewhat different.

A marine cable containing four conductors is shown in Fig. 178.

The four conductors, with their cross-shaped paper diaphragm, are seen at the center of the cross-section. The group is then wrapped with paper, as previously indicated. This is then sheathed in a lead tube, which is afterwards supplied with an additional insulation in the shape of a double coating of gutta-percha. The armor of the cable, instead of being ordinary iron wire, is formed of galvanized

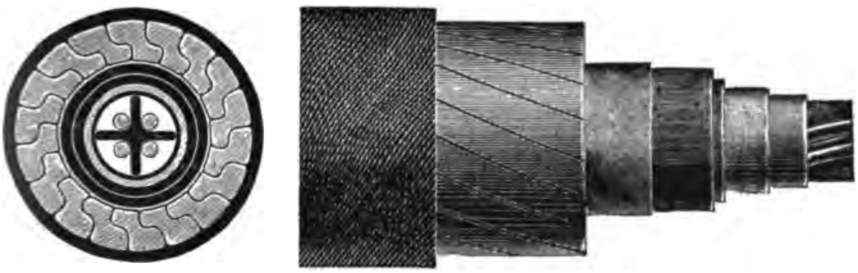


Fig. 178. The Guilleaume Submarine Telephone Cable.

wires so arranged that they lock into each other, forming an envelope that is exceedingly firm and incompressible, and which effectually protects the paper of the cable core from becoming compressed, and the conductor short-circuited.

210. The Herrmann Beaded Cable. — One of the early attempts looking toward the reduction of electrostatic capacity for telephone cables was an invention by Herrmann, who conceived the idea of incasing the several conductors of cables in a series of wooden beads, and then sliding them inside of the common leaden sheath. This construction is indicated in Fig. 179.

While this invention did secure a considerable reduction in the capacity over ordinary rubber insulators, it is more expensive, and has a still greater capacity than the present form of paper cables.

211. Cable Joints and Splices. — The operation of joining underground conductors having a solid core consisting of a single strand, or splicing multi-circuit cables, is one which requires the exercise of extreme care and the employment of the very best skill and workmanship, in order to make splices which shall be durable and lasting, and which shall continuously preserve the conductors from the incursion of moisture.

212. CASE 1. *Single-Conductor Cables.* — For splices or branches in single-conductor cables, the work should be performed as in Fig. 180, Nos. 1 to 14 inclusive. In order to splice a single stranded cable, the insulation should be carefully laid bare for a length of from 3″ to 18″, depending upon the size of the cable core. At either end the insulation should be carefully tapered away to form a long

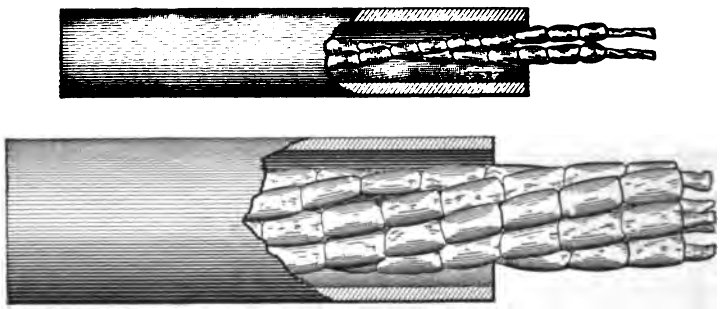


Fig. 179. The Herrmann Cable.

cone. The strands of the cable should next be tightly twisted together and dipped in solder, to secure the ends of individual wires. The ends may then be beveled, as indicated in No. 6, with a long scarf, which should then be thoroughly and carefully soldered together, under no circumstances using any acid as a solder flux. When the scarf is thoroughly soldered, it should then be wrapped with a continuous tight serving of copper wire, as indicated in No. 2. The joint is then completed, as shown in Nos. 3, 4, and 5, by wrapping layer after layer of okonite tape around the joint, until a smooth, conical splice is obtained, as indicated in No. 5.

Solid conductors may be spliced in a similar manner, as indicated in Nos. 6, 7, and 8. A branch in a cable may be taken off in a manner similar to that indicated for splicing, excepting in so far as the description of that process refers to the actual joining of the conductors in the cable. The process for taking off a branch is indicated in Nos. 11 to 14 inclusive. Here the insulation of the cable is laid bare for a couple of inches, the insulation being carefully tapered away on each side. The branch is then firmly lashed to the cable by means of a serving of copper wire, as indicated in No. 11, the whole being

securely and firmly soldered to the cable core. The insulation is then replaced in a manner similar to that for making splices, excepting that the layers of tape must be served over the branch as well as

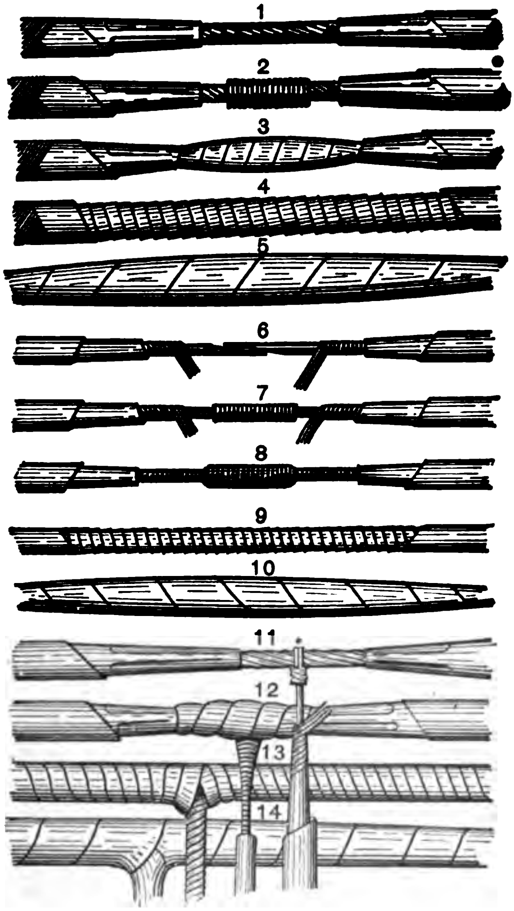


Fig. 180. Cable Splices.

over the core of the cable itself, the final completed joint being finished as shown in No. 14.

213. CASE 2. *Multi-Conductor Cables.*—For all of the special forms of cables, such as those made by the Siemens Bros. and the Edison Company, special methods of splicing are used, which have been indicated in the accounts of the respective styles of cables.

For multi-conductor cables of rubber insulation, a length of the cable from 8″ to 2 ft. in length, depending upon the number of circuits, must be laid bare of insulation. The separate circuits must then be carefully untwisted from each other, each circuit being properly tagged to preserve its identity. The insulation must then be removed from each of the individual wires. The splice is effected by twisting together and soldering each conductor to the conductor to which it is assigned in the new piece of cable. Insulation, usually consisting of okonite tape, or some equivalent rubber compound, is then replaced upon each of the individual circuits, the circuit replaced in as compact a form as possible, and the whole splice completed by three or four layers of okonite tape serving the entire cable and binding the circuits into position. With special skill, splices may thus be made in okonite cable, which can hardly be detected from the regular cable.

Lead-covered cables with paper compound cores may be spliced by cutting away the lead sheath, exposing the conductors, and splicing them, as has already been indicated. As soon as the splice is completed, a piece of lead pipe of sufficient size, previously slipped over the cable, may be soldered to the lead sheath on either side of the splice, making an absolutely water-tight joint.

For dry-core paper cables, an additional process is necessary to seal either end of the cable to prevent the entrance of moisture while the splice is being made. To this end, as soon as the cable is opened, it is thoroughly heated and dried by immersing it in a bath of boiling paraffine oil, and then hot paraffine is poured into the cable to entirely fill it up, and seal it for the space of some 2 or 3 ft. This is done on each end of the pieces to be spliced; and then the conductors are connected, and are insulated by being covered with paper tubes, the whole core bound together with tape, and a lead sleeve soldered over the joint. With careful workmanship, splices of this kind can be made without injuring the cable in any respect, and without increasing its diameter at the splice more than fifty per cent over that of the original cable.

214. The Connection of Underground and Aerial Systems. — The connection of underground and aerial systems is a problem of great practical importance. It is customary to construct at the junction between the pole-line and the conduit system, a vault or man-

hole of the requisite dimensions, directly at the base of the anchor pole forming the end of the pole-line. From this vault, iron pipes of sufficient diameter to permit easy introduction of the necessary cables are run up alongside of the poles to such a distance above the street as to secure the cable from malicious injury. The pipes extend through the earth, and are built through the wall of the vault, with a curve of some 5 or 6 ft. radius, to permit of the easy introduction of the cable. The necessary cables are then passed through the iron pipes, up the side of the pole, and are terminated in cable heads, usually placed upon a balcony or platform, set directly under the lowest cross-arm, Fig. 181.

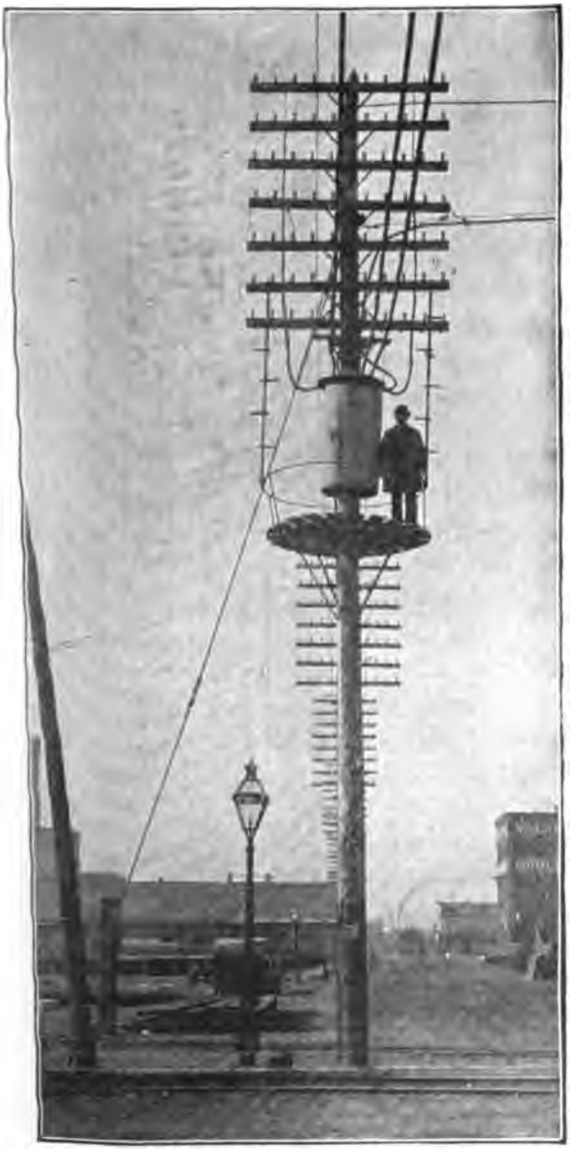


Fig. 181. Cable Terminal Pole.

The cable head is a rectangular cast-iron box, represented in Fig. 182, about 4″ in thickness, some 8″ or 9″ in width, and varying from 18″ to 4 ft. in length, in order to accommodate from twenty-five to one hun-

dred pairs of wires. The lower extremity of the box terminates a brass tube, which, being threaded into the casting, forms a w tight joint. The sleeve of the cable is soldered to the brass thimble, thus completing the connection between the box and cable. The office of the cable-box is to afford a water-tight compartment having a sufficient number of binding-posts to correspond to the number of wires in the cable. Upon the completion of the soldering of the cable sheath to the brass thimble at the base of the box, the wires composing the cable are untwisted inside of the cable-box, and each one soldered to the interior terminal of the binding-post. By this means the wires from the cables are extended through the cable-box to the exterior, in such a manner as to make a waterproof connection, and to afford an easy and rapid means of distribution to the pole-line. The appropriate number of cable-heads, corresponding to the number of cables ending in any pole-line, are placed in a circular wooden compartment built around the pole above the top of the balcony. The entire construction is indicated in Fig. 181, showing the balcony, cable-box, and cables to the aerial line. The sides of the cable-head, as represented in Fig. 182, are supplied with lightning arresters, of the pattern shown in Fig. 132 A. As aerial lines are particularly subject to the incursion of strong currents, these protectors are a necessity, to guard the cable wires from injury that would be much more serious than any damage resulting to the open-wire conductors.

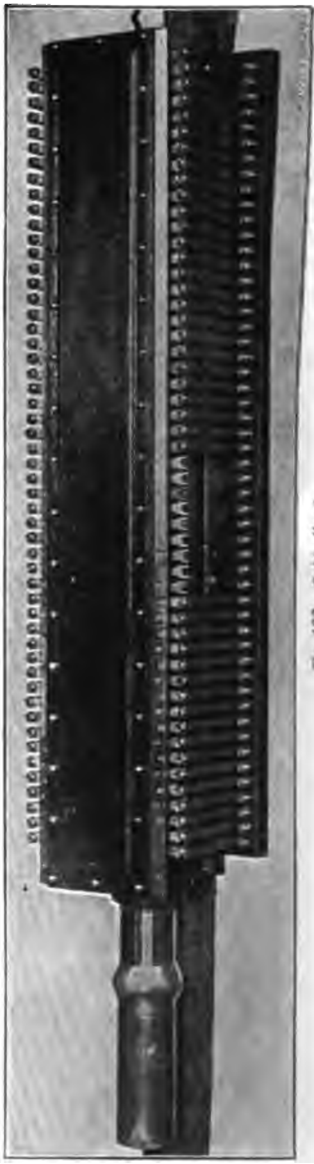


Fig. 182. Cable-Head.

CHAPTER VIII.

SPECIAL RAILWAY CIRCUITS.

THE INTERURBAN TRANSMISSION LINE. THE THIRD RAIL. THE URBAN CONDUIT.

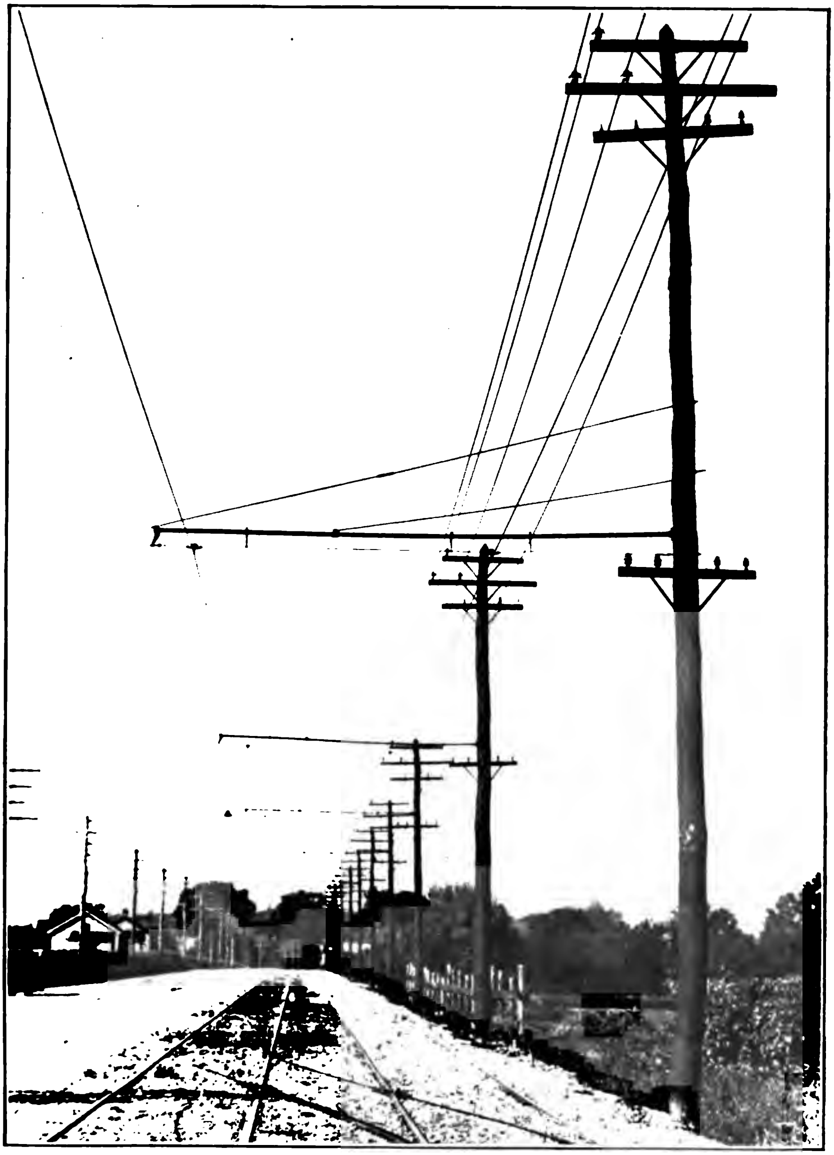
Art. 215. At first the electric railway was merely a substitution of motive power on a street railway. That its projectors had the faintest idea of the transportation systems which would develop from the application of electricity is improbable, for one of the most famous of the early railway men is credited with enunciating the rule: "As two horses can pull a car, put on a three horse-power motor and you can go fifty per cent. faster." The street-railway car of '90 was 16 ft. to 18 ft. long, weighed possibly 3 or 4 tons, and contented itself with a speed of 4 to 5 m. p. h. Now electric cars are equipped with motors aggregating 400 H.P., are 50 ft. to 60 ft. long, weigh 40 to 50 tons, and achieve a running speed of more than a mile a minute. Originally merely a street railway handling urban transportation, the electric railway is now spreading a network of lines over the entire continent, so that to-day one may travel from Portland (Me.) to Chicago (Ill.) with but a few short gaps.

To handle the necessary quantity of force to propel heavy cars at high speed over long distances with sufficient economy, presented weighty problems to electrical and mechanical engineers, the solution of which has resulted in the development of three special forms of electric circuits designed solely for railway purposes. These are:

1. The Interurban Transmission Line.
2. The Third-rail System.
3. The Urban Railway Conduit.

216. The Interurban Transmission Line.—By an unwritten law based on experience the electric railway is usually operated by direct-current motors at a potential of from 500 to 600 volts, and though many efforts have been made to increase potential, or to substitute

alternating-current motors, or to devise other systems, none save the alternating motor seem likely of commercial adoption. So when electric railways commenced to extend interurbanwise the first problem to be solved was a system of transmission capable of carrying sufficient energy over great enough distances to move heavy cars at high speed without an excessive investment in copper and without too great a multiplicity of power stations. The amount of energy which can be sacrificed in the conducting system will depend on the relative prices of fuel and labor, the nature of the load which the system bears, and the price of conductor material; and may for any particular case be calculated according to the principles developed in Chapters XIV and XV. Irrespective of any question of cost, it is undesirable to operate motors at a voltage other than that for which they are designed. A drop of 10 per cent. or 50 to 60 volts is as much as the best practice sanctions. Doubtless many instances can be cited where a drop of 100 or even 200 volts exists but such cases are (or should be) cases of "what not to do." Therefore the design of a railway transmission system demands that the location of the station and size of the line shall be so proportioned as to transmit the required current with a drop of something under 100 volts. This condition, with the burdens electric roads now carry, restricts the working radius of a direct-current station to about 8 miles; it rarely falls below 6 miles, and equally rarely rises above 12 miles. To serve a large district the plan most frequently adopted is to build a power station containing machinery for producing direct current to feed the immediate vicinity at 600 volts, and apparatus giving alternating currents at a relatively high potential, which may with small loss be transmitted many miles and then rectified for car service. The so-called double-current generators are now becoming common for this purpose. These are machines giving a triphase alternating current on one side and direct current on the other. By means of step-up transformers the pressure is raised to from 10,000 to 25,000 volts for the transmission line. At various points in the territory *rotary substations* are placed. Here step-down transformers lower the line potential to from 300 to 500 volts and deliver the current to a *rotary*, a dynamo whose office is to transform the alternating current into direct current at about 550 volts. The substations are so placed that they may feed a district of about 8 miles radius. This plan has developed the inter-



Side Pole Line, Boston & Worcester Ry.

To face page 252.

An Example of a Direct Current Line.

To face page 253.

urban transmission line into a very complex piece of circuit construction.

217. The interurban transmission line is usually of the side-pole type, though sometimes built with span wire. It must carry four

Fig. 183. On the Detroit, Ypsilanti, Ann Arbor & Jackson Railway.

entirely distinct circuits: 1st. A high-potential alternating-current line supplying power for the entire territory; 2d. The direct-current 500-volt feeds to operate a particular section of the railway; 3d. The trolley-wire to distribute energy to the cars of this section; 4th. Telephone, telegraph, or other signal circuits used in operating the road. The methods of building such lines are best exhibited by some

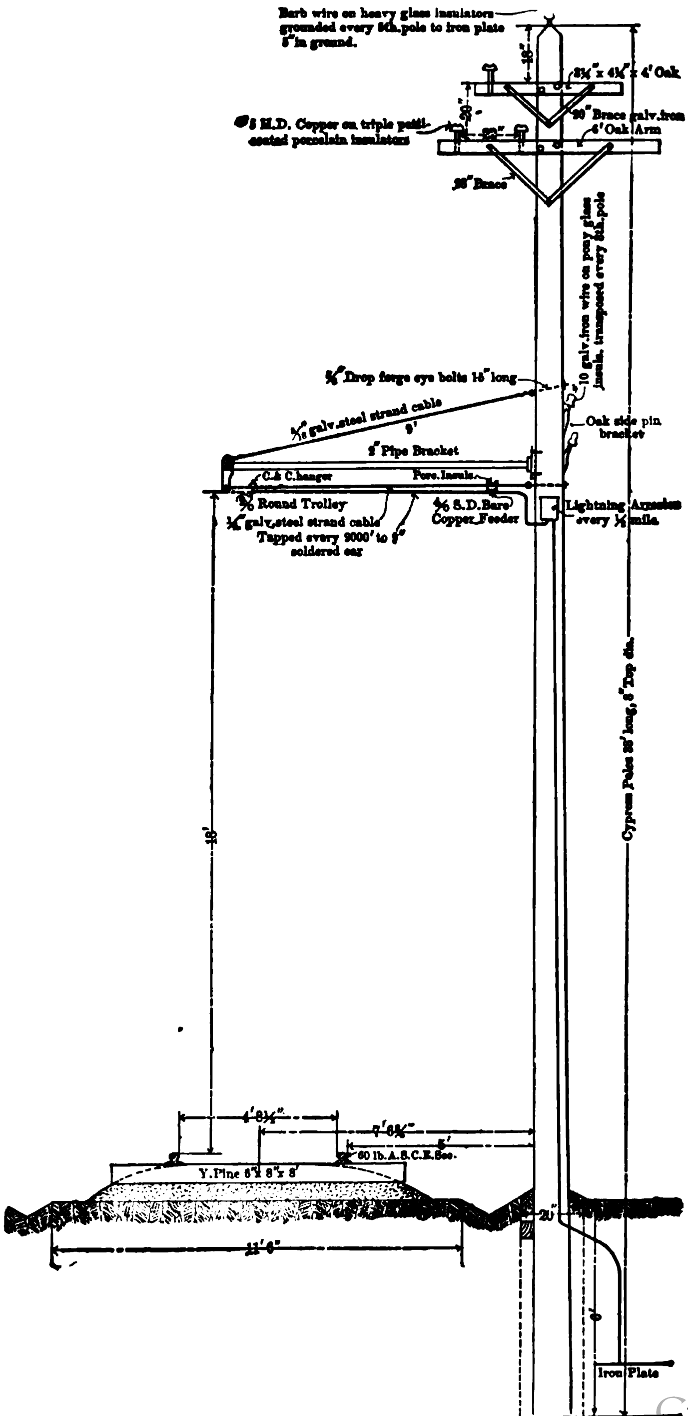


Fig. 184. Details of Transmission Line. Single Bracket.

examples from actual practice. The frontispiece is from a photograph of a large interurban road in southern Ohio, while Fig. 183 represents a well-known road in Michigan. The line is usually built of substantial poles from 35 ft. to 45 ft. in height. About 18 ft. above the ground

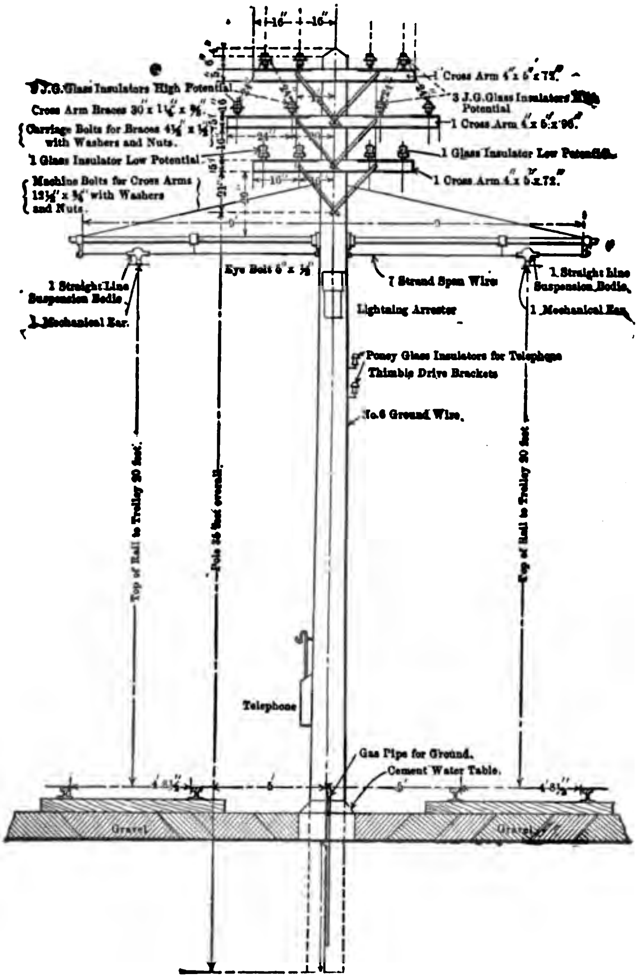


Fig. 185. Details of Transmission Pole. Double Bracket.

the bracket for holding the trolley-wire is placed, as will be seen in dimension drawings of Figs. 184 and 185. The railway feeds, and telephone or other signal wires, are carried on a cross-arm placed near the trolley-bracket, sometimes above and sometimes below, according

to the fancy of the designer, or even supported on brackets on the side of the pole as in Fig. 184. The top of the pole is devoted to the high-tension lines. Large roads usually use two or even three circuits, while the small ones content themselves with one. Several designs

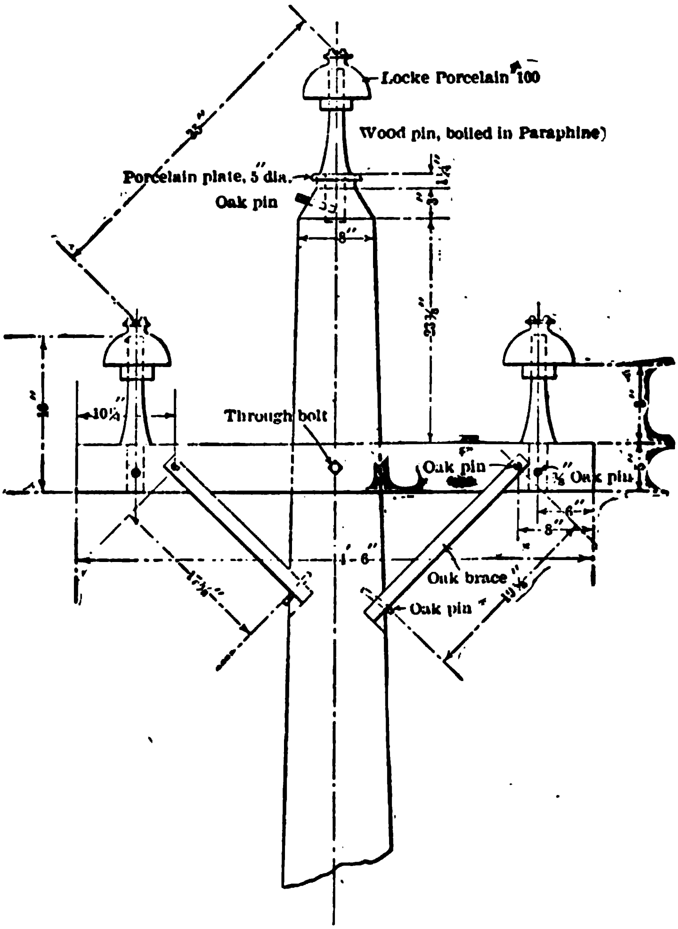


Fig. 186. Pole Top.

are common, some of which are shown by dimension drawings in Figs. 186 and 187. At *A*, Fig. 187, is a design for a 35-ft. pole carrying single triphase circuit. One wire is usually placed on the apex of the pole, as shown in detail in Fig. 186, while the other two phases are supported on a cross-arm 4 ft. 6 in. long, set about 2 ft. below

the top of the pole. *B* and *C*, Fig. 187, are designs for 40-ft. and 35-ft. poles for two triphase circuits. For convenience in repairing these designs show two phases located on a cross-arm placed on the top of the pole. While this plan has the disadvantage of making the pole appear top-heavy, it obviates the necessity for linemen to crawl through the high-tension circuits when repairs become necessary. Fig. 184 shows a design for a 38-ft. pole in which the long cross-arm

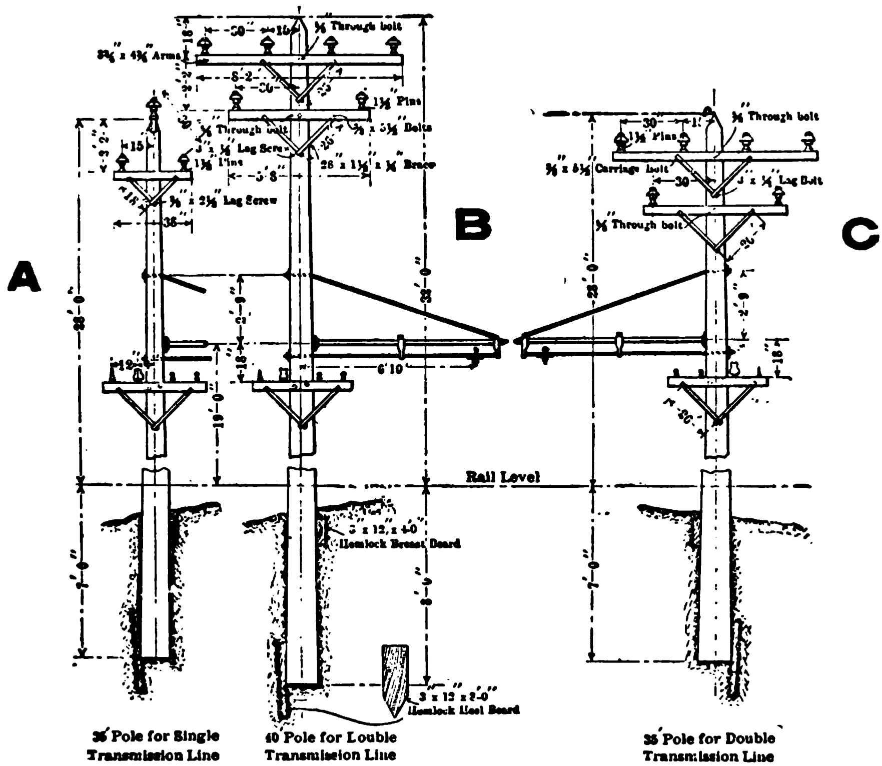


Fig. 187. Transmission Line Details. Poles for One and Two Circuits.

is placed lowest on the pole. The apex of the pole of this design is supplied with an insulator to carry a barb-wire lightning-guard.

When the apex of the pole is not occupied by one of the circuit wires it is quite common to equip it with a pony insulator and string thereon a strand of barb fence wire, which operates as a lightning-guard, and has been proven by experience to be one of the best methods of protection which can be adopted.

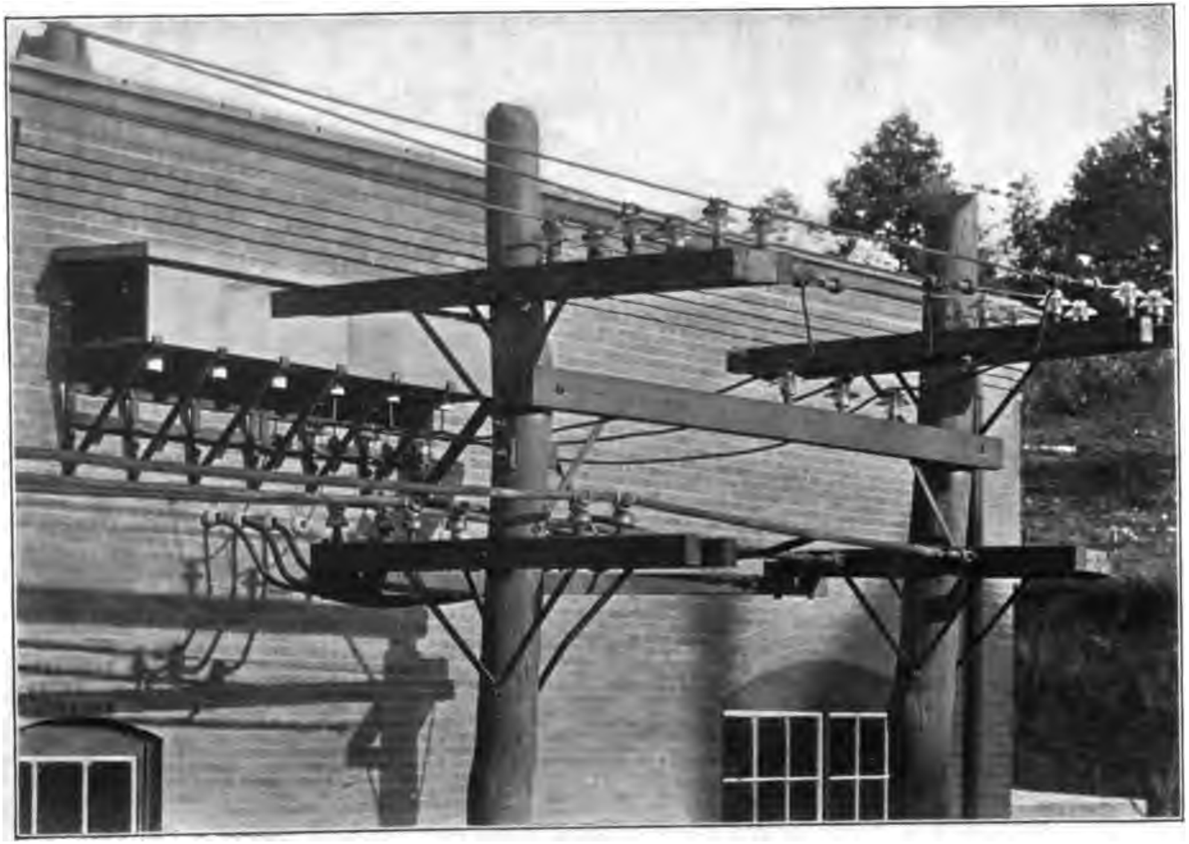
Plates 4 and 5 are illustrations of typical railway lines of recent construction. Plate 4 shows double track, center pole and bracket construction along the lines of the Boston & Worcester Railway. The railway is set upon one side of the highway, the poles occupying the center between the two tracks. Each pole carries a pipe bracket, from which the trolley wire hangs, immediately beneath which is a cross-arm, carrying the telephone or signal wires. The pole-top is equipped with three cross-arms, the lower one of which carries the direct-current feeds, while the two upper ones are intended for two triphase circuits. Plate 5 is another photograph from the Boston & Worcester lines, exemplifying a case where it became necessary to use bracket construction for double track and yet place the poles upon one side of the railway. Here 50-ft. 8-in.-top poles are employed to carry the high-tension lines at a desirable height above the roadway. A pipe bracket of sufficient length supports the trolley wires, which is reinforced by two tension-guys, sustaining the end of the bracket from the pole. Otherwise the arrangement of circuits is not different from that of Plate 4. The neatness, compactness, and symmetry of these examples are in striking contrast to the slovenly work which is often found upon the older direct-current installations, for in such lines poor engineering, a pennywise and pound-foolish desire to economize copper, or bad judgment as to the probable traffic demands, is often responsible for a mess of wiring that is a disgrace to electrical science. For example, Plate 6 is a bit of line construction within a very few miles of New York City, and illustrates forcibly the happy-go-lucky fashion that has prevailed in the construction of some electric-railway properties.

The entrance to the power station or substation, as the case may be, deserves most careful planning in order that the conductors may be permanently and properly secured and carried within the walls of the building so as to become neither a fire risk nor an architectural offense. Plate 7 is the cable entrance to a power station of the Union Railway Co. and shows the kind of *rat's nest* that inevitably develops when cable after cable is strung in an attempt to bolster up the circuit with sufficient copper to carry the load, when no adequate first-hand design has been provided to care for the extension of the system which normal growth of business demands. Plate 8, in contrast with Plate 7, shows the method adopted to carry the circuits into one of the substations of the Boston & Worcester Railway. The lines from this substa-

Power Station Entrance D. C. Line.

To face page 258.

PLATE VIII.



Substation Entrance A.C.- D.C. Line, Boston & Worcester Ry.

To face page 259.

tion deliver over twice the energy that circulates in the cables shown in Plate 7, and the neatness and trimness of the installation are a sufficient commentary upon the desirability of good design.

218. The Third-Rail System.—While the overhead trolley has been eminently successful on urban railways, there are signs that the interurban road taxes it beyond its capacity. Thus on a car making 60 m. p. h. the rotative speed of the trolley-wheel is from 3000 to

Fig. 188. On the Chicago, Elgin & Aurora Railway.

6000 r. p. m., and to provide a successful bearing for such a wheel on the top of a long pole exposed to all sorts of grit is a difficult mechanical problem. The area of contact of the wheel and wire is a very small one through which to convey the heavy current demanded by large swift cars. The most promising plan to remedy these difficulties yet proposed, and which has already received successful commercial applications of magnitude, is that of the *third rail.* This system is constructed by making every fifth or sixth tie of the permanent way two or three feet longer than the others. An ordinary T rail, mounted

on insulators, is supported by these ties, and serves as a conductor. Each car is furnished with one or more sliding contacts usually flexibly suspended from the trucks, so adjusted as to bear upon the surface of the third rail. Thus a conductor of low-priced material is secured, which may be made of ample size, and as solidly and substantially constructed as the designer may desire. Also it is easy to make the shoe, as sliding contact is termed, of any dimensions and enable it to carry as much current as the heaviest of trains can demand. The general appearance of third rail is shown in Fig. 188, a view taken upon the Chicago, Elgin & Aurora Railway, showing the running rails in the immediate foreground with the third rail upon its insulators at the right hand.

219. The problem of insulating the third rail has presented the greatest difficulty. It must be set very close to the track, and only

Fig. 189. Third-Rail Insulator. Umbrella Pattern.

Fig. 190. Block Insulator for Third Rail.

a few inches above the ground. The rail itself is quite rigid, but the insulators rest on the ties, which spring under the impact of the car, so a connection more or less flexible must be provided between rail and tie. Fig. 189 shows one pattern. An iron pedestal supports on a pin a block of insulating material in the form of a cone. On the cone is an umbrella of iron with two lugs between which the third rail rests, by its own weight. Another form is as in Fig. 190, and consists of a non-conducting block bolted to the ties, having a pair of pivoted clips that embrace the rail flange and hold it in place. For the insulator itself a trade compound known as *Reconstructed Granite* has been found valuable. This material is made by pulverizing

PLATE IX.

A View on the Lackawanna & Wyoming Valley Ry.

To face page 260

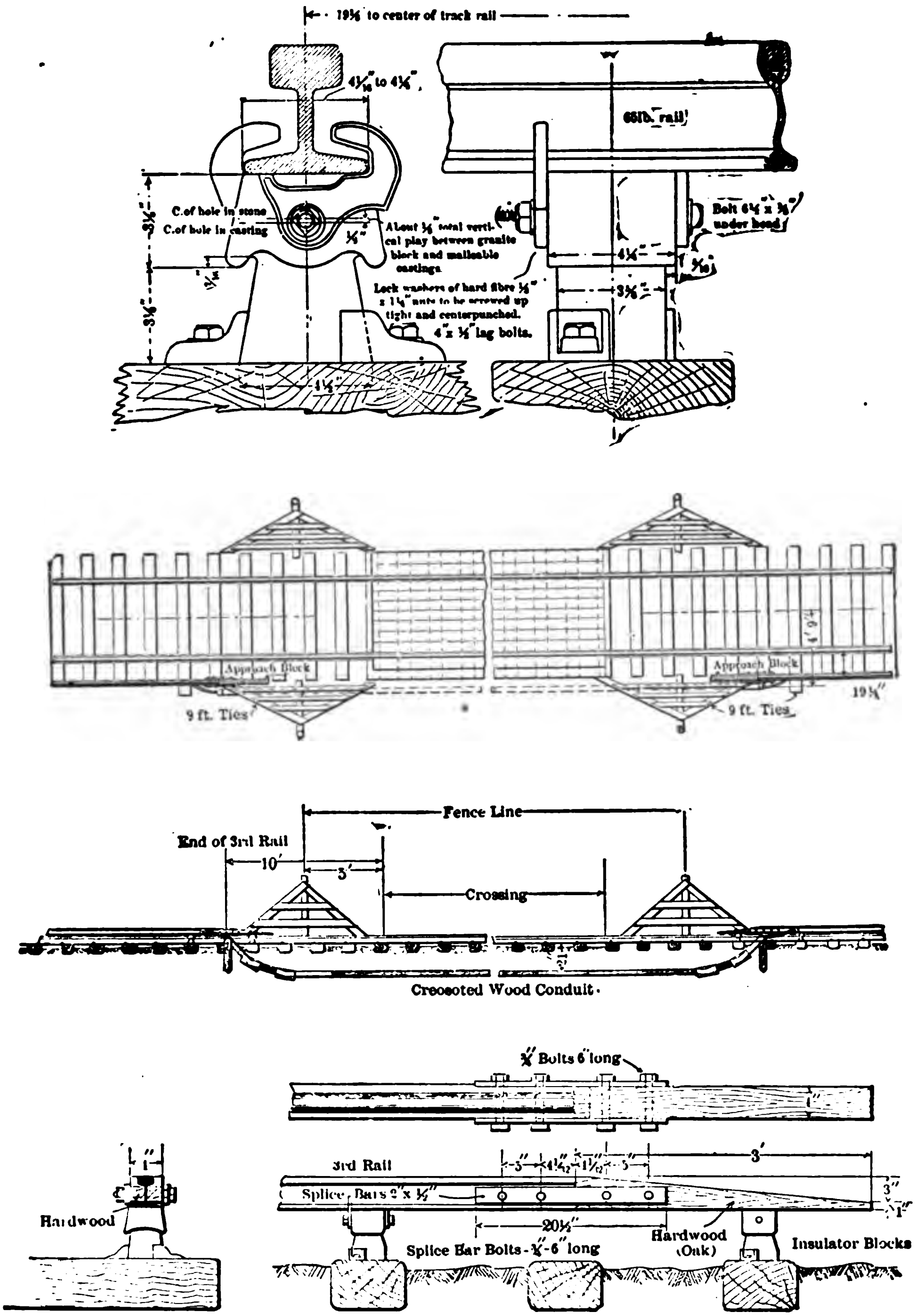


Fig. 191. Details of Third-Rail Construction, showing Third-Rail Insulating Support, Highway Crossing, and Approach Blocks.

granite rock, compressing it hydraulically in a suitable mould, and baking after the fashion of porcelain, obtaining an article of any desired form, of good insulating qualities, and great mechanical strength. Various pottery compounds are also used. (See Plates XIV and XV.)

Fig. 191 gives the mechanical details of the third-rail installation employed on the Grand Rapids, Grand Haven & Muskegon Railway. The upper part of the illustration shows a section and elevation of the mounting. Wherever a highway is intersected it is necessary to open the third rail and allow cars to cross such a portion of the road by momentum, for it is impracticable to place a naked conductor charged with 500 volts along a wagon road.

The lower portion of Fig. 191 indicates the method adopted at such points. On either side of the wagon road a cattle-guard is placed and the third rail stopped beyond this guard; beneath the highway a creosoted wooden conduit is installed in which a cable is placed to connect the ends of the rail on either side, preserving electrical continuity.

220. The third rail is an ideal method for power to elevated railways and so has been adopted in Chicago, New York, and Boston.

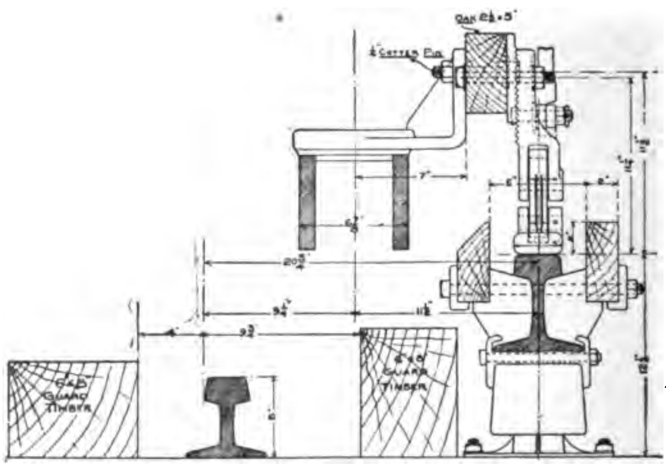


Fig. 192. Section through Third Rail, Manhattan Railway Company.

Fig. 192 is a cross-section through the third rail and one running rail on the Manhattan Elevated Railway, and Fig. 193 a side elevation of the contact shoe in working position on the rails. On either side of the third rail in Fig. 192 a wooden string-piece is placed. This

Car on the Lackawanna & Wyoming Valley Ry.

To face page 262.

is designed to act as a protector, to prevent workmen or others from short-circuiting the rail. The contact-shoe in Fig. 193 is suspended by a pair of toggles from a U-shaped casting bolted to the bolster of

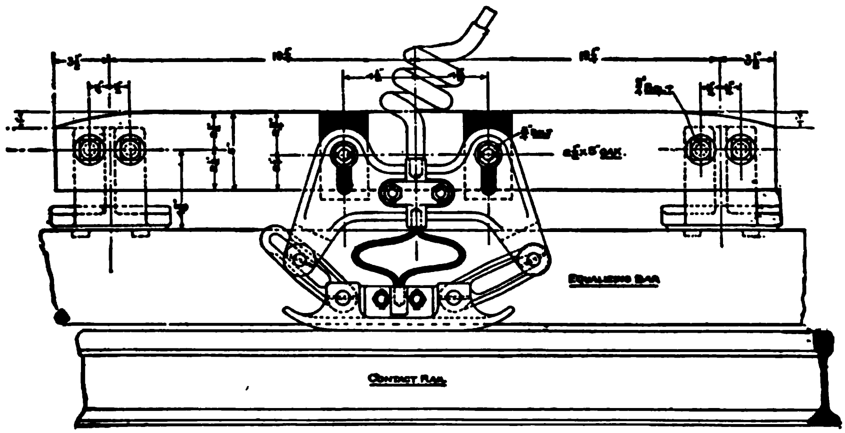


Fig. 193. Elevation of Shoe in Contact with Third Rail.

the truck, two heavy flexible copper cables providing an electrical connection that is independent of all joints.

221. A great objection to the third rail is offered by its upturned unprotected surface, which forms a convenient lodgment for sleet and snow, that may seriously interrupt traffic. Efforts have been made to protect the rail, one of which is shown in Fig. 194, giving a general view of a track equipped with protected third rail, while Fig. 195 is a detail. The rail is supported in the usual manner, but in addition, at each insulator there is a gooseneck casting that holds an inverted channel-iron set a few inches above the top of rail. This channel forms a roof, keeping the rails dry, and free from ice and sleet, while the space between the channel and the rail permits a properly shaped plow to enter and run on the rail, as shown in Fig. 195.

Plate 9 is a photograph showing the construction of the Lackawana & Wyoming Valley Railway—one of the best types of the modern electric interurban, which differs in no wise from the best-built steam railways, save in the motive power employed. The line extends from Scranton to Wilkesbarre, and is double-tracked with 80-lb. rails laid in a ballast of crushed stone that forms a road-bed that compares favorably with those of either the Pennsylvania or the New York

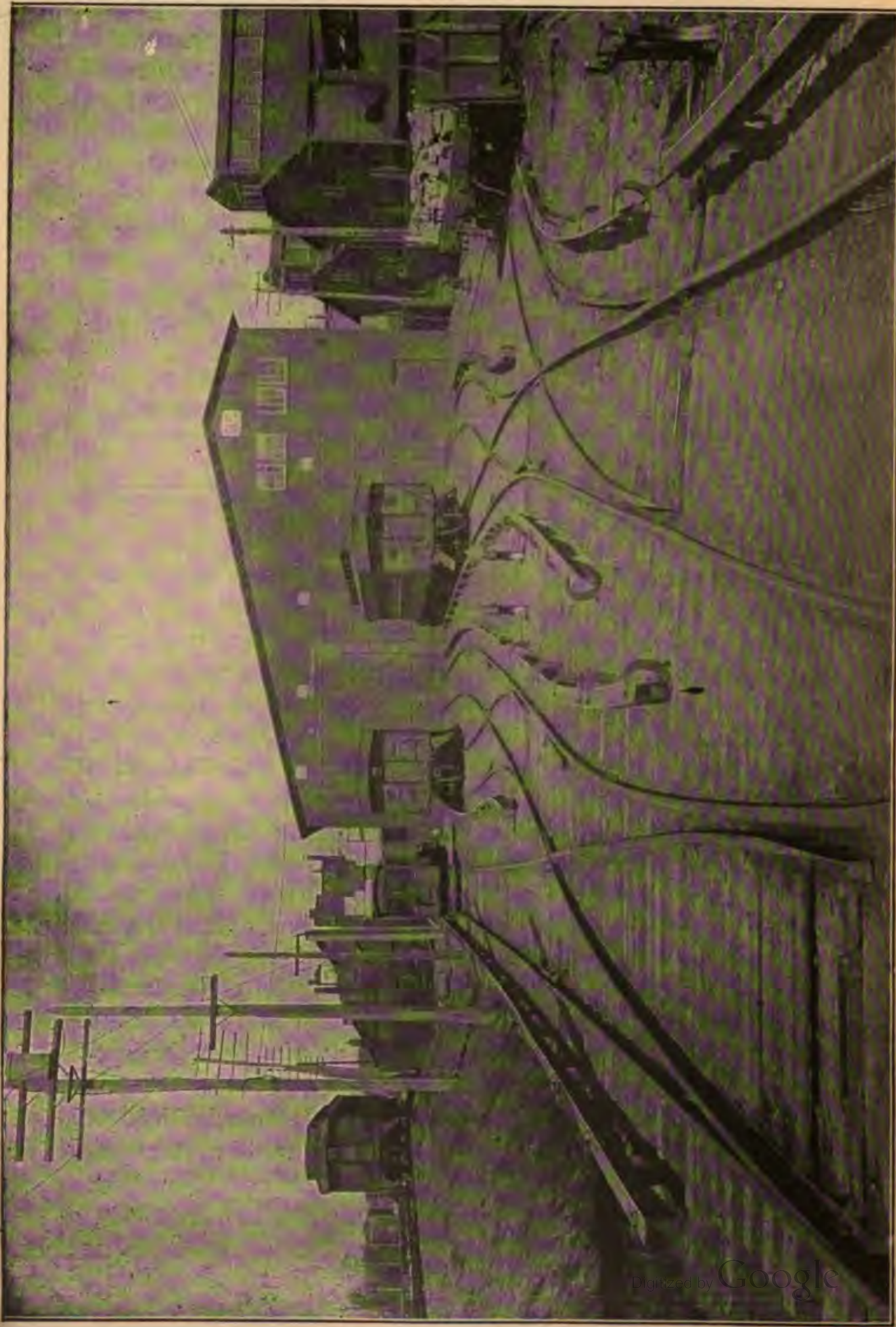


Fig. 194. Protected Third Rail, Wilkesbarre & Hazelton Railway.

PLATE XI.

Berlin Branch N. Y., N. H. & H. R. R.

To face page 265.

Central. In the center between the running-rails the third rails are placed, supported upon reconstructed granite insulators, set upon every fifth tie. Plate 10 is a view of a car upon the Lackawana & Wyoming Valley Railway, showing the method of attaching the current-shoe to the bolsters of the truck, and indicating the relation between the running-rails and the conductor-rail, and the position of the shoe when ready to receive current.

Modern practice almost universally adopts the plan of setting the third rail at a short distance outside of the running-rails. To prevent

Fig. 195. Detail of Protected Third Rail.

an accumulation of snow upon the third rail and to increase the insulating distance between the top of the tie and the base of the rail, it is desirable to elevate the latter a few inches above the road-bed.

The demand for larger and heavier cars to be operated at higher speeds has necessitated increasing the motor capacity with which cars are equipped, and to secure the necessary motive power has required machines which occupied every possible inch between the tops of the ties and the under side of the car body. This practice has caused the third rail to be placed on the outside of the running-rails rather than between them, though some of the earlier roads adopted this

location, for example the Berlin branch of the N. Y., N. H. & H. A third-rail road, constructed several years ago, adopted the plan of placing the conductor-rail in the center between the running-rails. The conductor-rail was formed of pieces of special angle-iron having a flat top along the apex of the angle.. The rail-lengths were supported upon insulators set upon the regular ties, the angle forming a roof to protect the insulator from snow and moisture. This design is shown in Plate 11 and has been very successful.

222. The Urban Conduit.—The poles and wires of the overhead trolley are too great an obstruction and a disfigurement to be allowed in crowded thoroughfares, and charged rails are impossible, so in the center of large cities the entire railway-conducting system must go underground. This necessity has developed the *electric-railway conduit*, examples of which may be seen in successful operation in London, Paris, Brussels, New York, Philadelphia, Washington, etc. In all of these cities the general plan is the same, though the details differ slightly, so an account of one will essentially fit all.

The electric-railway conduit is modeled after its predecessor, the cable railway. The foundation is a bed of concrete, in which iron yokes are set at intervals of 4 ft. to 5 ft. The yokes carry on their outer extremities the running rails and in the center a pair of Z bars that form a slot in the street. Beneath the slot a long tube is molded in the concrete, in which two conductors are suspended from insulators set at intervals of 15 ft. From the car an iron rod hangs between the slot rails and presses two sliding-contacts against the conductors in the conduit, so the conduit railway is a complete metallic line. On either side of the conduit it is customary to install a number of ducts in which both a low- and high-tension railway distributing system may be placed with room for telephone and signal wires.

223. In Fig. 196 sections are given of the Washington conduit, by which the general dimensions and arrangement of parts are so well shown that further description is superfluous. In Fig. 197 the Broadway, New York, structure is given, which is seen to be very similar except in the details of the yoke. The first steps in conduit-building consist in opening the street, digging the requisite excavation, introducing the concrete, and setting the yoke, as shown in Plate XI and Fig. 198. Then all the rails are set and the structure is ready for concrete, as indicated in Plate XII. The next operation is to place a

PLATE XII.

Building a Conduit Road.

To face page 266.

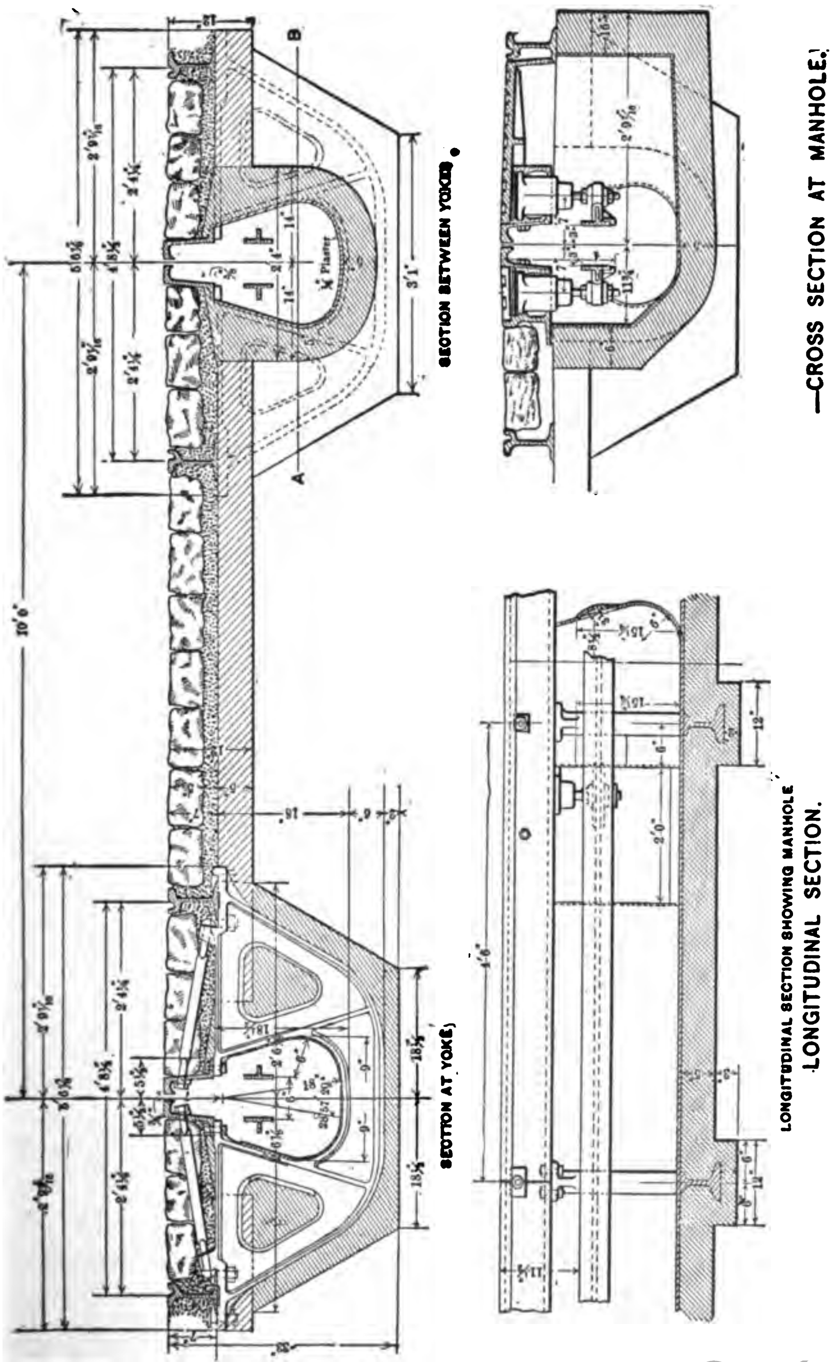


Fig. 196. Details of Railway Conduit, Washington, D. C.

wooden mold around which the concrete may be rammed to form the tube to hold the conductors. This mold is made in sections to be

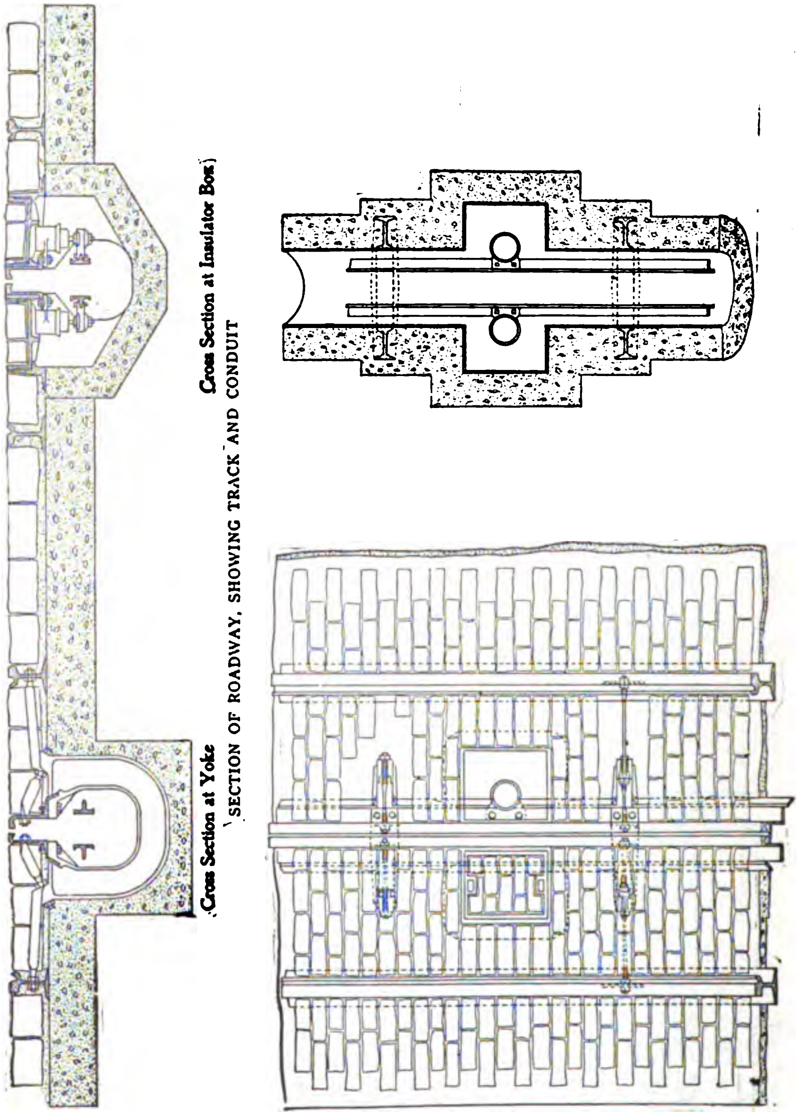


Fig. 197. Broadway Conduit.

easily moved when concreting is done. Fig. 199 shows the mold and arrangement of concrete. Finally, when the concreting is done, the structure is ready for replacement of paving, as indicated.

The insulator problem is an interesting one. There can be no question about adopting a porcelain insulator, or at least a material which has the same general characteristics. The first line in New York City used a built-up insulator, a bolt covered with an insulating compound; it failed, and the type of insulator successful at Washington was adopted. In Brussels the insulator used is a bolt

Fig. 198. Conduit-Building, Fourth Avenue, New York.

surrounded by a rubber compound, and subjected to a severe test with spark-coil before acceptation. These insulators have been fairly successful, but are expensive, and deteriorate by exposure. The iron-clad porcelain insulator is strong, durable, and cheap. The insulators should be set vertically, so they may not accumulate dirt.

Assuming a vertical iron-clad porcelain insulator, the method of attaching it to the conduit becomes important, the depth of the tube

being somewhat dependent upon this. Insulators should be fastened to the metallic structure, which has the advantage of keeping the conductor-bars at the same distance from the wheel-rails. The extreme

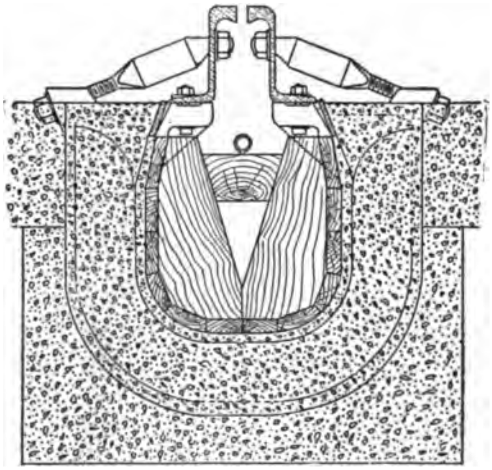


Fig. 199. Section showing Wooden Mould for Conduit.

positions of the insulator are, first, with the upper surface of the insulator as near the street surface as it can be for mechanical protec-

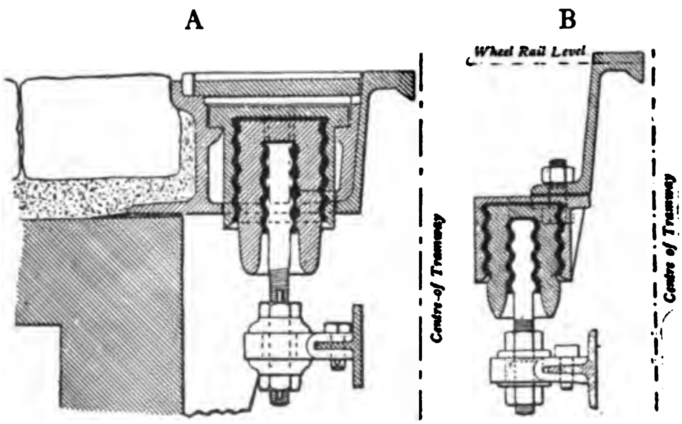


Fig. 200. Conduit Insulation.

tion, say 2 in.; second, with the upper surface of the cast-iron cover bolted directly to the bottom flange of the slot-rail. Fig. 200 gives an example of the first case at *A* and of the second at *B*, both with

Conduit Ready for Concrete.

To face page 271.

the same slot-rail. The illustrations show the general method of making the insulator and connecting to the feeder system as often as may be desirable.

Between the running rails two, so-called, slot-rails are provided, which, as is indicated in the several illustrations, are composed of Z bars of such shape and height as to permit of bolting the lower flange to successive projections on the yokes, while the upper flange forms a guard for the slot at the level of the street. The Z bars provide a wall against which the paving material is placed, and as the bars are set about five-eighths of an inch apart the space between them affords a slot through which the plow suspended from the bottom of the car enters the conduit and bears against the conductors. The chief points in the design of an electric-railway conduit are the provision for adequate drainage facilities, and ready accessibility to the insulators and conductors for repairs. Wherever the sewer system of the street is below the level of the bottom of the conduit, drainage may be secured by building, at intervals of from 300 to 400 ft., a manhole of sufficient dimensions to receive and collect the water which drains through the slot. The bottom of this manhole is connected to the sewer system, and as fast as water collects therein it flows away. This manhole also affords convenient room in which to splice the feeder cables which are usually extended along the line of the railway in terra-cotta ducts built into a conduit upon one side of the yokes. Where the sewer level is above that of a conduit a difficult problem is presented, the only adequate solution being to construct a special sewer for the purpose of draining the railway conduit. In other respects such conduits must possess great strength and solidity in order that they may be able to successfully resist the destroying influences of the heavy street traffic which is constantly traversing the surface of the streets under which the conduits run. In addition to removing the unsightly overhead line from the surface of the streets, the conduit road possesses the advantage of operating upon a completely metallic circuit. Therefore it removes from the consideration of the railway all questions of electrolysis, or of possible interference with other electrical circuits, and the conductor material may be designed of such size as to secure such a drop as appears most economical, considering the cost of producing electrical energy, the average load upon the railway system, and cost of conductor material.

CHAPTER IX.

ELECTRICAL INSTRUMENTS.

Art. 224. No exposition of the methods of distributing electrical energy would be complete without such reference to the various electrical instruments, and methods of measurement, as will enable the designer to accurately inspect the condition, and determine the performance, of electrical circuits. The principal electrical instruments may be divided into five classes : —

First. Instruments for the measurement of resistance.

Second. Instruments for the measurement of the quantity of electricity.

Third. Instruments for measuring electrical pressure.

Fourth. Instruments for the measurement of capacity.

Fifth. Instruments for the measurement of the energy delivered by a circuit.

INSTRUMENTS FOR THE MEASUREMENT OF RESISTANCE.

225. The Wheatstone Bridge. — The most widely known instrument for resistance determinations is the Wheatstone Bridge, the theoretical arrangement being shown in Fig. 201. Four resistances, a, b, d, and x, are arranged in the form of a parallelogram, a battery being placed in series with one diagonal and a galvanometer in the other. When the four resistances forming the sides of the bridge are so adjusted that no current flows through the galvanometer, these resistances bear a certain definite relation each to the other. When there is no current between the points A and C, the galvanometer may be removed without altering the current in the arms of the bridge. Also, the points A and C may be short-circuited without interfering with the balance. Suppose the points A and C to be separated; then the joint resistance of the four arms of the bridge between the points B and E will be, $\frac{(a + x)(b + d)}{a + x + b + d}$. If now the

points A and C be joined, the resistance may be expressed as $\frac{ab}{a+b} + \frac{dx}{d+x}$. These two expressions are evidently equal to each other, and may be stated in the form of an equation, —

$$\frac{(a+x)(b+d)}{a+x+b+d} = \frac{ab}{a+b} + \frac{dx}{d+x}, \tag{22}$$

which, by simplification, may be reduced to the form

$$x = \frac{ad}{b}. \tag{23}$$

Therefore, if three of the quantities of this equation are known, the fourth can be easily determined. Usually two of the arms consist of fixed known resistances, the third is an adjustable resistance formed of a number of coils whose value has been previously determined, while the fourth is the unknown resistance which it is desired to measure. By the simplest method, a and b would be of equal value, in which case x would be equal to d; or, in other words, the resistance between A and E, when the equilibrium is obtained, gives the value of the resistance to be measured. It is essential that some resistance should be in the arms a and b; for otherwise the galvanometer is short-circuited, and equilibrium will always be apparently produced. Instead of using equal resistances in a and b, one of the two may be 10, 100, or 1000 times as great as the other; or, in fact, any multiple that may be desired. Multiples of ten, however, are those which are most commonly used. If b is made ten times as large as a, the resistance in d will be ten times as large as x, and thus every unit of resistance in d will represent one-tenth of a unit in x. By this means it is practical to determine the value of the unknown resistance to the tenth of a unit. Similarly, by making d 100 or 1000 times as large as a, the value of

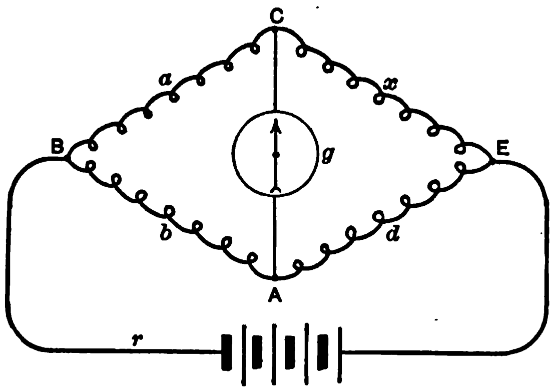


Fig. 201. Diagram of Wheatstone Bridge.

x may be correspondingly ascertained to the $\frac{1}{100}$ or $\frac{1}{1000}$ of a unit. If, on the contrary, a be made 10, 100, or 1000 times as large as b, each unit in d must be multiplied by the corresponding factor of 10, 100, or 1000, to give the value of x. By this means it is practical to make the bridge measure very small or very large resistances, with fair accuracy. It is obvious that the sensitiveness of the galvanometer employed to detect the current flowing between A and C

Fig 202. Portable Testing Set

forms a large factor in the accuracy of bridge measurements. The more sensitive the galvanometer, the smaller the current it will be possible to detect, and the nearer the bridge arms can be brought to an exact balance. A very convenient and portable form of testing-set, embracing resistance coils, bridge, and galvanometer bridge, is shown in Fig. 202.

At the extreme right of the cut is shown a small D'Arsonval galvanometer, having the advantage of being dead-beat. Next to the galvanometer is the resistance-box, containing four sets of coils, units, tens, hundreds, and thousands; and on the extreme left hand,

Types of Third-Rail Insulators.

Protected Third Rail.

Third-Rail Insulator.

the arms of the bridge, *a* and *b* are seen, the arm *a* having coils of 1, 10, and 100 ohms, and arm *b* with coils of 10, 100, and 1000 ohms. By means of pegs, the arms can be arranged either to multiply or divide at pleasure. With the coils in the arms *a* and *b*, ratios of 1 to 10, 100, or 1000 can be obtained, either multiplying or dividing; and as the resistance coils measure from 1 ohm to 1111 ohms, the set can measure from $\frac{1}{1000}$ of an ohm to 11 megohms.

226. The Slide Wire Bridge. — While the previously described form of bridge is capable of detecting a thousandth of an ohm, very low resistances are more conveniently measured by a modification of this instrument, termed a "slide wire bridge," as shown in Fig. 203.

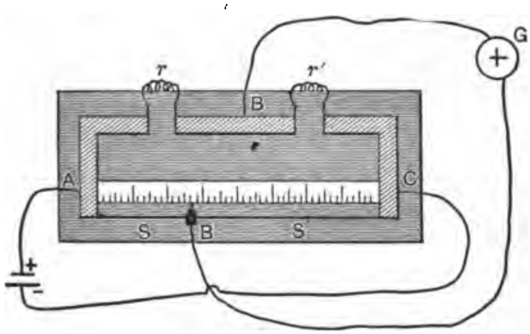


Fig. 208. Slide Wire Bridge.

The illustration indicates the simplest and cheapest form of the apparatus, consisting of a baseboard of insulating material upon which are placed three heavy copper bands, A, B, and C. Between the bands A and B, and B and C, are gaps into which any desired resistance coils may be placed. The other ends of the pieces A and C are joined by a uniform wire, having a resistance proportioned to the capacity of the measurement that it is desired to make. Parallel to this wire a scale is placed, having its initial and final points exactly opposite the places where the measuring-wire is connected to the heavy copper bars.

The scale should be graduated to read both ways; and on the assumption that the wire is of uniform resistance throughout, and also that the scale properly corresponds to the beginning and end of the wire, the ratios of the resistances *r* and *r′* may be read from the two segments into which any point, such as "B′," divides the wire

and the scale. The point B′ forms a sliding contact on the wire, extending from the middle of the bar B to the wire, and including the galvanometer in its circuit.

By examination of the illustration, it will be easy to trace the similarity of the circuits to those of the ordinary Wheatstone bridge. Thus it is evident that the "slide wire bridge" is merely such a modification of the ordinary Wheatstone arrangement as will permit the introduction of any desired low resistances at the points r' and r, and the use of a uniform wire for the variable resistance arm, in order that the variable resistance may be obtained in sufficiently small fractions of a unit.

227. The Ohm-Meter. — It is an obvious consequence from Ohm's law that the resistance of any circuit, or portion of a circuit, may be calculated by measuring the electro-motive force operating, and the amount of current flowing. It is not uncommon to measure the insulation of heavy circuits by ascertaining the difference of potential at the terminals of the dynamo supplying circuit, and then, by means of a milliammeter, determining the leakage between the circuit and the ground, the quotient of these quantities being the desired resistance. The objection to this method is that it requires a simultaneous reading of two instruments, which, in cases of varying currents and varying potentials, is difficult to obtain. As an improvement, an instrument termed the ohm-meter has been devised, that consists of two circuits, one of fine wire and another of coarse wire. At the intersection of the two coils a magnetized needle is suspended, carrying a pointer, that, playing over a scale, serves to determine the readings of the instrument. Using the apparatus, the fine wire coil is connected across the terminals of the circuit, serving, by its effect upon the magnetized needle, to determine the electric pressure; while the coarse wire is connected in series with the circuit whose resistance is desired, and affects the needle proportionally to the amount of current flowing. By the combined action of the two coils, the needle assumes a position of equilibrium, which is in proportion to the resistance of the circuit. An instrument of this description forms an exceedingly useful auxiliary for all circuits carrying heavy currents, as by means of its aid the insulation or resistance of the circuit may be continually determined, even during the time that the plant is under full operation.

228. Another form of ohm-meter, very useful for measuring low resistances, may be constructed by arranging a differential galvanometer so that the coils of the instrument may be moved either toward or away from the needle, by means of a micrometer screw, so arranged that the position of each coil may be accurately determined.

To measure a resistance, a known resistance is placed in one-half of the differential galvanometer circuit, and the unknown in the other half. The coils are then adjusted until no deflection is produced on the needle. The relative positions, then, of the two coils, give accurate indications of the unknown resistance in terms of the known resistance. With a sensitive reflecting galvanometer arranged in this manner, it is perfectly practicable to measure one-millionth of an ohm with accuracy.

229. An instrument known as an ohm-meter has recently come into vogue which, while it is really a modification of the Wheatstone

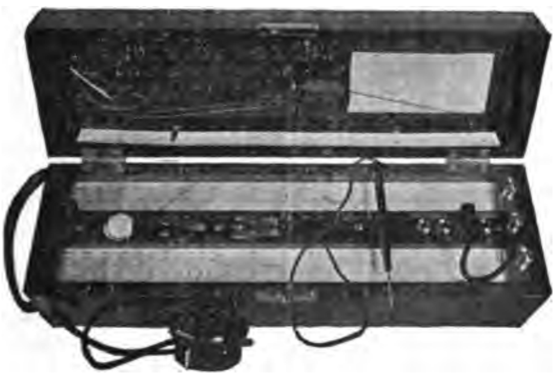


Fig. 204. Ohm-Meter.

bridge, is an exceedingly valuable piece of apparatus. A general plan of the ohm-meter is shown in Fig. 204, while the circuits are given in Figs. 205 and 206. In the circuit of Fig. 205 suppose X to be an unknown resistance, R a variable rheostat containing coils of 1, 10, 100, and 1000 ohms. Let X and R be connected at the point D and joined across the other ends A and B by a uniform conductor of moderate resistance. Let a battery be joined to A and B, a telephone or galvanometer provided with a sliding contact E be connected to D as shown in the illustration. Between A and B there is evidently

a fall of potential, and if the wire AB be touched with the point E a sharp click will be heard in the telephone or the galvanometer will be deflected, but it is also evident from the principles of the Wheatstone bridge that with any moderate ratio between X and R a point upon

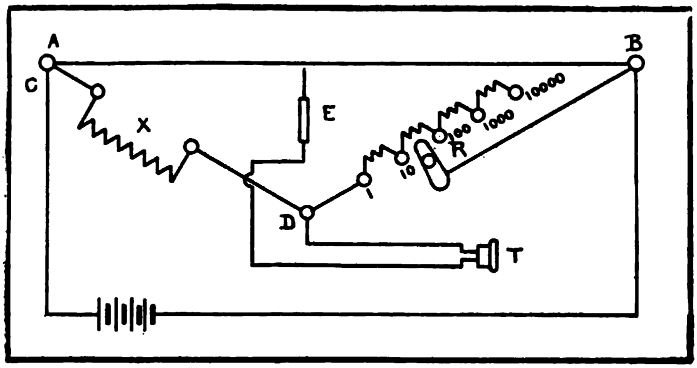


Fig. 205. Ohm-Meter Circuit for Measuring Resistance.

AB can be found of the same potential as the point D, and when the contact E touches this point there will be no sound or no deflection. If the battery be replaced by any alternating current, such as a small induction coil, the telephone will emit a continuous sound as long

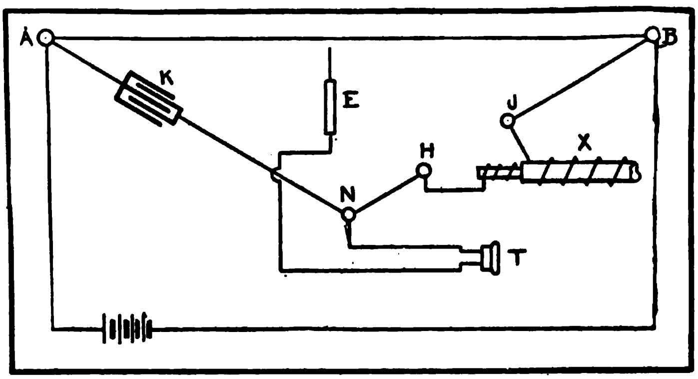


Fig. 206. Ohm-Meter Circuit for Measuring Capacity.

as there is a difference in potential between D and E. By sliding the contact to and fro on AB the point of no difference of potential can be quickly found, and then the formula of the Wheatstone bridge shows that $X = R\frac{AE}{EB}$. As the wire AB is of uniform resistance it is

easy to graduate a scale of equal parts, from which the unknown resistance can be immediately read. This instrument possesses many desirable features in addition to its low price ($65.00) and the speed with which measurements can be made. The form represented in Fig. 204 is capable of measuring resistance from .002 to 2,000,000 ohms, an ample range for all ordinary electric testing. This instrument lends itself with the greatest facility to the measurements of capacity and inductance, for by using a standard condenser or a standard inductance as shown in circuit, Fig. 206, it is easy to measure, by exactly the same process of manipulation, any capacity or inductance. Lastly, the ability to use an alternating current enables the resistance of electrolytes to be rapidly measured and thus the determination of battery resistance or of electro-motive force of batteries is greatly simplified.

INSTRUMENTS FOR MEASURING ELECTRICAL QUANTITY AND PRESSURE.

230. Galvanometer.—Nearly all practical instruments for estimating either current or electro-motive force are based on the mutual reactions developed either between a coil of wire and a magnetic field, or between two coils of wire, when arranged to form a part of the circuit it is desired to measure, the only notable exception being in the case of the hot wire and electrostatic voltmeters, to which special reference will be made. *Galvanometers*, as these electro-magnetic instruments are broadly termed, may be used in three distinct ways:—

First. Simply to detect the presence of a current.

Second. When constructed so that their indications are proportional to the electro-motive force, they become voltmeters or pressure indicators.

Third. When the readings correspond to the quantity of current they are termed ammeters.

As a current indicator, the Thomson Reflecting Galvanometer is too well known to need more than passing reference. It is the instrument universally employed for all accurate work involving currents of small magnitude, such as insulation, resistance, and capacity tests. The present forms of this instrument are made to permit the use of a number of interchangeable coils; so, by proper calibration, the galvanometer may serve either as a voltmeter or an ammeter. As the Thomson instrument is very sensitive tc the slightest varia-

tion in the external magnetic field, and as it is not dead-beat, its use is almost restricted to laboratory work, and the most refined methods of testing.

231. The D'Arsonval Galvanometer.—In the D'Arsonval Galvanometer, Fig. 207, an instrument is obtained, which, while it lacks

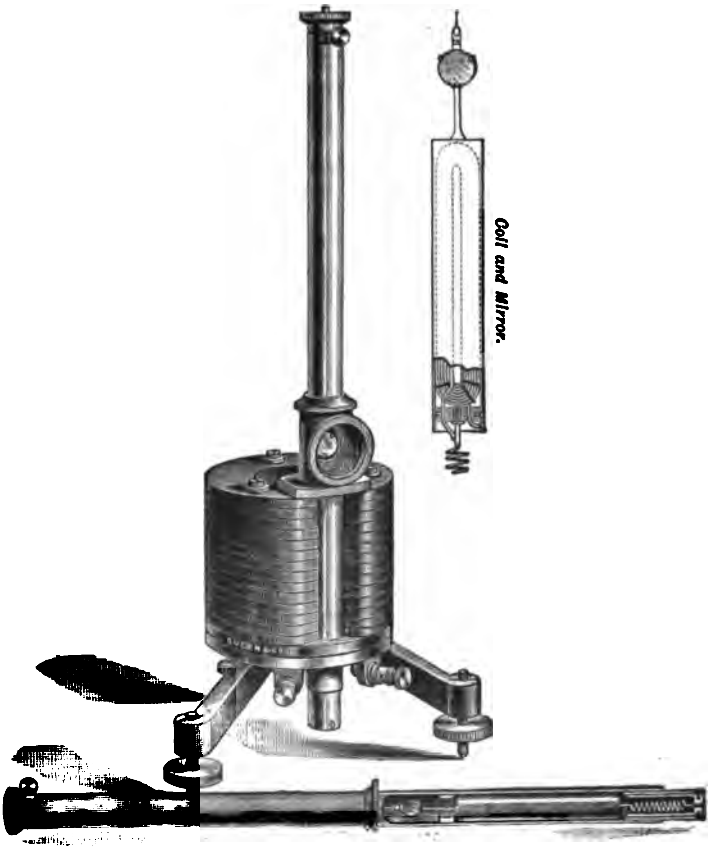


Fig. 207. The D'Arsonval Galvanometer.

the extreme sensitiveness and delicacy of the Thomson, is better adapted to the general practice of the electrical engineer. In this instrument the magnetic system is fixed, the poles of which are hollow and inclose a suspended coil of very fine wire, through which the current to be measured circulates. By this arrangement the instrument is rendered independent of any surrounding magnetic

fields, and can be used in close proximity to the largest dynamos; and when used with a short-circuiting key, is practically dead-beat. Supplied with a reflecting mirror, reading telescope, and scale, sufficient accuracy and sensitiveness may be obtained for everything but the most delicate tests.

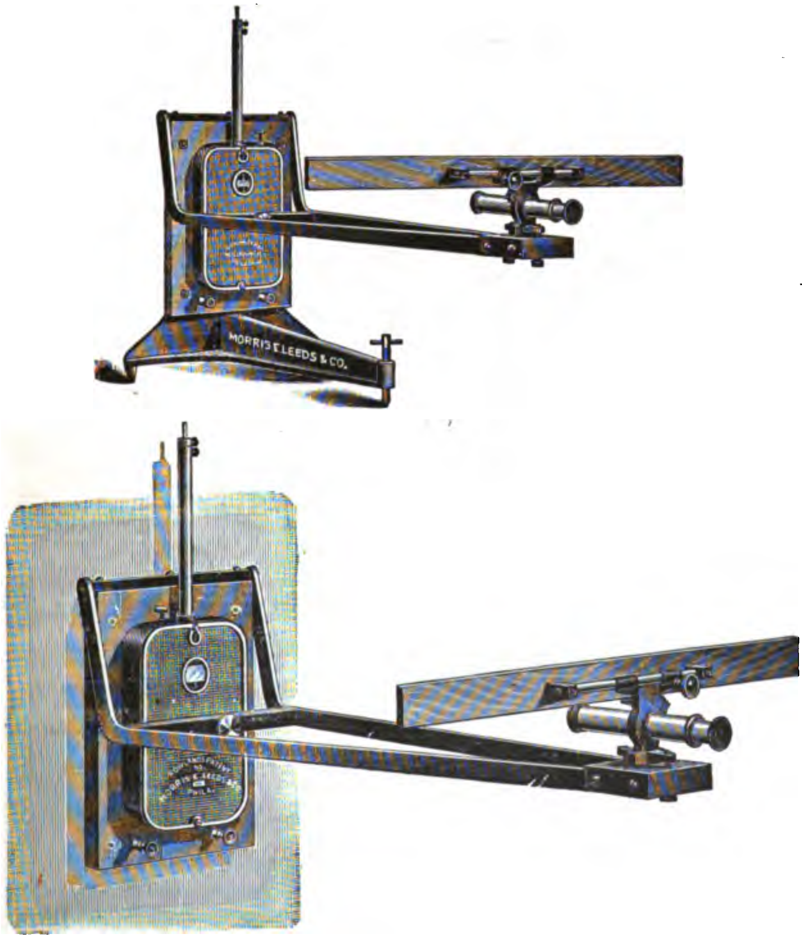


Fig. 208. Testing-Room D'Arsonval Galvanometer.

The D'Arsonval Galvanometer has recently assumed new and desirable forms to secure increased portability. A favorite type is that shown in Fig. 208, in which the whole instrument is mounted upon a baseboard which carries a bracket arm upon which the telescope and scale are supported. This model can be hung on the wall

Fig. 210. A Street Galvanometer.

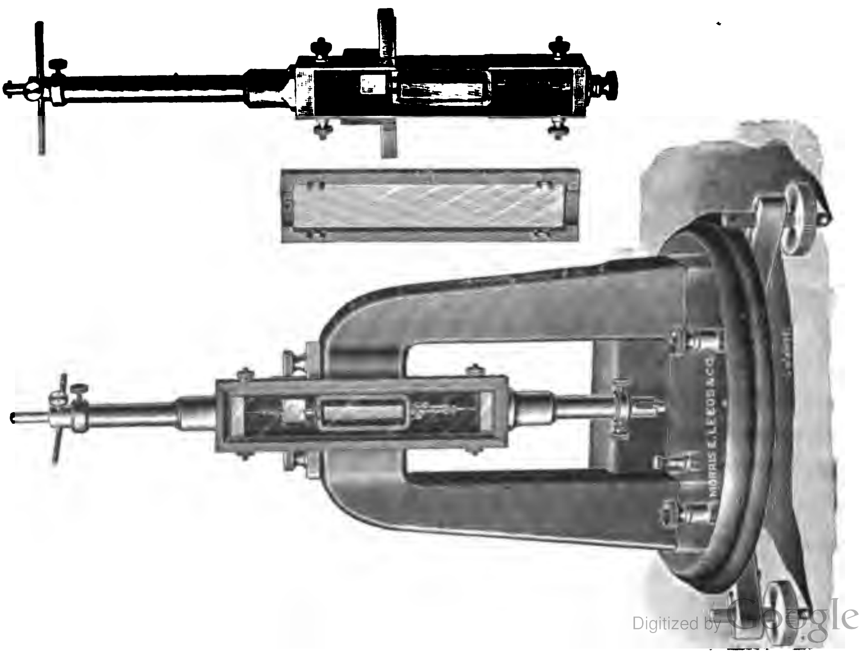


Fig. 209. Marine D'Arsonval Galvanometer.

or instantly dismounted and set upon a tripod. A marine type of instrument is shown in Fig. 209, so contrived that without undue loss of sensitiveness it may be tilted through 20° or 30°. For the field work of cable-testing the galvanometer may be mounted as in Fig. 210, upon the top of a tripod, much after the fashion of a surveyor's instrument, and is built to be carried from place to place upon the shoulder with as little compunction as a transit or level.

232. The Ballistic Galvanometer. — For some measurements, notably in capacity testing, it is essential to employ a certain modification of the above instruments, usually termed a ballistic galvanometer. The object of this device is to enable accurate determinations of transient currents, such as are produced by the discharge of a condenser, or currents developed by electro-magnetic induction. In the ballistic galvanometer the needle system is so arranged that movement does not (practically) take place until the transient current has ceased. Thus, as its name indicates, the instrument is intended to measure impulses. It is now customary to supply the best Thomson and D'Arsonval instruments with interchangeable needle systems for the purpose of ballistic work. The ballistic needle is usually a thimble or bell-shaped magnet, so arranged that its rotation may be as little retarded as possible. The relation between an electric discharge and its effect on a ballistic galvanometer is given by the following formula: —

Let Q = the quantity of electricity in coulombs.
T = the time in seconds required for one complete oscillation.
D = deflection with Q coulombs.
F = figure of merit with a constant current.

Then it can be shown that —

$$Q = \frac{TFD}{2\pi}. \tag{25}$$

For example. Suppose the discharge from a certain condenser gives a throw of 120 divisions on the scale of a ballistic galvanometer, the figure of merit of which is .0002082 amperes, and time of oscillation 6 seconds; what was the amount of electricity? Here

$T = 6$
$D = 120$
$F = .0002082$

$$Q = \frac{6 \times 120 \times .00020082}{2 \times 3.14159} = .0239 \text{ coulombs.}$$

For the full demonstration of this formula, the reader is referred to any of the extended works on testing, particularly that of Kempe.

233. The *constant* of a galvanometer may be defined as the relation existing between the deflection indicated on the scale, the current, and resistance of the circuit.

For example. Suppose a galvanometer having a resistance of $r = 10{,}000$ ohms is connected with a battery having an internal resistance $r' = 200$ ohms, and an external resistance $r'' = 100{,}000$ ohms, giving a deflection of 20 divisions. The total resistance of the circuit is 110,200 ohms; therefore, as the current in the circuit is inversely proportional to the total resistance, and as the deflection is assumed to be directly proportioned to the current, the constant would be $110{,}200 \times 20 = 2{,}204{,}000$. As any change in the resistance of the circuit will change the deflection, it is possible to use the constant to determine unknown resistance. Indeed, this is the most common method used to measure high resistances, such as the insulation of circuits.

234. The *figure of merit* of a galvanometer is the *amount* of current which will produce a deflection of one division or one degree upon the scale. To find this current, it is only necessary to connect up the galvanometer and battery, in series, with a known resistance, and then to measure the deflection produced. Having the total resistance, it becomes a simple matter to calculate the amount of current flowing, and from this to deduce the quantity of current necessary to produce a deflection of one division, which, by definition, is the figure of merit.

For example. Suppose a galvanometer having a resistance of 1,000 ohms to give a deflection of 100 divisions when joined with a battery of 250 ohms and an external resistance of 10,000 ohms, the battery having an electro-motive force of 100 volts. As the total resistance of the circuit is 11,250 ohms, the electro-motive force of 100 volts will produce a current of .0089 of an ampere. Under these circumstances, as the deflection is 100 divisions, the figure of merit of the galvanometer will be .0089 divided by $100 = .000089$ of an ampere. That is, .000089 of an ampere will produce a deflection of one division.

235. A galvanometer having a high figure of merit is one the needle of which will deflect from zero with a very small amount of

current. This, however, does not necessarily convey the idea of sensitiveness, for by a sensitive galvanometer is meant one whose needle *when deflected* under the influence of a current will *change* perceptibly with very small variation in the current itself. To attain a truly sensitive instrument, it is essential that the needle system should have a fiber suspension, as it is impossible to obtain sensitiveness with any other means.

236. Reduction to Zero. — The *angular* deviation of a needle system in reflecting galvanometers is so small that it is usually customary to assume that the number of divisions in the deflection is proportional to the current that produced it. While for instruments of this class this assumption is essentially true, it is not mathematically correct. For precise work, therefore, it is desirable, as far as possible, to use methods involving "reduction zero;" as in this case the final balance obtained is with a zero reading, which must necessarily be precisely accurate.

237. Inferred Zero. — In a reflecting galvanometer, the angle of maximum sensitiveness is the largest deflection that can practically be obtained; as, however, under any circumstances, the deflection is only a few degrees, the true maximum angle of sensitiveness can rarely, if ever, be reached. The method of inferred zero here comes into play, by means of which increased sensitiveness can readily be obtained. By moving the controlling magnet so that the needle is turned away from the scale to a considerable distance, the readable deflection of the galvanometer can be largely increased.

For example. Suppose the needle normally on the zero of the scale, and that a given current causes it to deflect through 300 divisions. Then an increase in the current of one per cent would increase the deflection $300 \times 101/100 = 303$, an increase of three divisions. Suppose now that the working zero be placed 400 divisions away from the *scale* zero, and that the current has been sufficiently strong to produce a deflection of 300 divisions on the scale, the actual deflection would therefore be equal to $400 + 300 = 700$, and an increase in the current of one per cent would increase the deflection to $700 \times 101/100 = 707$, or a deflection of seven additional divisions, for the same small increase of current. It will thus be apparent that the sensitiveness of the instrument may in this manner be very largely increased. An additional use of the inferred

zero is to be found in making insulation or capacity measurements, when the standard, by means of which the galvanometer constant is determined, produces a current through the instrument which is widely different from the current used in making the test.

238. Galvanometer Shunts. — The deflection of a galvanometer being proportional to the current traversing its coils, and the scale being of limited extent, it frequently happens that the current under examination is sufficient to carry the index off the scale, giving a deflection that is unreadable. It is usual, in such cases, to place in parallel between the terminals of the galvanometer an amount of resistance sufficient to permit of a readable deflection. Such a resistance placed in parallel with the galvanometer is called a "Shunt." In Fig. 211, let G be the resistance of the galvanometer, S that of the shunt, I the total current, and i and i' the currents in the galvanometer coils and shunt respectively, then —

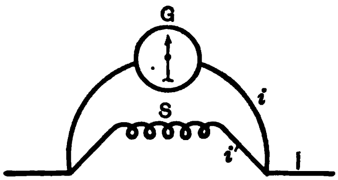


Fig. 211. Diagram of Shunt Connection.

$$I = i + i'. \tag{26}$$

As the electro-motive force is the same in both branches, the respective currents will be inversely as the resistance of each branch; hence —

$$\frac{i'}{i} = \frac{G}{S} \ (27), \qquad \text{and } i' = i \times \frac{G}{S}; \tag{28}$$

replacing i' by its value found from equation (27),

$$I = i \times \frac{G + S}{S}. \tag{29}$$

Knowing the deflection given by the galvanometer with a known current, the current i is determined; and from the known resistance G and S, I is readily calculated. The deflection that would be produced on the scale with the current I, assuming the law of proportional deflection to hold true indefinitely, is evidently the deflection given by i multiplied by the factor $\frac{G + S}{S}$. This factor is termed the multiplying power of the shunt, and is frequently symbolized by m. Thus —

$$\frac{G + S}{S} = m. \tag{30}$$

Suppose a galvanometer of 6340 ohms, when shunted with 10 ohms, to give a deflection of 125 divisions, then —

$$125 \times \frac{6340 + 10}{10} = 125 \times 635 = 79375 \text{ divisions.}$$

In this case $m = 635$. Any known resistance may be used as a shunt, though for rapid work easy multipliers should be selected. As, —

$$m = \frac{G + S}{S} \ (31), \qquad S = \frac{G}{m - 1}. \qquad (32)$$

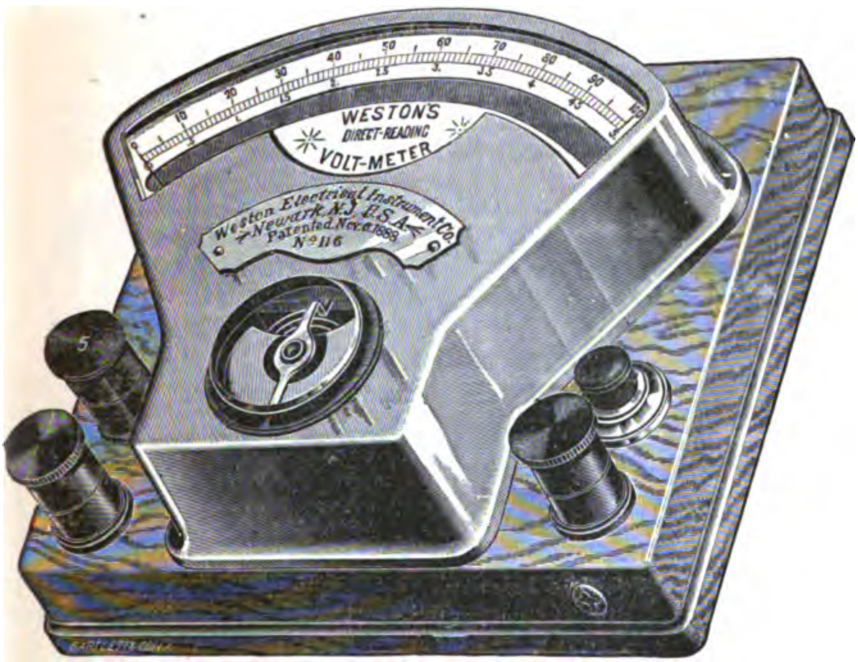


Fig. 212. The Weston Voltmeter.

an expression giving the amount of resistance to be placed in a shunt to give any desired multiplying power m. The best galvanometers are accompanied by shunt-boxes having multiples of 10, 100, and 1000.

239. The Weston Instruments. — For the field-work of electrical engineers, the series of instruments manufactured by the Weston Electrical Instrument Co. are eminently desirable. The general form of the Weston instrument is shown in Fig. 212, from which it will

appear that the instrument consists of a small, square mahogany box, about 6″ on each side, and about 1″ in thickness, which carries a raised brass framework, under which may be seen a graduated scale, over which a pointer travels. The mechanism of the instrument is shown in Fig. 213, and consists of a powerful horseshoe magnet carrying two enlarged pole pieces. Between the polar extensions a fine wire coil is delicately pivoted upon jewel bearings. To this movable coil is attached the pointer, or index, which plays over a graduated scale. If a coil of wire carrying an electrical current is placed between the poles of a magnet, it will tend to set itself at right angles to the lines

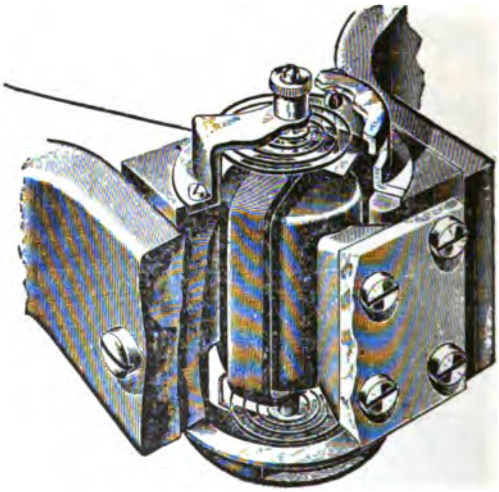


Fig. 213. Needle-Bearing Weston Instrument.

of force. In order to make a measuring-instrument, it is necessary that this tendency to turn be opposed by some well-known graduated counter-force. In the Weston instrument this is accomplished by introducing two flat spiral springs, fastened to the ends of the coil above and below, close to the bearings. When no current is in the instrument, these springs will keep the coil at a certain zero position, from which it will be deflected in proportion to the current through the coil. The pointer may therefore be arranged to move over an appropriately graduated scale, giving indications, which, by proper calibration, will be of great precision. Long experience and great care in workmanship have served to refine the details of the Weston instruments until they are exceedingly reliable. They are now made

to cover every practical range of capacity, and are designed to be used either as ammeters, voltmeters, or wattmeters. They are further arranged to be used either on direct or alternating currents.

As the Weston instruments are perfectly dead-beat, and will give fairly reliable readings in even so unfavorable a location as a jolting electric car, they form an essential part of the electrician's outfit. For work requiring particular accuracy, the instruments should be recently calibrated, set quite level, carefully oriented, removed from powerfully varying magnetic fields, and corrected for temperature.

The instruments thus far described have all been of the galvanometer type, and are open to the objections of requiring frequent calibration; of sensitiveness to surrounding magnetic fields; of introducing a slight error by consuming in themselves a small fraction of the energy of the circuit to which they are applied; and of requiring

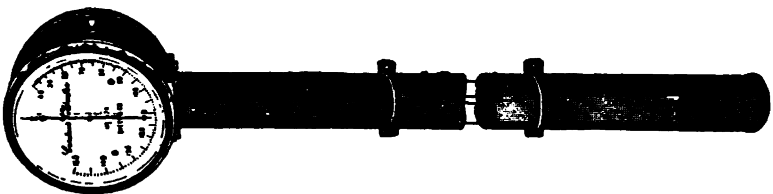


Fig. 214. The Cardew Voltmeter.

temperature corrections when accurate work is desired. To obviate these difficulties many devices have been proposed, among the most successful of which the Cardew voltmeter and the electrostatic voltmeters of Lord Kelvin may be mentioned.

240. The Cardew Voltmeter. — The operation of the Cardew instrument depends upon the expansion produced in a long fine wire, due to the amount of heat developed in the wire by the current flowing through it. The heating effect of a circuit is proportional to the square of the current, and to the resistance of the circuit. By making the resistance extremely high, the amount of current becomes proportional to the electric pressure. In the Cardew voltmeter a long fine wire gives the necessary resistance. The instrument is shown in Fig. 214, and a view of the mechanism in Fig. 215.

The wire is stretched under the action of a spring, and by suitable mechanism is connected with a registering pointer. When applied to a circuit, a small fraction of the current passes through the

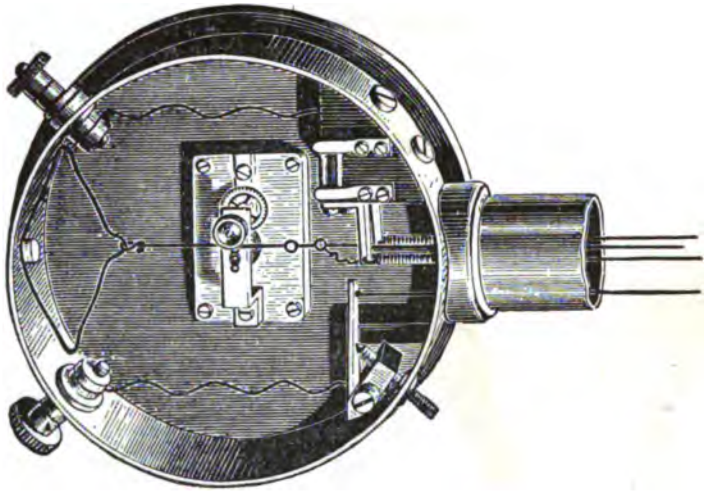


Fig. 215. The Mechanism of the Cardew Voltmeter.

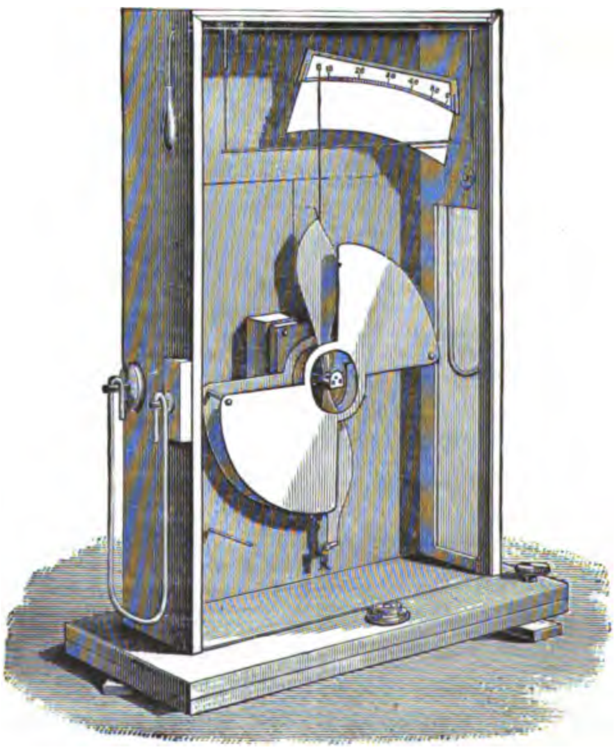


Fig. 216. The Electrostatic Voltmeter.

fine wire, and, being transformed into heat, expands it. The movement of the needle really records the amount of this expansion, which by proper calibration may be made to read in terms of the voltage of the circuit. This instrument is perfectly dead-beat; absolutely insensitive to all magnetic fields; and, when carefully made, forms within a limited range an instrument of great precision.

241. The Electrostatic Voltmeter. — The Kelvin voltmeters are constructed on the principle of an air condenser, one of the sets of plates of which is movable about an axis in such a manner that the capacity of the apparatus may be either increased or diminished. The instruments are so designed that, under the influence of an electrostatic stress, the pointers indicate the tension produced. They cover the widest range, having capacities to measure from 40 to 100,000 volts. As they take no current, they are insensitive to changes in temperature and to varying magnetic fields, and introduce no errors into the circuits to which they are applied. One form of the instrument is shown in Fig. 216.

The instrument consists of quadrant-shaped plates inclosed in a glass case with metal back, the plates being in metallic connection, and nearly surround an aluminum plate suspended between them. The movable aluminum plate carries a pointer which indicates on a scale at the top of the case the difference of potential between the two parts of the condenser. When the fixed and movable plates are connected to two points of an electric circuit, between which there exists a difference of potential, the movable plate places itself in such a position as to augment the capacity of the instrument, and the magnitude of the displacing force is proportional to the square of the difference of potential acting upon the plates. This force is counterbalanced by means of a weight which can be hung upon a knife-edge at the lower extremity of the movable plate. In order to economize time in making readings, there is a device for checking the oscillations of the movable plate, and stops are introduced to limit its range of motion, and prevent damage to the indicator. The scale of the instrument is graduated from 0 to 60, the divisions indicating equal differences of potential. The actual value of any division depends upon the weight that is placed upon the knife-edge of the movable plate. With each instrument a set of three weights is usually supplied, having ratios of 1, 2, and 4. When the smallest

weight is used, each division indicates 50 volts; with the second, 100; and with the third, 200 volts.

242. Siemens Dynamometer. — This instrument consists of two coils of wire, one fixed and one movable, which are arranged as in Fig. 217, so the movable coil surrounds the fixed coil placed in the center of the instrument. By the means of binding-posts on the base, the current to be measured may be caused to flow through both the fixed and the movable coil. The movable coil is suspended from the top of the instrument by means of a spiral spring attached to a knob which carries a pointer playing over a graduated scale. When a current is passed through the instrument, there is mutual attraction set up between the movable coil and the fixed coil. The movable coil, therefore, swings about its axis, and, by means of the spiral spring and milled head, may be brought back to its original position at right angles to the fixed coil. The number of degrees or divisions of the scale through which it is necessary to turn the head forms a measure of the current. It is usual to accompany these instruments with a tabular statement showing the value of the divisions on the scale. When wound with coarse wire, this instrument may be used for measuring current strengths of any desired amount up to several thousand amperes. When wound with fine wire, a similar instrument may be used as a voltmeter; and by winding the fixed coil with coarse wire, and the movable coil with fine wire, the instrument becomes a wattmeter whose indications are proportional to the total energy flowing through the circuit.

Fig. 217. Siemens Dynamometer.

243. Condensers. — When one conductor is adjacent to another it possesses the property of storing upon its surface a quantity of electrical energy. This quality is called the capacity of the conductor, and plays an important part in the development of electrical circuits. The capacity of a conductor may be numerically defined as

the number of coulombs of electricity required to be given to the one conductor in order to produce a difference of potential of one volt between it and the other. The capacity of a conductor depends upon its geometrical shape, upon its position relatively to the neighboring conductor, and on the characteristics of the dielectric separating the conductors. The Leyden jar is a familiar example. In this case a glass jar, coated inside and out with tin-foil, gives the two conductors which are separated by means of the glass of the jar as a dielectric. The capacity of circuits is usually measured by comparing the quantity of electricity which may be stored upon them with that of a standard condenser. The condenser usually consists of a box of insulating material in which are preserved a number of alternate layers of tin-foil, separated by paraffine paper or mica as an insulator, the paraffine paper serving in the condenser precisely the same office as the glass in the Leyden jar. A condenser may be charged by connecting its terminals with the poles of a battery, the amount of electricity stored being in proportion to the size of the condenser and the electromotive force of the battery.

Fig. 218. Standard Condenser.

The unit of capacity is the "farad." Inasmuch as this unit is too large for ordinary work, condensers are usually made in fractions of one or more millionths of a farad, termed a microfarad.

The standard condenser usually takes the form of a carefully finished box, having upon its top a series of plates that may be connected by means of plugs to respective divisions of the condenser. In fact, the apparatus very closely resembles a resistance or megohm box. An exceedingly convenient size is given in the illustration, Fig. 218, having a total capacity of one microfarad, subdivided into five parts of .5, .2, .2, .05, and .05 microfarad each.

244. The different subdivisions of a condenser may be combined either in series, in parallel, or in the various combinations of series-parallel. Thus: supposing that $C' C'' C'''$, etc., be the respective capacities of the various subdivisions of a condenser. They may be grouped

in parallel as represented by $C' + C'' + C''' + C'''' +$ etc. Under this condition the total capacity will be equal to the sum of the respective capacities. This condition may be expressed graphically as shown in Fig. 219.

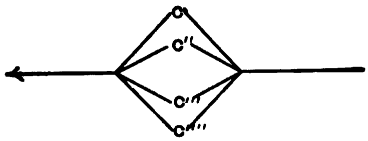


Fig. 219. Diagram of Condensers in Parallel.

When the combination is made in series the joint capacity follows the law of the resistance of parallel circuits, the capacity being the reciprocal of the sum of the reciprocals of the respective parts. Analytically this is expressed by:—

$$C = \frac{1}{\frac{1}{C'} + \frac{1}{C''} + \frac{1}{C'''} + \frac{1}{C''''} + \text{etc.}}$$

Graphically the relation may be indicated by Fig. 220.

$$\text{———} \; C' \; \text{———} \; C'' \; \text{———} \; C''' \; \text{———}.$$

Fig. 220. Diagram of Condensers in Series.

Further combinations may be made by uniting the parts of a condenser in any of the possible series-parallel arrangements. Such combination may be expressed either symbolically or graphically. For example, one combination of a three-part condenser is:—

$$C' + \frac{C''C'''}{C'' + C'''},$$

or graphically, Fig. 221.

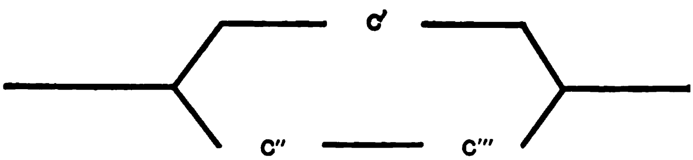


Fig. 221. Diagram of Condensers in Series-parallel.

Thus, with two divisions in a condenser, four combinations may be made; viz.:—

$$C', \; C'', \; C' + C'', \qquad \text{and} \qquad \frac{C'C''}{C' + C''}.$$

With three divisions fourteen combinations may be made, the possible combinations increasing in a geometrical ratio with the number of parts of the condenser.

WATTMETERS.

245. The wattmeter forms one of the most valuable measuring instruments at the command of the electrician; for by its use the total energy delivered at any point of a circuit may be measured, irrespective of the variations in potential and current. Instruments of this kind are divided into two classes, known as the Indicating Wattmeters and the Integrating Wattmeters. Instruments of the first division are typified by the Weston Wattmeter and the Siemens Electro-dynamometer, which have been already described. Their province is simply to indicate the instantaneous value of the product of the volts and amperes traversing any part of the circuit. The second class, or integrating instruments, embrace nearly all the various devices known as electric meters, of which the Thomson Recording Wattmeter is a representative example. These instruments do not indicate instantaneous values, but integrate the total energy delivered to the circuit during the time through which they are attached to it. Thus the readings of these devices are in watt-hours or watt-minutes. To obtain the average instantaneous value of the energy, the reading of the instrument (if in watt-hours) must be multiplied by 3,600, and divided by the time expressed in seconds during which the meter has been in circuit.

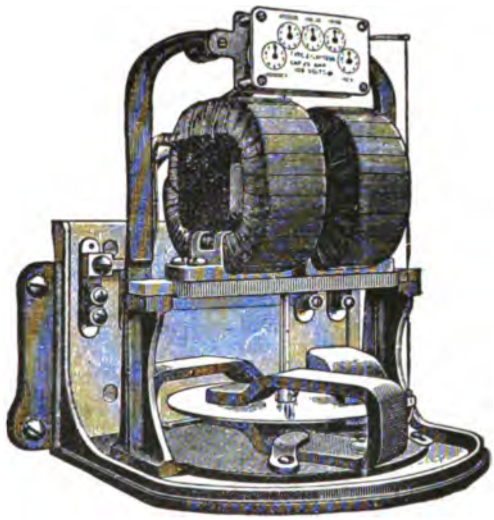


Fig. 222. Thomson-Houston Wattmeter.

246. The Thomson-Houston Wattmeter. — One of the most valuable forms of wattmeter is that devised by Professor Thomson, and is shown in Fig. 222.

It consists of an iron frame carrying two heavy coils of wire, through which runs a light shaft attached, near its base, to a copper plate revolving between the poles of three magnets. The shaft also carries a coil of fine wire placed inside of the coarse wire coils. This instrument is therefore an electrical motor, in which the coarse wire coils form the field, the fine wire coils the armature, the rotation of the shaft being proportional to the product of the current in the coarse and fine wire coils, which, in turn, is proportional to the total quantity of electricity and to the pressure in circuit. The rotation of the copper disk between the poles of the magnets experiences a constant retarding force, which tends to check the motion. The dial at the top serves to register the rotation of the shaft, and is calculated to give readings in watt-hours. Thus this instrument sums up the entire energy which flows through a given circuit between any two intervals of time at which readings may be taken.

Fig. 223. Short-circuiting Key.

247. Keys. — To complete a set of testing instruments, a number of keys should be provided for readily manipulating the circuits. The most important keys are the short-circuiting key, the reversing key, and the discharge key. See Figs. 223, 224, and 225.

By means of the short-circuiting key, the galvanometer coils may be closed upon themselves at the instant of opening the circuit, thus checking the oscillations of the needle, and tending to render the instrument dead-beat.

By means of the reversing key the direction of the current in a given circuit may be quickly and conveniently changed.

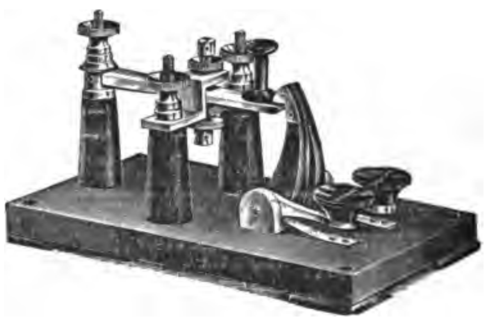


Fig. 224. Discharge Key.

The discharge key is a device for connecting the condenser alternately with the charging battery and the galvanometer, and is a necessary adjunct for all capacity tests.

248. The Magneto. — The apparatus termed "a magneto,"

frequently used in making tests of electrical machinery, is a very convenient rough-and-ready instrument. It consists of a small box, Fig. 226, carrying a bell furnished with a small alternating current dynamo. By means of a crank at the side of the box the armature of the dynamo can be rapidly rotated, thus generating an alternating current. If the circuit of the machine is closed, the current flows through the bell, and by causing the bell to ring, gives indication that the circuit is continuous. The magneto is chiefly used to detect low insulation. For this purpose the little generator is wound to be able to ring the bell through a resistance of from twenty to twenty-five thousand ohms. It thus forms a very handy and convenient detector for the purpose of determining short circuits or defective insulation. After considerable practice with a particular instrument, it becomes quite possible to make a rough approximation of the resistance of a circuit by judging from the strength and clearness of the ring which is given.

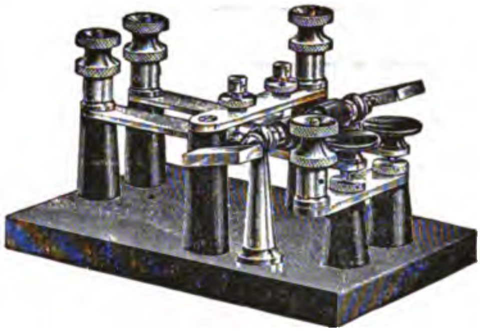


Fig. 225. Reversing Key.

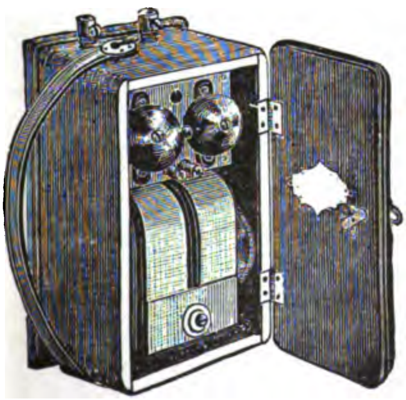


Fig. 226. The Magneto.

249. Ground Indicators. — Important lines carrying heavy currents should, as a matter of safety, be provided with constantly acting telltales to instantly indicate any fault in the insulation. The most simple of these contrivances is arranged in the following manner. Two similar incandescent lamps, of an appropriate voltage to be fully illuminated when placed in parallel across principal conductors, are connected in series as shown in Fig. 227, at L and L′ (p. 298).

Under these circumstances the lamps will burn at a dull red. As long as the circuit is completely insulated, no current will traverse the wire f. The galvanometer, or bell, gives no indication, and the aspect of the two lamps is identical. If now, however, a leak occurs at any other point of the line, a current will flow through f. One of the lamps, therefore, will find itself shunted by the circuit through the earth, and will consequently burn less brightly, while the light of the other lamp is augmented.

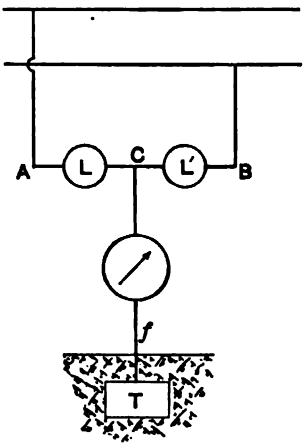


Fig. 227.
Continuous Current Ground Indicator.

250. Upon lines carrying alternating currents, an analogous arrangement can be used; and as it is inadvisable to permanently ground any part of the circuit, it is better to interpose in the ground wire a switch, by means of which connection may be made whenever it is desired to test the insulation of the line. Moreover, it is advisable to make the test-wire a part of the primary circuit of a transformer, in the secondary of which the telltale lamps are placed. The arrangement of this apparatus is shown in Fig. 228.

251. These contrivances, however, are defective to the extent that they do not give continuous and automatic indications of the insulation of the circuit, but require the presence of an operator to obtain results. The following modification of the ground indicator may be used for alternating circuits, from which continuous indications can be obtained. The principle of this contrivance is shown in Fig. 229.

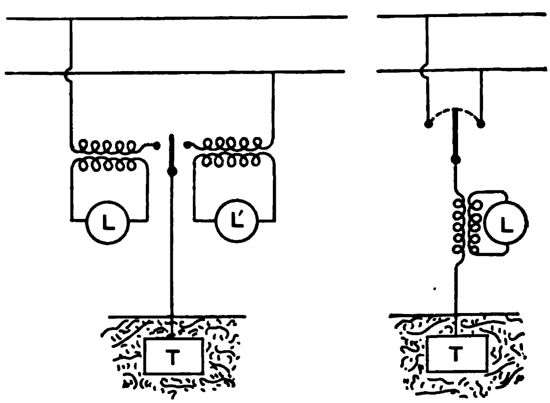


Fig. 228. Ground Indicator for Alternating Currents.

Between the principal conductors two large metallic plates are connected, C and C′, forming the armatures of a condenser. The other plates are connected to the ground by means of a wire, into which is placed a telephone, *t*. As long as the insulation remains perfect no current flows through the grounded wire. As soon, however, as a fault occurs, an alternating current is set up in the wire, which manifests itself by so loud a hum in the telephone as easily to be perceived throughout a large room. The sensitiveness of the telephone is sufficient to make this apparatus work successfully with condensers of very small capacity.

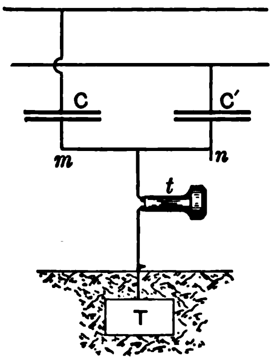


Fig. 229. Telephonic Ground Indicator.

252. The Boyer Speed Recorder. — The Boyer Speed Recorder, Fig. 230, is an instrument for obtaining instantaneous values of, and recording the curve of speed of any axle on any machine, and consists of a little rotary oil-pump supplied with a gauge, recording pencil, and a cylinder carrying a roll of cross-section paper, which is moved by clockwork. For determination of car speed, indications are obtained by attaching the machine by means of a belt to the car-axle, the motion of the axle being transmitted to the pump, and producing a pressure upon a piston attached to the pencil, thus causing the piston to rise and fall in proportion to the speed attained by the car. As a result, the instrument traces a curve, whose abscissæ and ordinates express, at any instant of time, the value of the car speed as a function of time. A curve, as described by this instrument, is given in Fig. 231.

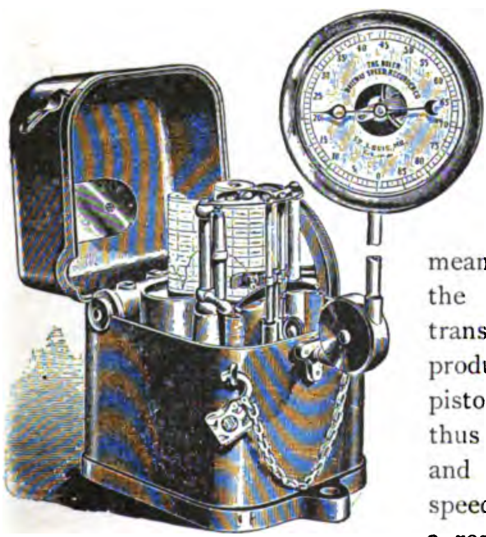
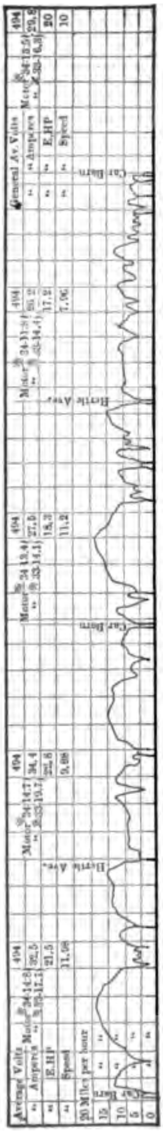


Fig. 230. Boyer Speed Recorder.

While the indications of the Boyer instrument give instantaneous values for car speed, it is frequently of use to obtain the mean speed. Assume on the horizontal axis of the diagram any points A and B, between which it is desirable to obtain the mean speed. The time required for the car to go any short distance dx, at a speed b, is dx/b. Consequently, the total time required for the car to go between points A and B is equal to the —

$$\int_A^B \frac{dx}{b}.$$



TEST OF CAR NO. 3, EQUIPPED WITH TWO MOTORS. LOAD ABOUT 22,000 POUNDS. TRACK NEARLY LEVEL.

Fig 231. Speed Curve.

The distance traversed is $B-A$, hence

$$\frac{B-A}{\int_A^B \frac{dx}{b}}$$

is the true mean speed.

The integral

$$\int_A^B \frac{dx}{b}$$

is the area of a reciprocal curve to that given by the Boyer indicator, between points A and B, which may be obtained as follows: —

Subdivide the base-line of the curve given by the indicator into equal parts, and set off upon the ordinates drawn to these divisions a series of lines, whose length will be, respectively, equal to the reciprocals of the ordinates to the Boyer curve at each proper point.

Drawing a curve from the vertices of these ordinates, a new curve is obtained, which is the reciprocal curve sought for. When the car stops, the value of the expression under the integral sign becomes infinity, which cannot be included in the calculation.

CHAPTER X.

METHODS OF ELECTRICAL MEASUREMENT.

Art. 253. To the practicing engineer, the various methods of electrical measurement are chiefly valuable as affording a means of inspecting the condition of the circuit of a plant for the electrical transmission of energy, with a view to the determination of the performance, in order to afford information as to the possibilities of increasing efficiency or remedying defects. In every electrical circuit, there are five elements which are worthy of consideration. To the line may be attributed the properties of resistance, capacity, and inductance, while, having regard to the energy conveyed, there are the factors of potential and quantity of current.

In strictly scientific investigation, all electrical quantities are, by means of the C. G. S. system, finally referred to fundamental units of length, mass, and time. For practical work, the more common commercial units are chiefly used, being readily deduced from the C. G. S. system.

254. Electrical Intensity. — The amount of energy transmitted by electricity is always measured by the product of two factors, namely, electrical intensity multiplied by electrical quantity. Electrical intensity, pressure, potential, or electro-motive force, as it is variously called, is that property of electrical energy by means of which it is enabled to overcome resistance. While the foregoing synonyms have not quite a parallel significance, when used in the most exact scientific sense, the terms are usually regarded as interchangeable for ordinary work. The commercial unit of electrical pressure is the *Volt*, and may be defined as that amount of electrical pressure which will produce a current of one unit of electrical quantity in a circuit having one unit of resistance in one second (or unit) of time.

255. Electrical Quantity. — The unit of electrical quantity is the *Coulomb*, and is defined as the amount of electricity which will flow through a circuit having a resistance of one unit in one

second of time, when the difference of electrical pressure between the ends of the circuit is one volt.

256. Unit of Current. — As a corollary to the two preceding units, a circuit having a resistance of one unit, and which, under a pressure of one volt, delivers in a second of time one coulomb of electricity, is stated to carry one unit of current. This unit is termed the *Ampere*. Usually all currents are measured in amperes.

The coulomb defines electrical quantity, pure and simple, while the ampere conveys the idea of rate of transfer; namely, one coulomb per second, the ampere differing from the coulomb by embracing the idea of time.

257. Capacity. — It is found that circuits of all kinds, and, in fact, all conductors and insulators, are capable of storing a certain amount of electrical energy; and the ability to thus contain electrical energy is termed "capacity." The unit of capacity is the *Farad*, and is that amount of electrical capacity which, under an electrical pressure of one volt, is enabled to store one coulomb of electricity. Unfortunately for practical use, this unit is altogether too large; and the *Microfarad*, or millionth of a farad, is the subdivision most commonly employed.

258. Resistance. — The unit of resistance is the *Ohm*, and is equivalent to the resistance of a column of pure mercury, one square millimeter in cross-section, 106 centimeters in length, at a temperature of 0° Centigrade. For practical purposes, units of resistance, in the form of resistance-boxes, as described in the last chapter, are commonly employed.

259. The Watt. — The amount of energy transmitted during a given time by an electrical current is equivalent to the product of the electrical pressure multiplied by the quantity of electricity. To measure the power of doing work of a given current, gives rise to the employment of a derived unit called the *Watt*, equivalent to the product of the volts and amperes, precisely in a manner similar to the determination of mechanical work by means of the foot-pound. Thus the rate at which any machine is capable of dispensing energy is measured by the number of foot-pounds per minute that it is capable of delivering; so, in an electrical circuit, the rate of doing work is equal to the number of volt-amperes, or "watts" per unit of time.

260. Ohm's Law. — In any electrical circuit, the generator may be regarded as a contrivance whereby, at one point of the circuit, the electrical potential may be raised; and if, for the sake of illustration, electricity be regarded for the moment as a material substance, the pressure is rendered useful by the amount of electricity which may be set in motion against the resistance of the circuit. In every part of a circuit the amount of work expended is equivalent to the quantity of electricity that passes this portion of the circuit, multiplied by the fall of potential, or loss of pressure, that takes place within the part of the circuit under consideration.

According to the law of conservation of energy, the amount of work done by the generator will be precisely equal to the sum of all the work delivered in the whole circuit. If E be the electromotive force between any two planes in any circuit, R the resistance between the planes, and I the quantity of current flowing, in amperes, the relation existing between these quantities has been stated by Dr. Ohm to be: —

$$R = \frac{E}{I}. \tag{33}$$

As above written, Dr. Ohm's law only applies to steady, non-pulsating currents; but if the quantities E, I, and R be assigned values expressing the instantaneous *effective* electro-motive force, *effective* current and impedance of the circuit, having due règard to the positive and negative effects of capacity and inductance, the formula becomes applicable to currents of all descriptions, whether continuous or alternating, and of whatsoever shape of wave.

261. Kirchhoff's Laws. — Kirchhoff has announced, under the form of two laws, principles that underlie many of the formulæ employed in electrical investigation.

First Law. — If any number of conductors meet at a point, and if all the currents flowing toward the point be considered as positive, while all those flowing away from the point be considered negative, and if equilibrium exists, that is, if the electrical potential at the point of junction remains steady, the algebraic sum of all the currents meeting at the point will be zero. Mathematically expressed: —

$$\Sigma I = 0. \tag{34}$$

Second Law. — In any network of electrical conductors forming a closed polygon containing varying currents, varying resistances, and

varying electro-motive forces, the algebraic sum of all the products of the currents and resistances is equal to the sum of all the electro-motive forces, or mathematically: —

$$\Sigma I \times R = \Sigma E. \tag{35}$$

Nearly all the formulæ for electrical measurements are based upon the laws of Ohm and Kirchhoff; and while the demonstration of the succeeding formulæ are not given in full, they may readily be deduced from the above cited laws by the ordinary algebraic processes.

262. It is now proposed to consider the determination of the various electrical quantities in the following order: —

Resistance.	Capacity.
Current Strength.	Inductance.
Electro-Motive Force.	

For the determination of these quantities, such a selection of methods is given as will enable the practicing electrician to choose an arrangement to fit the apparatus commonly to be found in all electrical installations. In the methods given, careful consideration has been exercised to include only those which are of the greatest practical value, leaving laboratory methods and tests at one side, as not being suitable for the field-work of the electrical engineer.

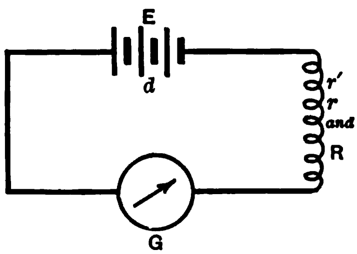


Fig. 232.
Diagram for Resistance by Deflection.

In all methods of measurement, it is necessary to compare a standard unit with the unknown quantity, in order to ascertain the ratio which exists between the two; and for this purpose it is essential to make use of some form of indicator, by means of which comparison between the standard and the unknown can be readily made. In most electrical measurements, the galvanometer is selected for this purpose.

All of the instruments necessary to the following methods have already been discussed and described in the previous chapter.

263. Measurement of Resistance. — *First, by deflection.* In Fig. 232, suppose G to be a galvanometer of any desired type, E a battery, or other generator of convenient voltage, and *r* a known

resistance, such as may be readily found in a standard resistance-box, and that it is further desired to measure the value of some unknown resistance, R. Connect up the galvanometer battery and the known resistance r, as shown in Fig. 232; that is to say, with the galvanometer, battery, and resistance all in series. Let G equal the resistance of the galvanometer, r the known resistance, and r' the resistance of the battery and the remainder of the circuit. In many cases, this latter quantity is so small in comparison with the unknown resistances, that it may be neglected. If E be the electro-motive force of the battery, and I the current in the circuit, a certain deflection d will be produced on the galvanometer. By Ohm's law,

$$I = \frac{E}{G + r' + r}; \tag{36}$$

$$I(G + r' + r) = E. \tag{37}$$

Now, for the known resistance r, substitute the unknown resistance R. Under these circumstances, suppose I' to be the current in the circuit, giving a new deflection d' upon the galvanometer.

$$I' = \frac{E}{G + r' + R}; \tag{34}$$

$$I'(G + r' + R) = E. \tag{39}$$

Solving for R, transposing, and arranging, —

$$R = \frac{I}{I'} \times (G + r + r') - (G + r'). \tag{40}$$

As the deflections d and d' are proportional to the currents I and I', d/d' is proportional to I/I', and may be substituted for it, hence: —

$$R = \frac{d}{d'}(G + r + r') - (G + r'). \tag{41}$$

264. The deflection readings on the galvanometer scale in degrees have been used in the formulæ. When the readings are small, or when no special accuracy is required, this assumption is sufficiently correct. When the deflections are of considerable magnitude, or when in using the scale readings, either of an ordinary galvanometer or of a reflecting instrument, great accuracy is desired, the tangent of d and the tangent of d' should be substituted for the actual reading in degrees. To measure resistance by this method, the resistance of the galvanometer, as well as that of the rest of the circuit, must be known, or neglected.

The galvanometer resistance is usually given by the maker. Knowing the galvanometer resistance, and neglecting that of the battery and connections, —

$$R = \frac{d}{d'}(G + r) - G. \tag{42}$$

By adjusting the resistance r so that $d' = 2d$, the preceding formula is simplified. Under this condition equation becomes —

$$R = \frac{r - G}{2}. \tag{43}$$

When it is inconvenient to make d' equal $2d$, simplification may be obtained by making d' any even multiple of d.

265. The quantity rd, obtained by multiplying a known resistance r by the galvanometer deflection produced with the resistance r in circuit, is called the galvanometer constant, and is much used in making line-insulation tests.

Thus a galvanometer, megohm box, and battery are joined up in series, and a deflection of d divisions is obtained with r ohms in circuit, rd being the constant. Any other high unknown resistances, R', R'', R''', etc., now are substituted for r, giving deflection d', d'', d''', etc. In case r is very small compared to R', R'', R''', etc., the method of the inferred zero may be advantageously applied. For great accuracy tangent d, tangent d', etc., should be used. In using a tangent galvanometer with methods in which only one deflection is concerned, it is best to make the deflection as nearly 45 as possible. If two deflection methods are employed, it is advisable to make them fall, as nearly as may be, at equal distances on either side of 45. If one deflection is to be double the other, then about 35 and 55 are convenient to employ.

For such measurements a battery of constant electro-motive force must be used, or corrections for change in pressure introduced. There must also be no change in the external magnetic field, or a redetermination of the constant is necessary.

266. Resistance by Wheatstone Bridge. — Resistance measurements by Wheatstone bridge are extremely simple. A battery is connected to the binding-posts, marked "Battery," of the testing-set, and the resistance to be measured connected to the posts marked "x." In each of the bridge-arms a peg is inserted in the coils that

are estimated to furnish the appropriate arm resistance; and then the pegs in the rheostat are shifted about until the needle of the galvanometer fails to move, indicating that a balance has been attained. The resistance indicated by the rheostat, multiplied by the appropriate factor due to the ratio of the bridge-arms, gives the desired resistance. To secure the best results, however, it is advisable to follow certain conditions.

Referring to Fig. 201, Chapter VI, assume the letters attached to the various parts of the bridge to represent the resistances corresponding to each one. Make a rough measurement to obtain an

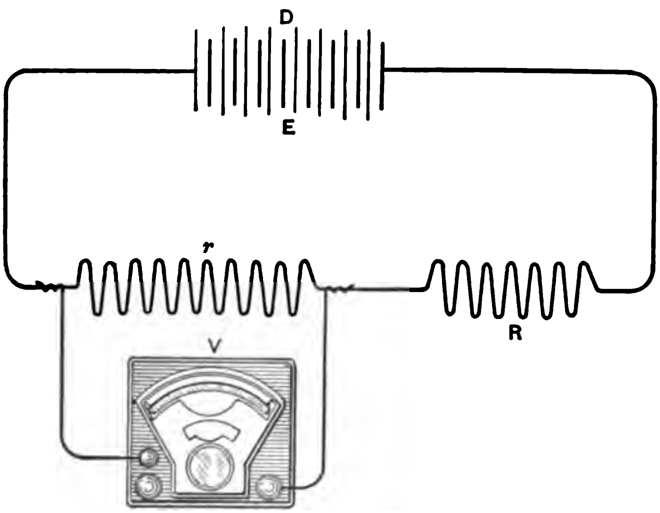


Fig. 283. Diagram of Resistance by Voltmeter Measurement.

approximation to the value of x. Then make the resistance in the arm d, as nearly as convenient, equal to $\sqrt{\frac{Grx + rx^2}{r + x}}$. It should not be less than this quantity, nor larger than $G + x$.

The electro-motive force of the battery should be as great as is convenient, compatible with safety to the testing-set, and the resistance of the exterior circuit as small as possible. Though these conditions are theoretically desirable, they can be attained practically very rarely, and only within the middle ranges of the capacity of the testing-set. If manipulated with great care and considerable skill, the ordinary testing-sets can be made to measure a thousandth of

an ohm; yet they are hardly reliable to so small an amount; so, if much accurate measurement on small resistances is to be done, recourse should be had either to the slide-wire bridge or to the differential galvanometer. So far as manipulation is concerned, the use of the slide-wire bridge is precisely the same as the common Wheatstone pattern.

267. Measurement of Resistance by Ohmmeter and Differential Galvanometer has already been described in the account of these instruments in Chapter VI.

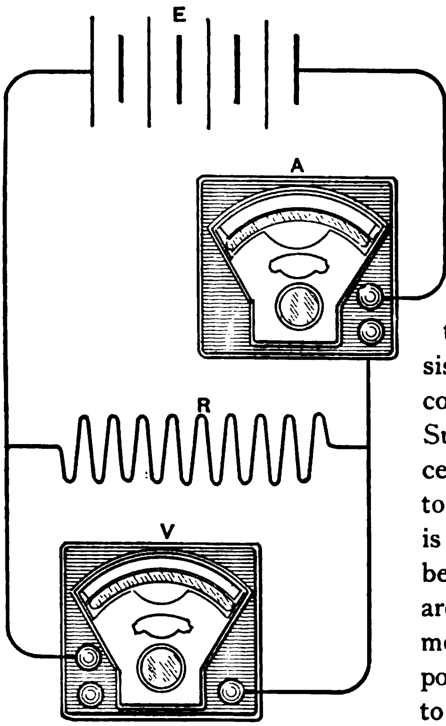


Fig. 234.
Diagram of Resistance Measurement with Volt and Ammeter.

268. Resistance Measurement by means of Voltmeter. — To measure resistance by means of a voltmeter, the apparatus should be connected as shown in Fig. 233, in which R is the resistance to be measured, E the battery or source of current, V the voltmeter, and r a known resistance. The voltmeter is first connected around the resistance r. Suppose, under these circumstances, the readings of the voltmeter to be V volts. After this reading is obtained, the voltmeter should be connected in a similar manner around the resistance R to be measured. In the latter case, suppose the readings on the voltmeter to be V' volts, then, —

$$V : V' :: R : r.$$

$$R = \frac{rV}{V'}. \tag{44}$$

By this method the measurement made by the voltmeter is the fall of potential through each resistance. As the electro-motive force is supposed to be constant, the fall is directly proportional to the resistance.

269. Resistance Measurement with Volt and Ammeter. — A modification of the preceding method may be made, when no con-

venient known resistance is at hand, by using, in place of the known resistance, an ammeter as shown at A in Fig. 234. In this method the current I, flowing in the circuit, is given by the reading of the ammeter; while the fall of potential E, through the unknown resistance, is given by the voltmeter. Thus two elements of Ohm's formula are obtained, from which R may be readily calculated. Thus:—

$$R = \frac{E}{I}. \qquad (45)$$

270. Small Resistance. — To measure very small resistances, the method indicated by the arrangement in Fig. 235 is to be used,

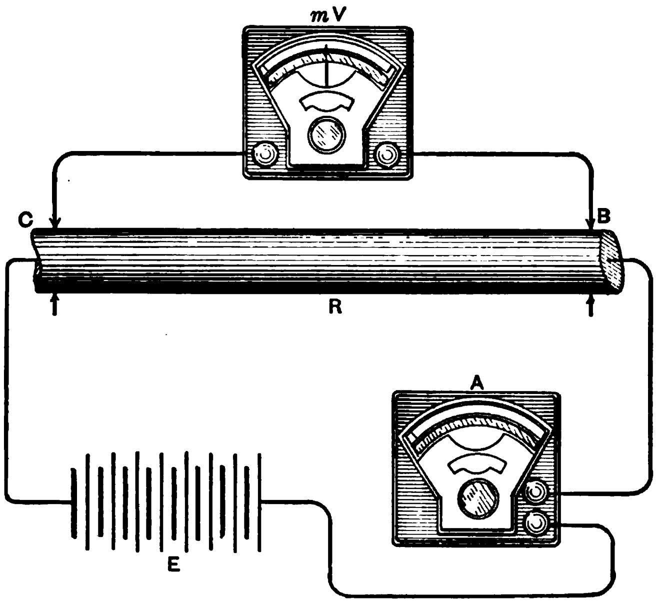


Fig. 235. Diagram of the Measurement of Low Resistance.

which is practically the same as in the preceding instance, except that, owing to the very small difference of potential to be estimated, a milli-voltmeter, *m*V, must be substituted for the voltmeter, in order to obtain readings of sufficient accuracy. A battery E, or other source, supplies requisite current, that is measured by the ammeter A. The unknown resistance R is placed in series with the ammeter and battery. The milli-voltmeter is applied to the points B and C, between which lies the resistance to be measured; the

reading giving the fall of potential between B and C. Care should be taken to make good contacts at B and C, that there may be no errors from loss of pressure at these points, and the reading at A and *m*V should be simultaneous. A very essential and practical application of this method is its adaptability to the measurement of resistance of armatures of dynamo machines. For this measurement the arrangement shown in Fig. 236 should be adopted, in which the milli-voltmeter is clamped to opposite sections of the commutator, while the battery and ammeter are placed in series with the same sections. The milli-voltmeter should be connected directly to the

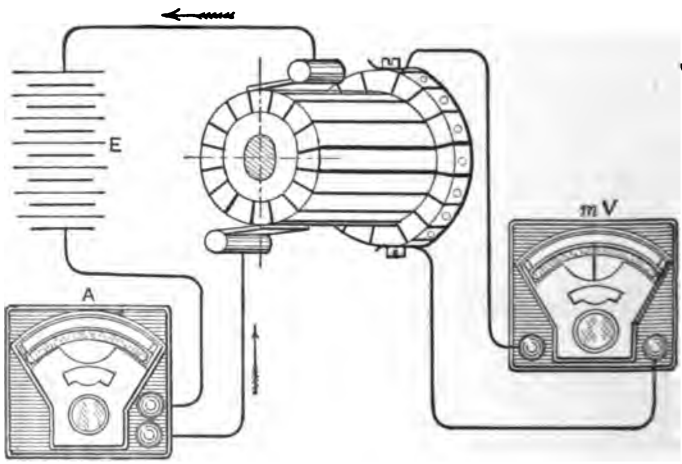


Fig. 236. Method of Measuring Armature Resistance.

sections, and not to the brushes, to avoid introducing the error of the contact resistance of the brush.

271. High Resistance. — For measuring high resistance, the connections shown in Fig. 237 are preferable. In this case the battery E, voltmeter V, and key K are so arranged that the resistance to be measured, *R*, may be either included or excluded from the voltmeter circuit. For measurements of this kind, as high an electro-motive force should be used as practicable, it being understood that in any event the potential is not higher than the highest reading of the voltmeter. With this arrangement, supposing *r* to be the resistance of the voltmeter, two measurements are made; first, with the switch closed, and then with the switch open. Sup-

pose V to be the reading with the switch closed, V' that with the switch open; then —

$$R = r\frac{(V - V')}{V'}. \tag{46}$$

272. A very convenient application of this method of measuring resistance occurs in making frequent trials of the insulation of high potential circuits. It is practical to make such tests with the line in full operation. The apparatus should be arranged as shown in Fig.

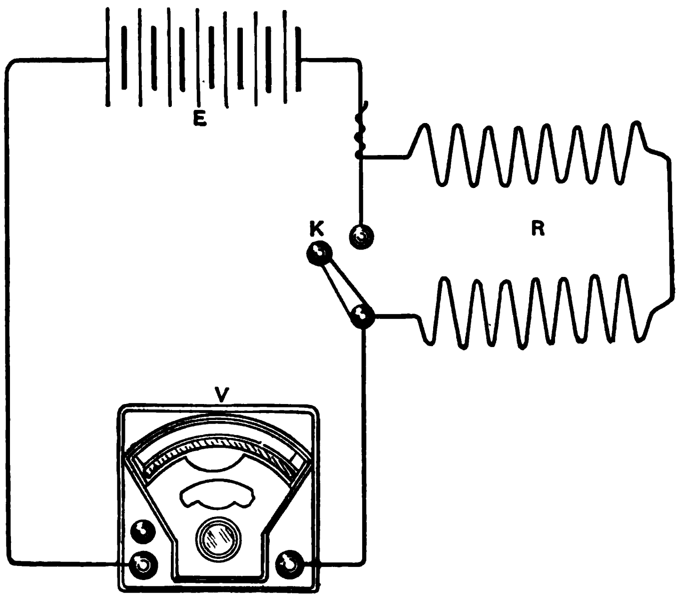


Fig. 237. Method of Measuring High Resistance.

238, in which the voltmeter V is connected first from one main, and then from the other main, to the ground. Under these circumstances, if V equals the difference in potential between the two sides of the line, V' the reading between either side of the line and the ground, and r the resistance of the voltmeter; then, representing the line insulation by R, its value is given by the equation —

$$R = r\frac{(V - V')}{V'}. \tag{47}$$

273. In the case of a dead ground, $V' = V$, and, consequently, $R = 0$. In case there is no current in the main, a test-battery may

be added, and the connection made first to one side of the circuit, and then to the other, as shown in Fig. 239.

As the resistances of the commercial voltmeter and milli-voltmeter

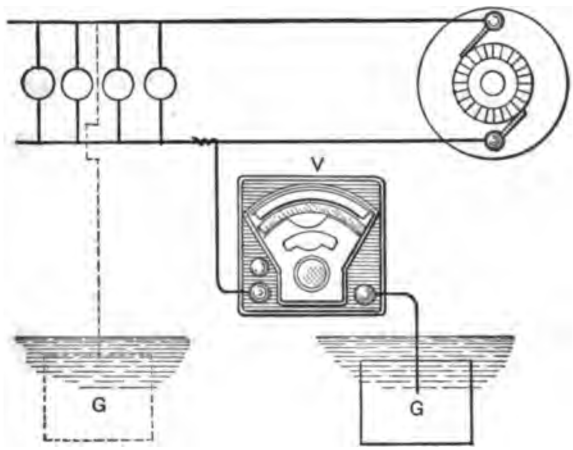


Fig. 238. Diagram of Method of Measuring Insulation Resistance.

vary from a fraction of an ohm to upwards of a megohm, this method may be used with great convenience and accuracy to determine all resistances that are commonly found.

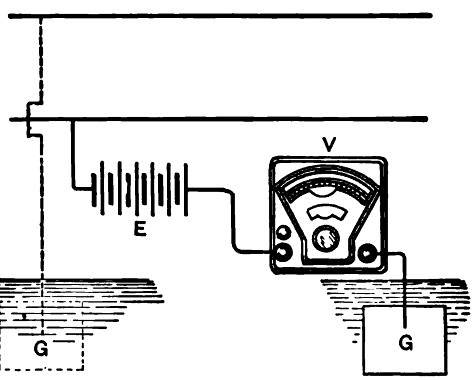


Fig. 239. Diagram of Test for Insulation.

274. Insulation Resistance by the Method of Loss of Charge. — If a condenser of a known capacity be charged and then be allowed to discharge through a resistance for such a period of time as will not completely exhaust its charge, and then, if the residual charge

remaining in the condenser be measured, it is possible to calculate from the charge lost the resistance.

To put this method into practice a condenser of known capacity having anywhere from ¼ to 2 micro-farads is charged by connecting it with a battery or other source of electricity. The terminals of the condenser are then connected to the resistance to be measured and allowed to remain in contact therewith for a definite number of seconds. When this time has elapsed the condenser is connected to a galvanom-

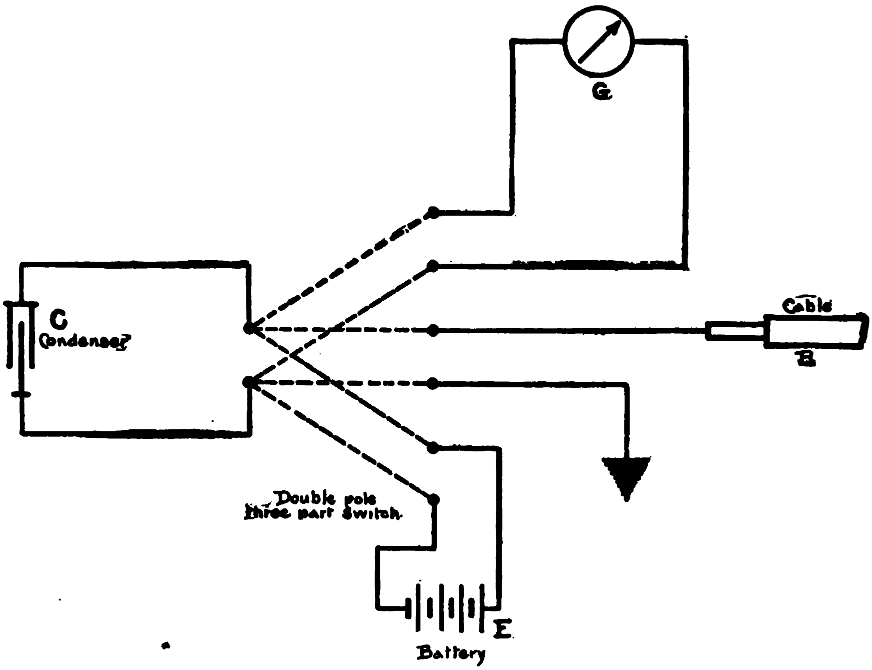


Fig. 240. Diagram of Circuit to measure Resistance by Loss of Change.

eter and the charge remaining in it measured by the deflection of the needle. The diagram, Fig. 240, shows the connections for this purpose. E is the battery of any convenient electro-motive force, preferably about 100 volts. Whenever an Edison lighting circuit is convenient it forms an admirable source for charging the condenser. G is the galvanometer, C the condenser, R the cable or other unknown resistance to be measured. By means of the key the battery is connected with the condenser for a few seconds. By throwing the key one pole of the condenser is then connected to the cable and the other to the earth (if any other high resistance is to be measured, one

terminal of the condenser is connected to one end of the resistance and the other terminal to the other end). The condenser is then allowed to discharge for a desirable length of time, and when this time has elapsed a second throw of the key connects the condenser to the galvanometer and the deflection of the needle is observed. Prior to making any test the deflection produced on the galvanometer by the full charge of the condenser should be measured. It can be shown that under these circumstances the resistance is given by the formula

$$R = \frac{.4343t}{C \log \frac{d}{d'}}. \tag{48}$$

In this expression R is the unknown resistance, t is the time in seconds that the condenser is connected with the unknown resistance, C is the capacity of the condenser in micro-farads, and $\log \frac{d}{d'}$ is the common logarithm of the quotient of the deflection d produced by discharging the fully charged condenser through the galvanometer divided by the deflection d' obtained after the condenser has been discharged for t seconds through the unknown resistance. The best condenser is one similar to that shown in Fig. 218, but good results can be obtained by using the common paper condenser made for telephone stations. Any of the galvanometers described can be used, though ballistic instrument is the most accurate. A high wound Weston voltmeter (25,000 ohms) is very convenient for this purpose.

A good rubber-mounted double pole switch forms an excellent key, so that the whole apparatus, condenser, voltmeter, and key may be placed in a box 8 in. wide, 3 in. deep, and a foot long. A flexible cord with a screw plug for attaching to a lamp socket, if Edison current is available, or a portable chloride of silver battery of seventy-five cells, completes the outfit. Theoretically if the insulation of the condenser is not perfect it will itself lose some charge during the time it is connected with the resistance to be measured, and for extreme accuracy the results must be corrected for this loss. This is easily done by deciding the length of time the condenser shall be connected with the cable, and measuring the deflection d after the condenser has been charged and insulated for this time, thus eliminating all question of loss of charge.

The preceding formula is not difficult to solve, but a graphical table may be made whereby all calculation can be saved, and as soon

as the deflections are read from the galvanometer the resistance in megohms is immediately obtained. Such a table is shown in TABLE 44 (see Pocket).

DIRECTIONS FOR USING TABLE.

Divide the deflection d, given by the galvanometer with the condenser fully charged, by the deflection d', given by the galvanometer after the same condenser has been connected to the cable for t seconds. If this quotient is less than 5, use the sheet with the scale A, in the left-hand corner. If greater than 5, use the sheet with the scale B, in the lower left-hand corner. On the left-hand vertical scale select the number corresponding to the quotient $\frac{d}{d_1''}$, and from this number follow a horizontal line to the curve marked A. Then follow a vertical line until this vertical intersects such a line radiating from the corner labelled "capacity factor" as bears the number representing the capacity of the condenser employed; thence follow a horizontal line to the right until this horizontal line intersects such a line radiating from the corner marked "time factor" as bears the length of time t that the condenser was allowed to remain in contact with the unknown resistance. Thence follow a vertical to the lower scale reading "unknown resistance in megohms." The method of using the table with the scale B in the left-hand corner is exactly the same.

275. Measurement of Line Resistance. — The resistance of lines may be readily measured by means of the Wheatstone bridge, under either of the following three methods:—

1. When the bridge can be grounded at one end and the line at the other, so thoroughly as to interpose essentially no ground resistance, and when no earth currents interfere with the measurement, good results can be obtained, the bridge measurement giving directly the desired resistance.

2. When a second wire of known resistance can be joined from the farther end of the wire to be measured and returned to the bridge. Under these circumstances, the resistance given by the bridge is that of the sum of the two wires, from which that of the known wire must be deducted, in order to obtain that which is unknown.

3. In cases where three wires, X, Y, and Z, are accessible, all of which may have unknown resistance. X and Y are to be joined at the farther end, and the resistance measured on the bridge, giving an

amount A. X and Z are then joined in a similar manner, and measured, giving a resistance B. Y and Z are then joined and measured, giving resistance C. Under these circumstances,

$$X = \frac{A + B - C}{2}; \tag{49}$$

$$Y = \frac{A + C - B}{2}; \tag{50}$$

$$Z = \frac{B + C - A}{2}. \tag{51}$$

276. Measurement of Ground Resistance. — The estimation of ground resistance could be accomplished similarly to the determination of any other resistance, were it not for the fact that frequently earth currents from extraneous sources, or polarization set up by the ground plates themselves, tend to vitiate the results. If these perturbating causes do not exist, the Wheatstone bridge may be used, and the resistance determined in the usual fashion. If two wires can be obtained, the measurement may be made as indicated in the preceding paragraph. If the earth currents are reasonably steady, an approximation to the true quantity may be obtained by making two bridge measurements; one with a positive and the other with a negative current, and taking the mean of the results. Otherwise the ground may be treated as a battery, and its resistance determined by any of the methods for measuring battery resistances.

277. Special Methods for Resistance Measurements. — The preceding methods for the determinations of resistance are adapted to all the general cases that will fall under the notice of the electrical engineer. For certain special cases, such as the measuring of galvanometer resistance and internal resistance of batteries, or other generators, some special methods are more expeditious and will now be noted.

278. Galvanometer Resistance by Equal Deflection Method. — Connect up galvanometer G, shunt S, known resistance r, and battery E of so low internal resistance that it may be neglected, as indicated in Fig. 241. Note the deflection of G. Now remove the shunt, and increase r to r', a second known resistance, until the *same* deflection is given by the galvanometer, then —

$$G = S \times \frac{r' - r}{r}. \tag{52}$$

This test is most accurately made by adjusting S, the resistance of the shunt less than G; the resistance r should be as large as possible, but not larger than —

$$r' \times \frac{S}{G+S},$$

r' being the largest attainable resistance. Low resistance battery power of sufficient quantity should be provided to give the deflection as nearly as possible at the angle of maximum sensitiveness.

279. Galvanometer Resistance by the Wheatstone Bridge. THOMSON METHOD. — Arrange the apparatus as shown in Fig. 242. Vary the resistance in the arm a or d, until the deflection on the

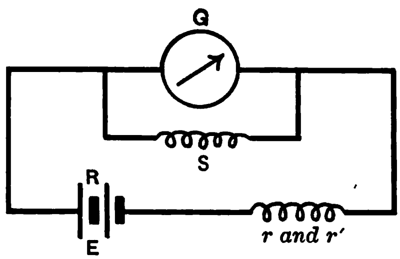


Fig. 241 Diagram of Resistance by Equal Deflection.

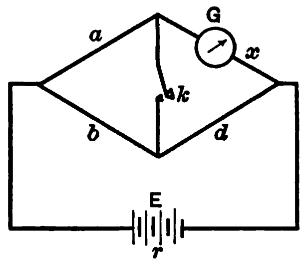


Fig. 242. Diagram of Galvanometer Resistance. (Thomson's Method.)

galvanometer G remains the same, whether the key k be up or down, then the value of G, the resistance of the galvanometer, is —

$$G = \frac{ad}{b}. \tag{53}$$

280. This test has the merit of being entirely independent of battery resistance, and of being very easily made. To attain the greatest accuracy, a should be about one-tenth of G, and b ten times as large as G. Vary d until it is nearly correct, and then change the battery power so that the final deflection shall be, as near as possible, at the angle of maximum sensitiveness of the galvanometer. Adjust d till the deflection remains unchanged on pressing the key.

281. Galvanometer Resistance by Condenser. — The connections for this method are shown in Fig. 243, in which G is the galvanometer, C the condenser, E the necessary battery furnished with the key K, and S a shunt that can at pleasure be placed around the galvanometer. The condenser is charged with the battery, and then

discharged through the galvanometer, giving deflection d. The condenser is again charged and discharged through the galvanometer, when shunted with a resistance S, giving a second deflection d', then —

$$d = \frac{E}{G};$$

$$d' = \frac{EGS}{G+S};$$

$$\frac{d}{d'} = \frac{S}{G+S};$$

$$G = S\left(\frac{d}{d'} - 1\right). \tag{54}$$

If S can be made so that $d' = d/2$, then $G = S$. (55)

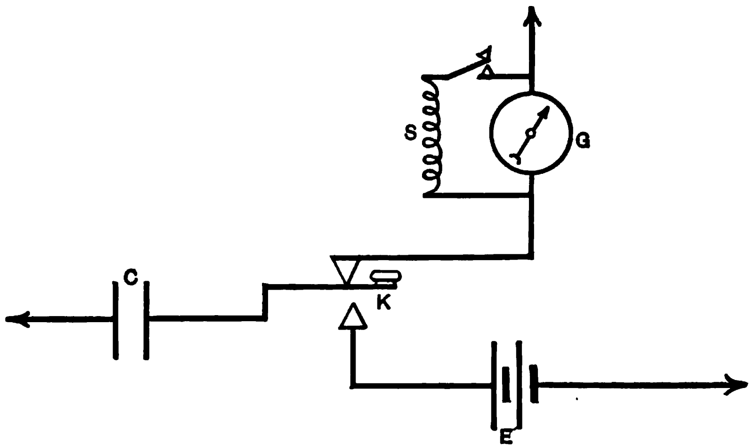


Fig. 243. Diagram of Galvanometer Resistance by Condenser.

282. Measurement of Battery Resistance by Voltmeter. — In Fig. 244 are given the connections for measuring battery resistance with a voltmeter. Suppose the battery to have an electro-motive force E. K is an appropriate key, and r a suitable known resistance. When the key K is open, the reading on the voltmeter indicates the potential of the battery. Upon closing the key K, the voltmeter indicates the difference in potential e existing between the ends of the resistance r. Under these circumstances, the resistance R of the battery is found from the equation, —

$$R = r\left(\frac{E-e}{e}\right). \tag{56}$$

283. Galvanometer Resistance by Deflection. — Connect the galvanometer to be measured in series with a known resistance r, as indicated in Fig. 232, obtaining the deflection d. Replace r by a second known resistance r', quite different from r, giving a second deflection d'. Then, in formula (41), substitute r' for R, and solve for the value of G, obtaining —

$$G = \frac{r'd' - rd}{d - d'}. \tag{57}$$

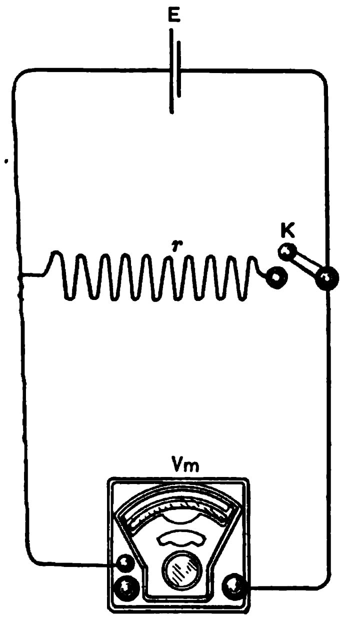


Fig. 244.
Diagram of Battery Resistance by Voltmeter.

284. Battery Resistance by Deflection. — If the galvanometer resistance be known, and two known resistances are at hand, the preceding method may be used to measure battery resistance. The connections are made as already described, the battery occupying the place of the unknown resistance. The known resistances are successively interposed in the circuit, the corresponding deflections obtained, and the necessary substitution made in formula (41).

285. In general, equation (41), involving six variables, of which two, d and d', are always measurable, may be used to determine any one of the remaining four, provided the other three are known, or may be neglected.

286. Battery Resistance by Condenser. — One of the best methods for the measurement of battery resistance is that involving the use of a condenser, for the reason that, while the connections are simple, the battery remains almost constantly upon open circuit, and is, therefore, free from errors due to polarization. The connections are shown in Fig. 245. A circuit is formed, comprising the battery E, the resistance of which, R, is to be measured, the condenser C, the galvanometer G, two keys k and k', and a known resistance r, arranged to shunt the battery. These are connected as shown, so that by depressing the key k the condenser may be charged through the galvanometer G, giving a deflection d on the galvanometer, cor-

responding to E, the electro-motive force of the battery. If now, while the key k remains closed, the key k' be also depressed, the battery is shunted through the resistance r; the potential at the poles of the battery falls to a value e given by the equation —

$$e = E\frac{r}{R+r}, \tag{58}$$

and a new deflection d' in the contrary direction is now obtained on the galvanometer, serving to measure the quantity $E-e$. As the deflections are proportional to the electro-motive force, —

$$\frac{E}{E-e} = \frac{d}{d'};$$

whence —

$$1 + \frac{r}{R} = \frac{d}{d'};$$

$$R = r \times \frac{d'}{d-d'}. \tag{59}$$

If the resistance r is so arranged that $d' = d/2$, then $R = r$. (60)

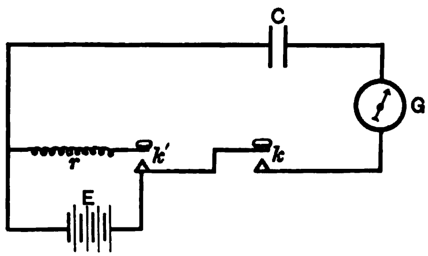


Fig. 245. Battery Resistance by Condenser.

287. Battery Resistance by Equal Deflection. — The equal deflection method for galvanometer resistance may be applied to determine battery resistance. The connections are shown in Fig. 241, in which R is the battery resistance to be measured, S a shunt of known resistance around the galvanometer G, and known resistance r in series with R and G. The circuits being connected as indicated, a deflection d is obtained on the galvanometer. S is then removed, and r increased to r', until the same deflection is obtained.

Then, $$R = S \times \frac{r'-r}{r+G}. \tag{61}$$

288. S should not be less than R; $r + G$ should not be larger than $s/G \times (G + r')$, r' being the greatest attainable resistance. The deflection should be arranged to fall nearly at the angle of maximum resistance sensitiveness.

289. Measurement of Potential Differences. — The most simple method of measuring potential differences is by means of a voltmeter; the instrument being directly connected to the poles of the generator, and the reading of the needle indicating at once the desired voltage. As various instruments are made to cover a range from .00001 of a volt to 10,000 volts, they are amply sufficient for all ordinary practice.

290. In the case of the Weston instruments, if the polarity of the generator be unknown, the instrument may also serve as a pole-finder; for if, on connecting the instrument, the needle is deflected toward the right, the binding-post on the right hand is the positive pole. If the needle should be deflected entirely across the scale, indicating that the potential difference is greater than the instrument is designed to measure, it is advisable to use one having a greater range. Yet, at the same time, by introducing an additional resistance into the voltmeter circuit, a reasonable approximation to the correct voltage may be obtained. For example: supposing voltmeter reading to 150 volts be the only one at hand, and it is desired to measure in the neighborhood of 600 volts.

Let E = the greatest number of volts to be measured.

Let e = highest reading on the scale of the instrument at command.

Let r = the resistance of this instrument, and

R = the additional resistance necessary to make it read E volts, then —

$$R = r\frac{E - e}{e}. \tag{62}$$

Under these circumstances, when R ohms are added to the voltmeter circuit, the readings on the scale of the instrument must be multiplied by the factor E/e.

291. It is seldom possible to add the exact quantity R ohms to the voltmeter circuit. Supposing R' ohms to be the nearest approximation to R that can be secured, then the scale readings of the voltmeter must be multiplied by $\frac{R' + r}{r}$ to give correct values.

292. Measurement of Electro-Motive Force. THE CONDENSER METHOD. — The arrangement of the apparatus for measuring electro-motive force by the condenser method is indicated in Fig. 246, in which G is the galvanometer, E the battery, or generator, to be measured, e the standard cell with which comparison is to be instituted, C the condenser, and S a shunt around the galvanometer. This method consists in charging a condenser having a capacity of about one-tenth microfarad, by means of the standard cell, and then discharging it through the galvanometer, and noting the deflection d. The condenser is now to be charged by the generator whose electro-motive force is to be measured, and again discharged through the galvanometer.

A second deflection d' is obtained, the deflections being propor-

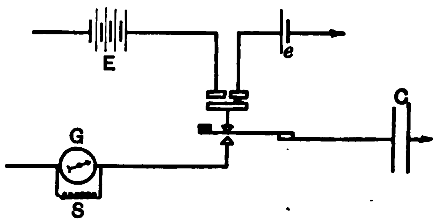


Fig. 246. Measurement of Electro-motive Force by Condenser.

tional to the electro-motive forces producing them; and if m be the multiplying power of the shunt,

$$\frac{E}{e} = m \times \frac{d'}{d}; \qquad E = \frac{med'}{d}. \tag{63}$$

293. Measurement of Electro-motive Force. WHEATSTONE'S METHOD. — The standard cell e is to be joined in series with a galvanometer and any known resistance, giving a convenient deflection d. The resistance is now to be increased by an amount r ohms, and a second deflection d' is obtained. The generator to be measured is now substituted for the standard cell, and the resistance of the circuit so adjusted that the deflection d is repeated. Additional resistance, r' ohms, is now added to the circuit, until the deflection d' is again obtained. Under these circumstances, the relative electro-motive forces are directly proportional to the additional resistances required to repeat the deflections, —

$$e : E :: r : r';$$

$$E = \frac{er'}{r}. \tag{64}$$

The first resistance should be as large as convenient; and the added resistance should be about double the original in order to get the best results.

294. Lumsden's Method. — Join the standard cell e and the generator to be measured, E, with the galvanometer G and the resistance r' and r, as shown in Fig. 247.

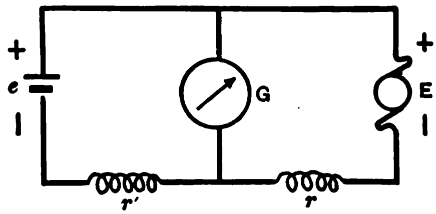


Fig. 247. Connections for Lumsden's Method.

Adjust r until no deflection is observed on the galvanometer. Under these circumstances, —

$$e : E :: r' : r;$$

$$E = \frac{er}{r'}. \qquad (65)$$

295. Measurement of Current Strength. — To measure the amount of current flowing in a given circuit, the direct reading

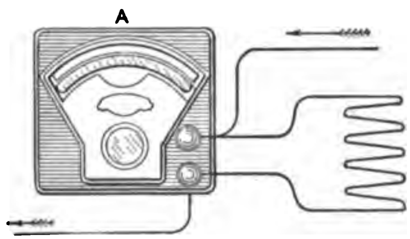


Fig. 248. Measurement of Current by Shunted Ammeter.

ammeter forms the most convenient instrument. Current strength is measured by interpolating instruments directly in the circuit, the readings on the ammeter giving the volume of current.

296. If the volume of current be too great for the instrument at hand, there are three methods for making the requisite measurements. A circuit may be arranged as shown in Fig. 248, in which the ammeter forms a shunt in connection with another circuit r. Under

these circumstances, knowing the resistance r, and the resistance of the ammeter, which is always to be found marked upon the case containing the instrument, the quantity of current flowing through the two branches of the divided circuit can be readily calculated by the formula for divided circuits, and the total current obtained by adding the respective quantities found in the branches. Suppose an ammeter having a maximum reading of "a" amperes is required to read to A amperes; let r be the resistance of the instrument, r' the resistance of the shunt to be added; then, —

$$r' = \frac{ar}{A-a}. \tag{66}$$

The scale reading must be multiplied by A/a.

As ammeters are always very low resistance instruments, great care must be taken to determine accurately the multiplying power of the shunt, and particular pains taken to see that no unknown or variable resistance is introduced in the various contacts.

297. Measurement of Current Strength by Voltmeter. — If the total resistance of the circuit or any portion of it be known, the measurement of the current strength may be made by means of a voltmeter. If, for example, the terminals of a voltmeter be connected across a circuit including a known resistance of r ohms and a reading of V volts be obtained, two quantities in the general equation of Ohm's law are given, from which the current strength may be calculated. By this method very large currents may be measured by the use of the milli-voltmeter. For this purpose arrange a circuit as shown in Fig. 235, containing a copper bar or strip, the resistance of which is known or can be approximately calculated.

The terminals of the milli-voltmeter are to be applied to two points of the strip, and the fall of potential taken by means of the milli-voltmeter between these two points.

Supposing the conductor to have a resistance of r ohms, and the reading of the milli-voltmeter to be E volts, the strength of the current I will be —

$$I = \frac{E}{r}. \tag{67}$$

The objection to this method lies largely in the difficulty in determining the resistance of the part of the circuit included between the terminals of the milli-voltmeter.

298. Measurement of Current Strength. DIFFERENTIAL GALVANOMETER METHOD. — In one half of a differential galvanometer G, Fig. 249, is placed a standard cell e, and known resistance r. Knowing the electro-motive force of the cell, and the resistance g of one half of the galvanometer, and r the resistance of the rest of the circuit, the current flowing may be calculated. The current to be measured is now passed through the other half g' of the galvanometer, and r varied to r', until the needle remains at zero. If the two coils of the differential galvanometer have equal resistance, the value of the unknown current is given by equation (68).

$$I = \frac{e}{r' + g}. \tag{68}$$

Should the two sides of the galvanometer be unequal, the preceding result must be multiplied by the ratio of the two sides. This ratio may be determined by passing the current from the standard cell simultaneously through both halves of the instrument, and varying the resistance of the respective circuits until equilibrium is produced. The desired ratio is then evidently the ratio between these resistances. If the unknown current is very large, a shunt may be placed in this circuit and the multiplying power introduced in equation (68).

299. SLIDE-WIRE METHOD. — In Fig. 250, AB is a wire of known resistance per unit of length, with a slide at B. The current to be measured is passed through this wire in a direction BA. The galvanometer standard cell and slide-wire are joined as indicated, so that the electro-motive force of the standard cell will oppose that of the current to be measured. The slide is then moved until the galvanometer remains at zero. Under these circumstances, $I = \frac{e}{r}$ (69), r being the resistance of AB, and e the electro-motive force of the standard cell.

300. Measurement of Electrostatic Capacity. — The most accurate and convenient method of measuring electrostatic capacity is to compare the unknown capacity to be estimated with that of a standard condenser. The arrangement of the circuits are given in Fig. 251.

Supposing the capacity to be measured is that of a cable, the apparatus is so arranged that either the condenser or cable, by

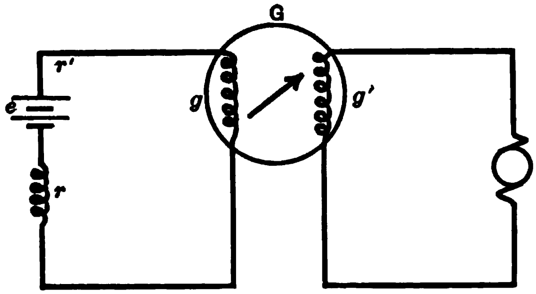


Fig. 249. Current Strength by Differential Galvanometer.

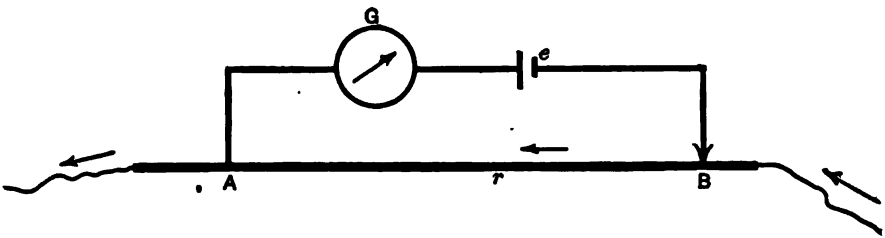


Fig. 250. Diagram of Current Strength by Slide Wire.

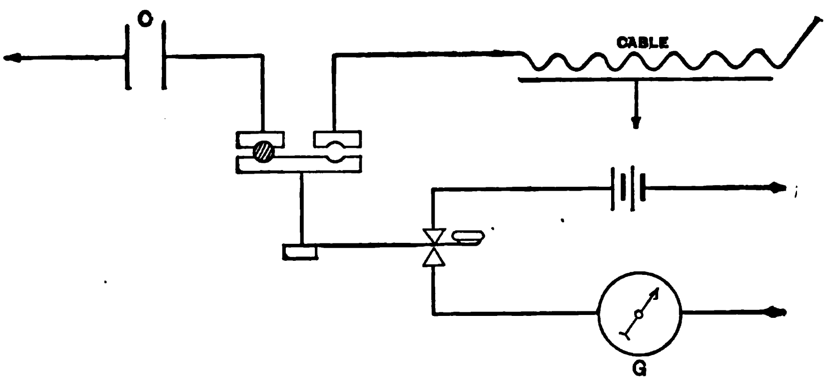


Fig. 251. Diagram of Circuit for Capacity Measurement.

means of a double key, may be charged from the same battery, and discharged through the galvanometer. Under these circumstances, the relative capacities are proportional to the deflections produced on the scale of the galvanometer. These deflections must be multiplied by the proper factor in case the galvanometer is shunted to bring the readings within the limit of the scale.

301. If a Ballistic galvanometer be employed, the scale readings can be used without correction. If an ordinary galvanometer is used, or one in which provision is made for checking the motion of the needle, a correction for the errors thus introduced must be made. This correction may be obtained by observing the first swing of the needle to the right, giving, for example, a deflection d', then the second swing also to the right, giving the deflection d''. The true deflection on the scale is obtained from the equation

$$d = d' + \frac{d' - d''}{4}. \qquad (70)$$

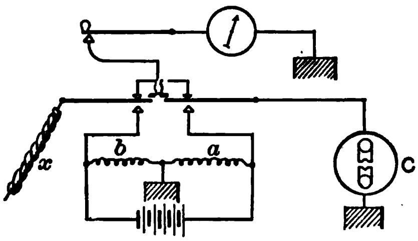


Fig. 252. Diagram of Circuit in Thomson's Method.

Should the deflections given by the discharge of the cable and that of the standard condenser be sensibly equal, no correction is needed. In cases where the cable or line to be measured is very long, or has very large capacity, it will not discharge itself instantly; and one of the succeeding methods must be employed in place of the above. It is also customary, in order to obtain uniform results, to allow the electrification by the battery, both of the condenser and that of the capacity to be measured, to proceed for a certain definite length of time, usually for one minute.

302. Thomson's Method. — The connections for Thomson's method for estimating capacity are shown in Fig. 252.

The resistances of two adjacent branches of a Wheatstone bridge are replaced by the capacity x to be measured on one side, and a standard condenser of appropriate capacity C on the other, while the remaining arms a and b are wired as in the cut. These capacities are then charged from the same battery during the same time, and are then simultaneously discharged through the galvanometer by

means of an appropriate key. When the resistances a and b are so adjusted as to produce equilibrium, and the galvanometer indicates no deflection, then

$$x = \frac{Ca}{b}. \tag{71}$$

in which x is the capacity to be measured, and a and b the known resistance of the bridge-arms.

303. GOTT'S METHOD. — The method here indicated is in some cases more convenient to apply. The capacity to be measured and the standard condenser C are mounted in series, as shown in Fig. 253, arranged so that they may be charged from the battery

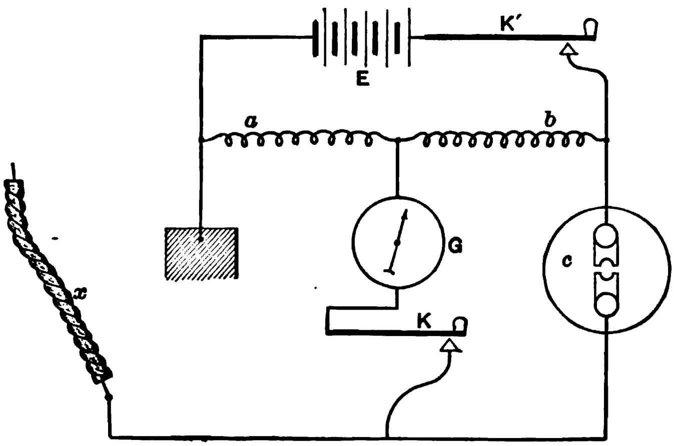


Fig. 253. Diagram of Circuit in Gott's Method.

E by means of the key K′, and discharged through the galvanometer by the key K. The resistances a and b are adjusted until no deflection is obtained on the galvanometer by closing the key K. Under these circumstances, using the preceding notation, —

$$x = \frac{Ca}{b}. \tag{72}$$

For these two methods it is advisable that the standard condenser and the capacity to be measured should have the same dielectric, as otherwise the different rate of absorption of different dielectrics may cause error.

304. DIVIDED CHARGE METHOD. — If a charged condenser be attached to a second condenser having no charge, the charge which

is in the first condenser will distribute itself between the two in proportion to their relative capacities. Thus, if a standard condenser containing a known charge be placed for a few seconds in communication with a capacity to be measured, and then if the residual charge in the standard condenser be determined, the unknown capacity can be calculated. Thus, if C' be the charge in a condenser of a capacity C which is connected to an unknown capacity C'', the quantity C''' which will remain in C will be —

$$C''' = C' \times \frac{C}{C'' + C}. \tag{73}$$

$$C'' = C \times \frac{C' - C'''}{C'''}. \tag{74}$$

305 The Localization of Faults. — Three kinds of faults are likely to occur in electrical lines: —

FIRST. — The conductor under consideration may be broken and entirely insulated from the other conductors and from earth. Under these circumstances, the resistance of the wire is infinite, or is equal to the original insulation of the circuit. If the previous capacity of the line is known, the localization of the fault may be determined by measuring the capacity of the wire; that is to say, the capacity from the testing-station to the point of rupture of the conductor. If the line is one having considerable capacity per unit of length, so that slight changes in length give rise to relatively large variations in capacity, this method of localization has a fair degree of accuracy.

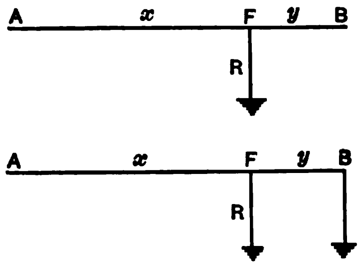


Fig. 264. Diagram of Blavier's Method.

306. SECOND. — If the faulty conductor is either crossed with a neighboring conductor, or with the earth, so that the fault has essentially no resistance, it is comparatively easy to locate its position by measuring the resistance of the conductor to the point of the cross. If the original resistance, or resistance per unit of length, be known, the localization of the fault becomes a mere matter of proportion between the measured resistance and that per unit of length.

307. THIRD. — Faults with resistance. Most frequently, however, considerable resistance is encountered at the fault itself; and in

order to locate the position of the fault, some method must be devised either to eliminate or to measure this amount. Blavier's method is shown in Fig. 254, in which A is the testing-station, B the end of the line, F the fault having a resistance R, and x and y the respective resistances of the segments into which the fault divides the line. The operation consists in measuring the resistance from A, when B is insulated, being the resistance of the part of the line x plus the resistance R of the fault, giving a quantity —

$$x + R = R'.$$

The end of B is then grounded or connected to a return conductor, and a second measurement taken, giving a quantity, —

$$x + \frac{yR}{y + R} = R'', \text{ or } x + \frac{1}{1/R + 1/y} = R''.$$

Also it is essential to know the original resistance of the line, —

$$x + y = R'''.$$

From these three equations the value of x can be calculated, and is shown to be given by the equation —

$$x = R'' - \sqrt{(R' - R'')(R''' - R'')}; \tag{75}$$

and

$$y = R''' - R'' + \sqrt{(R' - R'')(R''' - R'')}. \tag{76}$$

308. The Overlap Method. — A convenient modification of the foregoing method may be employed when the measurements can be made from each end of the faulty line. Under these circumstances, A measures when B is insulated, and B measures when A is insulated. In the latter case, when B insulates, a measurement from A gives —

$$x + R = R'.$$

When B measures, A insulating, —

$$y + R = R'', \text{ and}$$
$$x + y = R''', \text{ the original resistance.}$$

Then the value of x is found from equation —

$$x = \frac{R''' + R'' - R'}{2}; \tag{77}$$

$$y = \frac{R''' - R'' + R'}{2}. \tag{78}$$

The location of faults existing in submarine cables presents problems of peculiar difficulty, owing to the fact that the rupture of the cable

usually admits sea-water to the interior, thus allowing a saline solution to come in contact not only with the core, but with the sheath of the cable, thus forming a battery that is capable of giving quite a perceptible current in the core of the cable. Many ingenious and successful methods have been presented for the determination of faults of this kind, for full description of which the reader is referred to works particularly devoted to the subject of electrical testing.

309. Loop Test. MURRAY'S METHOD. — When both ends of the faulty conductor are accessible to the same testing-station, as, for example, a cable on reels, or if another perfect conductor can be obtained for testing-purposes, the loop-test forms one of the most accurate and convenient of methods. The connections should be

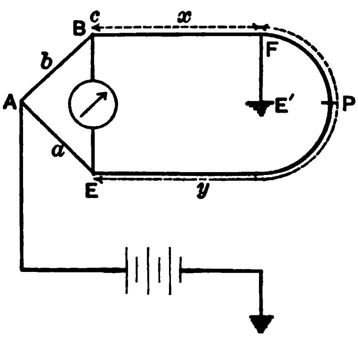


Fig. 255. Circuits for Murray's Method.

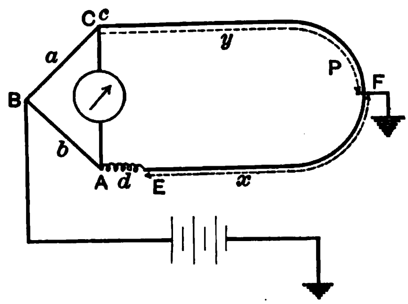


Fig. 256. Circuits for Varley's Method.

made as shown in Fig. 255, in which a and b are the arms of a bridge at the testing-station; F the location of the fault; and x and y represent the respective resistances of the segments into which F divides the conductor from c and E (the ends of the bridge-arms) to the fault. cF is the faulty conductor, and EP the perfect one looped with it. AB and AE are adjusted until equilibrium is attained, then —

$$by = ax. \tag{79}$$

Assume R to be the total resistance of the loop, then —

$$R = x + y \text{ and } y = R - x,$$

substituting this value of y in equation (79), and solving for x, —

$$x = \frac{Rb}{b + a}. \tag{80}$$

b and a should be made as high as possible to give great range of adjustability. A heavy battery should be employed, especially if the fault has high resistance. The galvanometer should have a resistance of not more than five times that of the circuits under test.

310. VARLEY METHOD. — This is a modification of the preceding loop-test, of which the connections are shown in Fig. 256. In the diagram $\overline{BC}$, $\overline{BA}$, and $\overline{AE}$ are the respective arms of the bridge, having the resistances a, b, and d, corresponding in notation to Fig. 201; a and b are the fixed resistances of the bridge, while d is the variable arm. F is the location of the fault, while x and y are respectively segments of the line extending from E and C. The resistance of $x + y = R$ is supposed to be known. The variable arm d is adjusted until the galvanometer indicates equilibrium.

$$a : b :: y : (d + x);$$

$$x = \frac{by}{a} - d, \text{ also, } y = R - x;$$

therefore, $$x = \frac{b(R - x)}{a} - d;$$

$$x = \frac{Rb - ad}{b + a}. \qquad (81)$$

If $b = a$, then, $$x = \frac{R - d}{2}. \qquad (82)$$

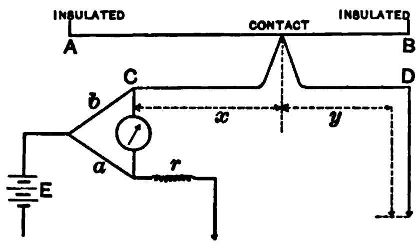


Fig. 257. Diagram for the Location of a Cross.

311. To attain the greatest accuracy, a should be as small as possible, but not less than

$$\frac{Gx}{G + x};$$

b should be so high that when d is a single unit out of balance there will be a perceptible movement of the needle.

312. Localization of Crosses. — To localize the position of a cross between two lines, the following method is sometimes convenient. Arrange connections between the lines, as shown in Fig. 257, in which AB and CD are the crossed lines. Adjust the arms of the bridge a and b and the resistance r to produce equilibrium. Then $x + y = br / a$. (83)

313. Rearrange the apparatus, making connections as indicated in Fig. 258, by placing the battery between A and the junction of

the bridge-arms, without making other changes, then $ax = by$, when $r = 0$. From these two equations

$$x = \frac{b}{a+b} \times \frac{br}{a}. \qquad (84)$$

314. Measurements of Coefficients of Inductance. — The determination of the coefficients of inductance may be easily made by means of a Wheatstone bridge, a condenser, and a variable non-inductive resistance. The apparatus should be mounted as shown in Fig. 259, in which A and B are the constant arms of the bridge, R the variable arm, S the variable non-inductive resistance, and R′L the inductance to be measured, of which R' is its ohmic resistance to a continuous current, while C is a condenser placed as a shunt across the arm of the bridge, in which S and R′L are inserted in series. The object of S is to bring the capacity required to balance the inductive resistance within reasonable limits. The balance is obtained by adjusting the mutual values of C, S, and R until no deflection is produced on the galvanometer when the battery circuit is interrupted. Under these circumstances, if A and B are equal, the value of L is found, from the expression $L = CR''^2$ (84), in which R'' is equal to the sum of S and R'.

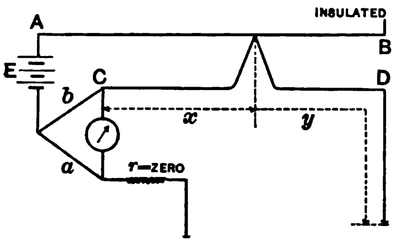


Fig. 258. Diagram for the Location of a Cross.

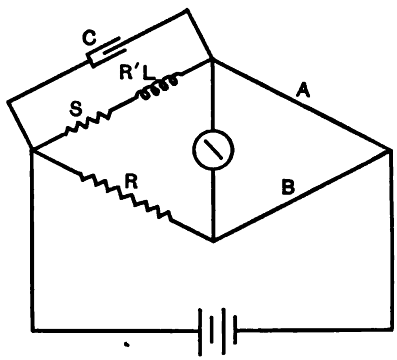


Fig. 259. Circuits for Measuring Inductance.

315. The value of this method may be extended over greater ranges by giving A and B any desired ratios, as in ordinary bridge measurements. The auxiliary resistance S is required to adjust the capacity within reasonable values to balance the inductance. If, for example,

L has a value of .4 Henrys,
R' has a value of 10 Ohms,
C must be equal to .01 of L, or 4000 *M.F.*

Such a capacity would be beyond ordinary apparatus. By increasing R' to 1000 ohms by the addition of the adjustable resistance S, C becomes equal to .4 *M.F.*, an easily obtainable capacity.

316. The Measurement of Self-Inductance with an Alternating Current of Known Period. — When an alternating current of known frequency can be obtained, the determination of the coefficients of inductance may be made; the apparatus needed being an alternating current dynamometer, a direct current ammeter, and a non-inductive resistance of known value. These instruments are all set up in series with the generator, in such a way that the current of known frequency may flow through the inductive resistance to be measured, and the known resistance. The direct current ammeter should be provided with a switch whereby it may be short-circuited at pleasure. The necessary measurements then consist in measuring the fall of potential with the alternating current dynamometer around the inductive resistance of which the inductance is desired, and also around the non-inductive resistance. A continuous current is then substituted for the alternating current; the amount of continuous current being varied until the dynamometer gives the same fall of potential across the known non-inductive resistance as was obtained in the first measurement. The amount of the continuous current is then obtained by reading the ammeter; and a measurement of the fall of potential across the inductive resistance, when supplied with a continuous current, is made with the dynamometer. The first and second dynamometer readings E and E' across the terminals of the inductive resistance give two *E.M.F.s*, the first of which is required to overcome the ohmic resistance plus the inductance, while the second is that required to overcome the ohmic resistance only. Knowing the amount of current I, in the second observation, and the frequency n, in the first, the value of L is determined from the expression —

$$L = \sqrt{\frac{E^2 - E'^2}{2\pi n I}}. \tag{85}$$

317. This method is subject to error, due to the current taken by the dynamometer, which must be of sufficiently high resistance as to be negligible in comparison with the resistance to be measured.

318. Measurement of Mutual Inductance. — The preceding method may be employed to measure the coefficient of mutual inductance M, of two coils. Let R_1 and R_2 be the respective ohmic

resistance of the coils, and L_1 and L_2 the respective coefficients of inductance. First connect the two coils in series, and measure the total inductance by the above method, obtaining a value denoted by L'. Then connect the coils in opposition, and again measure the total inductance, and denote the quantity thus obtained by L''. It can be shown that

$$L' = L_1 + L_2 + 2M;$$

also,

$$L'' = L_1 + L_2 - 2M;$$

hence,

$$M = \frac{L' - L''}{4}. \tag{86}$$

319. Measurement of Mutual Inductance. — To determine the mutual inductance of two coils, a circuit should be arranged, as indicated in Fig. 260, in which the first coil A is placed in series with

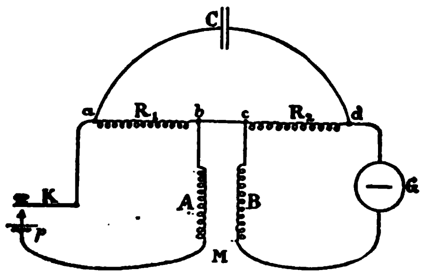


Fig. 260. Circuit for Measuring Mutual Inductance.

the key K, and the battery P, and resistance R_1, while the second coil B is placed in series with the galvanometer G and the resistance R_2. Between the points *a* and *d* a condenser C is placed as a shunt. The other extremities *b* and *c* of the resistances R_1 and R_2 are placed in series. Opening and closing the key K produces induced currents in the coil B, giving deflections on the galvanometer which are proportional to $M - CR_1 R_2$. By varying the capacity of the condenser, different deflections are obtained, which have the following relation: —

$$\frac{M - CR_1R_2}{d} = \frac{M - C'R_1R_2}{d'}. \tag{87}$$

in which C and C' are the different condenser values, and d and d' the different corresponding deflections, from which the coefficient M is obtained by equation —

$$M = CR_1R_2. \tag{88}$$

when d reduces to zero.

MEASUREMENTS ON ALTERNATING CURRENT CIRCUITS.

320. Measurements of Potential. — Measurements of potential upon alternating current circuits may be readily made by means of hot wire voltmeters, Siemens dynamometers, or electrostatic voltmeters. With the electrostatic instruments sufficient range can usually be obtained so that pressure determinations on any ordinary alternating circuits may be made directly by interpolating the voltmeter across the circuit. With the Siemens dynamometers or the Cardew voltmeters, the instruments rarely have sufficient range to permit of a direct determination; and recourse is usually had to the method of using a small step-down transformer, by means of which the voltage of the circuit is reduced in proportion to the ratio of the windings of the transformer. Under these circumstances, to obtain the actual voltage of the circuit, it is necessary to multiply the readings of the voltmeter by the ratio of transformation.

321. Measurement of Current. — The determination of current quantity may be made upon alternating circuits by means of a Siemens dynamometer, a Thomson balance, or other instruments of similar construction and based upon parallel principles. The operation consists in inserting the measuring instrument directly in the circuit, and obtaining the desired readings. In measurements of this kind, as well as those described for obtaining the pressure of the circuit, the readings of the instruments indicate what is termed "the effective current, or potential," being the square root of the mean square of the instantaneous values of current or pressure.

322. Measurement of Power. — The method to be used in determining the power transmitted by an alternating circuit depends upon whether the circuit under examination is inductive or non-inductive. In the case of non-inductive circuits, it is simply necessary to measure the virtual pressure and virtual current, as already described, taking the product of these two quantities as the amount of power transmitted. When the inductance of the circuit is considerable, the power measurements may be made with an electrodynamometer, either of the Siemens or the Kelvin type — the coarse wire coils being connected in series with the circuit, while the fine wire is placed across the mains. Under these circumstances, to secure accuracy, the following conditions are essential: —

First. The ratio of the inductance of the instrument to its resistance must be very small.

Second. The period of vibration of the movable coil must be very great compared with the period of the circuit.

Third. When an auxiliary transformer is used for reducing the voltage, the current required for the fine wire coil must be very small.

323. Power Measurement by Two Voltmeters. — Messrs. Ayrton & Sumpner are the authors of the following method for the measurement of power of an alternating current by the employment of two voltmeters and a non-inductive resistance. The circuit is arranged so that the inductive resistance of the circuit and the non-inductive resistance "r" are placed in series with each other. Then, by means of two voltmeters, the fall of potential across the inductive resistance e_I, and across the non-inductive resistance e_R, is measured.

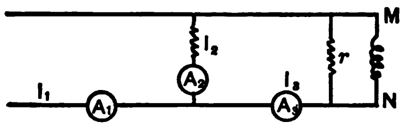


Fig. 261. Three-Ammeter Method.

The total fall across both resistances is also measured and denoted by e.

The power transmitted by the circuit, denoted by W watts is then, —

$$W = \frac{e^2 - e_I^2 - e_R^2}{2r}. \tag{89}$$

324. Method employing Three Ammeters. — J. A. Fleming is authority for measuring the power in an alternating circuit by the employment of three ammeters, as arranged in the accompanying illustration, Fig. 261. The inductive resistance is supposed to be placed at MN, and the known non-inductive resistance r, while the ammeters are shown at A_1, A_2, A_3. The reading of the ammeters gives three currents, from which the power in watts, represented by W, is obtained from the formula, —

$$W = \frac{r}{2}(I_1^2 - I_2^2 - I_3^2). \tag{90}$$

MEASUREMENTS ON POLYPHASE CURRENT CIRCUITS.

325. Diphase Circuits. — CASE 1. — To determine the power transmitted by diphase circuits, two conditions must be considered.

First. — Circuits containing four wires. Under these circumstances, each circuit may be measured separately and entirely independent of the other circuits, and the results considered either alone or in conjunction with the results obtained from the second circuit.

Second. — Three wires with a common return.

To determine the power delivered by such a circuit, two wattmeters are necessary, and should be placed with the coarse wires in series with the separate parts of the component circuits, while the fine wire is placed across the common return and each of the exterior wires.

326. Triphase Currents. — Measurements upon triphase circuits for current and potential may be made in the same manner as described for ordinary alternating circuits. To determine the power delivered by a triphase circuit, three cases must be considered.

327. CASE 1. — Where the circuits supply non-inductive resistance without current lag. Under these circumstances, the power is equal to $\sqrt{3}$ times the product of the current intensity in each circuit, multiplied by the effective difference of potential between the wires. This method holds good indifferently, whether the arrangement of circuit is the star or the triangle method.

328. CASE 2. — Case of equal lag and equal current. One wattmeter is arranged with its coarse wire in series on one of the circuits; and two readings are made with the fine wire successively, between the circuit under measurement and each of the other branches. The sum of the results thus obtained is the total power transmitted.

329. CASE 3. — The general method for any current and any lag. Two wattmeters are employed, arranged with the coarse wires inserted in two of the three circuits and the fine wires placed respectively between the third circuit and the other two. The sum of the readings thus obtained gives the total power transmitted by the circuit.

330. Eectrical Railway Testing. — By means of the foregoing methods the electrical engineer will be able to make such selection

as to enable him to thoroughly investigate the electrical properties of any ordinary line construction. No data or methods are given, either for the examination of dynamo machinery or for the determination of special factors, being beyond the scope of this volume. There remains, however, the special case of electric railway testing, which, having chiefly for its object the determination of the electrical properties of the conducting system, necessarily embraces within the measurements made for this purpose a large amount of data applicable to ascertaining the performance of the car-motors and the generating-station. A necessary adjunct to the examination of an electric railway plant is a reasonably accurate plan and profile of the entire line. If not already in existence, a transit survey may be rapidly made with sufficient accuracy, covering from ten to twenty miles per day's work. The tangents may be run out with great rapidity by stadia measurements, the location and amount of all gradients being simultaneously determined by means of a grade-screw on the vertical circle of the transit. The curves may be rapidly located by chord deflections. A testing-car should now be provided, which should be equipped with the following instruments: an integrating wattmeter, a Weston voltmeter, ammeter, and milli-voltmeter, a Boyer speed recorder, a revolution counter, a stop-watch reading to quarter seconds, and a gong. A separate observer should be provided for each instrument, with appropriate note-books having numbered lines, so that all observations may be correlated by corresponding numbers. As far as possible recording instruments should be used.

The instruments may all be appropriately arranged on the car-seats, being protected as much as possible against jarring by extra cushions and rubber springs. The voltmeter and ammeter are introduced in the motor circuit, so as to measure the amount of current and pressure. The wattmeter is similarly introduced, in order to integrate the total energy expended. The general connections of these instruments are indicated in Fig. 262.

The speed counter is to be connected to the driven axle of the car, provided only one motor is used; or if the car is a double motor equipment, one may be temporarily thrown out of service. The object of the counter is to determine the number of revolutions of the car-wheel, that, being multiplied by the wheel circumference, will give accurately the distance traveled by the car. Indeed, so

accurate is this method of measuring that repeated trials over a six-mile stretch of road have checked within an error of fifteen feet. It is obvious that, to prevent error, the counter must be attached to a *driven,* not a *driving* axle. The Boyer speed-recorder may be attached to the same axle, and, being a self-recording instrument, may be placed in charge of the same observer who records the counter. The instruments being in readiness, the car is arranged to start from one end of the line, one of the observers being detailed to strike the gong at the instant each line-pole passes the center of

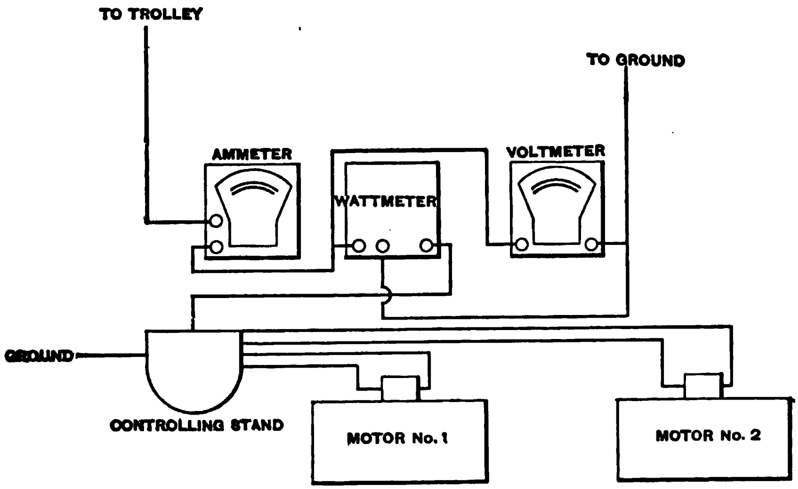


Fig. 262. Diagram of Circuits in Testing-Car.

the car. At each gong-stroke each observer records the reading of his particular instrument.

331. The records will then show readings corresponding to successive points along the line, as marked by each pole, consisting —

1st. . . .	Time in seconds	4th. . . .	Wattmeter
2d. . . .	Voltmeter	5th. . . .	Revolution-counter
3d. . . .	Ammeter	6th. . . .	Pole number

On the conclusion of the run, the information from each of these records should be plotted as a curve upon the sheet of profile-paper containing the plan and profile of the road, as developed from the previously mentioned surveys.

332. Contemporaneously with the trip of the inspection-car, station voltmeter readings should be obtained, either by a self-recording instrument, or by five-second interval observations. These should likewise be plotted as a curve on the profile-sheet. The line should now be short-circuited at the extreme end, through sufficient resistance not to overload the generator, but yet to permit a heavy current to pass through all the wiring, and the inspection-car again sent over the road, hauled by horses, so that the car will take no current. During this trip voltmeter readings should be made at each pole, together with a repetition of the station voltmeter observations. These readings should likewise be plotted on the profile-sheet. From this test, the behavior of the line under a steady load may be

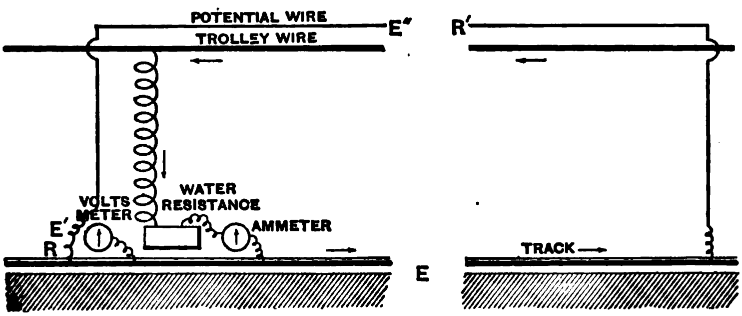


Fig. 263. Diagram of Test-Circuit for Electric Railway.

contrasted with previous curves of variable load. During this trip the milli-voltmeter should be connected with the fore and aft wheels of the car. Then the readings of the instrument will indicate the fall of potential in the rails in each car-length, affording a ready means of detecting any discontinuity in the return current, such as poor bonding, etc.

333. The examination may now be completed by measuring from the station the insulation and conductivity, jointly and separately, of the ground return feeder system and trolley wire. This is best accomplished by stringing a test-wire of about No. 14 or 16 gauge parallel with all the lines, and arranging the stations and testing-instruments as in Fig. 263. By this means, the line resistance, as well as the ground resistance, can be separately determined. A careful consideration and comparison of the curves to be developed from

this information will, from a maintenance standpoint, be richly rewarded; for in this way only is it practicable to so thoroughly and carefully adjust the conducting system of a railway line to the loading thrown upon it, as to secure a proper distribution of energy with reference to the demands introduced by grades, curves, variation in moving load, and the demands caused by the stopping and starting of the cars, in order that the line and station shall work together harmoniously in the endeavor to attain a maximum efficiency.

334. The Capacity of Aerial Lines. — Though the preceding methods are applicable to the determination of electrical quantities under all circumstances, when applied to the measurement of transmission lines, special precautions have sometimes to be taken. The capacity of an aerial line is a difficult quantity to measure, for the reason that lines of this kind are usually not highly insulated, and for this reason will discharge themselves in an extremely short period of time. It is possible, however, to obtain quite accurate results for aerial line capacity by arranging the circuits as shown in Fig. 264, in which $\overline{AB}$ is a lever pivoted at C, that by means of spring *r* is kept constantly in contact with the terminal *a* of the battery key M.

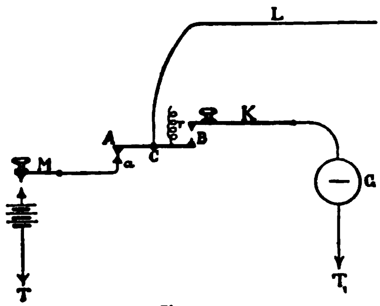


Fig. 264.
Connection for Measuring Capacity in Aerial Lines.

The line L is brought to the center of this lever at C. A second key K is mounted in series with the galvanometer, the depression of which makes contact with the lever $\overline{AB}$ at B. It is apparent that the action of this key is to open the battery circuit and close the galvanometer circuit, approximately, at the same instant. The line is charged for one minute by closing the battery key M. Then, by depressing the key K, the battery circuit is opened and the line circuit closed through the galvanometer at the same instant. The readings of the galvanometer, in comparison with those of a standard condenser, by any of the methods already given, furnish the necessary data for calculating the line capacity. This galvanometer reading, however, must be corrected for two errors.

335. FIRST. — It is usually found that different readings are ob-

tained when the line is charged with a positive current than when it is charged with a negative current. This difference is owing to the presence of the earth currents, which always manifest themselves upon aerial lines of any magnitude. Two deflections, therefore, obtained with currents of different sign, will differ by an amount due to the presence of such a foreign current. The deflections, therefore, must be corrected by subtracting or adding to the galvanometer reading obtained by the battery discharge the amount of deflection due to the earth current. This correction may be readily obtained by closing the galvanometer key with the battery key open for a few moments, and reading the deflection given by the earth current.

336. SECOND. — The deflection obtained upon the galvanometer is not exact, unless the opening of the battery circuit and the closing of the galvanometer circuit occur at mathematically the same instant; and the apparatus can rarely, if ever, be adjusted to accurately accomplish this. Therefore, usually the battery is short-circuited through the galvanometer for a very short interval of time. To determine the value of the error thus introduced, substitute for the line L three standard condensers, the capacities of which are known quite accurately to be in the ratio of 1½, 2, and 4, and by closing the key K, measure the galvanometer deflections obtained with these condensers in the place of the line, exactly in the same way as the line measurement is made. If the source of error alluded to does *not* exist, the following relation would be true: —

$$\frac{d'}{1.5} = \frac{d''}{2} = \frac{d'''}{4},$$

in which d', d'', d''', are the respective deflections. If equality does not exist in the above equation, the following relation evidently will hold, —

$$\frac{d' + x}{1.5} = \frac{d'' + x}{2} = \frac{d''' + x}{4},$$

in which x is such a quantity as will satisfy the equation. From the known value of the standard condensers with which these readings are made, it is possible to calculate the value of x, and thus determine the error introduced by the momentary short-circuiting of the battery through the galvanometer. Having obtained this figure with standard condensers, it may be applied to correction of the galvanometer reading, as obtained from the experiments upon the line.

337. An example may perhaps render the subject more clear. Suppose an aerial line, when tested with a positive current, to give a deflection of 73 divisions on the galvanometer scale, and with a negative current, of 113 divisions, also, that the deflection due to earth current is found to be 20 divisions. The true deflection on the galvanometer evidently then should be $d = 73 + 20 = 113 - 20 = 93$, the earth current evidently opposing the positive current. To introduce the second correction, assume three condensers, having the ratios of $1\frac{1}{2}$, 2, and 4, to give on the galvanometer scale deflections of 72, 88, 152. In order that the three numbers representing the deflections shall stand in the same ratio as the capacity of the condensers, it is necessary to subtract from each one 24. Correcting the line galvanometer deflection by the same number, the value of 69 remains as the true deflection.

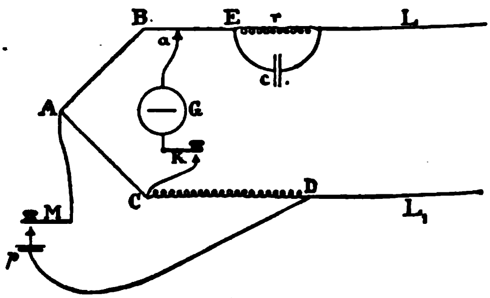


Fig. 265.
Circuit for the Measurement of Inductance on Aerial Lines.

338. The capacity of aerial lines in reference to the earth, as usually measured, is considerably greater than that which would be theoretically indicated. To account for the discrepancy between the measured figures and those given by theory, it is usually assumed that the insulators, poles, and cross-arms possess a sensible capacity which is inevitably measured in all trials made upon aerial lines. Confirmation of this hypothesis is obtained in the fact that in dry weather the line deflection falls, and agrees much more closely with the results indicated by theory.

339. The Inductance of Aerial Lines. — The estimation of the inductance of aerial lines may be made by any of the methods given; those that employ the Wheatstone bridge, as indicated in Fig. 265, being particularly convenient. The two parts of the line L and L′ are looped into the station, L being connected to $\overline{AB}$, one of the bridge-arms, and C connected to $\overline{AC}$, the other. In the line L the variable resistance *r*, and variable condenser *c* are arranged, while the contact *a* represents the slide of the bridge, as, for this experi-

ment, a slide-wire bridge is a convenient piece of apparatus. After adjusting a to obtain a balance for constant current, the capacity of the condenser c is increased or diminished, until the needle of the galvanometer remains at zero on interruption of the current. The inductance of the line is then given by the expression, $L = c\, r^2$. (91)

340. It must not be forgotten that the line itself has always a capacity; so from the above expression the true inductance of the line is not obtained, but a quantity equal to $L - \frac{1}{3} CR^2$, in which R is the resistance and C the capacity of the line itself. To demonstrate this, suppose, in Fig. 266, the two parts of the line to be

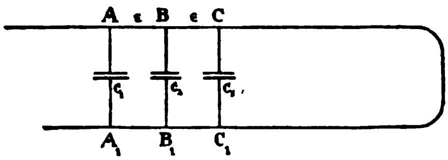


Fig. 266. Diagram of Line Capacity.

represented by $\overline{AC}$ and $\overline{A_1C_1}$, and assume the line to be divided into n equal parts, $\overline{AB}$, $\overline{BC}$, etc., and $\overline{A_1B_1}$, $\overline{B_1C_1}$, etc. At each section of the line imagine a small condenser to be placed, whose capacity c_1 c_2 etc., is the capacity of the section under consideration. Represent the resistance of each section by ρ, and the capacity of the condenser at each point by ϕ. The condenser c_1 placed across the points $\overline{AA_1}$ acts as an inductance of the value $-\phi\, n^2 \rho^2$; the next condenser c_2 at $\overline{BB_1}$ acts as an inductance of the value $-\phi\,(n-1)^2 \rho^2$, and so on for all the n sections into which the line is divided. All of the condensers are equivalent to an inductance of the value $-\phi\, \rho^2\, (1+2^2+3^2+\ldots\ldots+(n-1)^2+n^2)$, but the sum of the squares of the numbers from 1 to n is

$$\frac{n\,(n+1)\,(2\,n+1)}{6},$$

and the value of the inductance equivalent to the condensers is

$$-\frac{n\,(n+1)\,(2\,n+1)}{6}\,\phi\rho^2.$$

As $\qquad \phi = \frac{C}{n}, \qquad$ and $\qquad \rho = \frac{R}{n},$

the preceding expression becomes

$$-\frac{n\,(n+1)\,(2\,n+1)}{6\,n^3}\,CR^2.$$

When $n = \infty$, $$\frac{n(n+1)(2n+1)}{6n^3} = \tfrac{1}{3},$$

and consequently the capacity of the line acts as an inductance of the value $-\frac{1}{3}CR^2$. Consequently the true value of the inductance of the line is obtained by adding to the value of L, as given in equation (91), the value of the negative inductance due to capacity, or

$$L = cr^2 + \tfrac{1}{3}CR^2. \qquad (92)$$

C and R being measured by any desired method.

341. Measurement of Mutual Inductance on Transmission Lines. — To estimate mutual inductance on a pair of lines, the apparatus should be arranged as shown in Fig. 267, in which L is the *primary* or inducing line, and L′ the circuit in which inductance is to

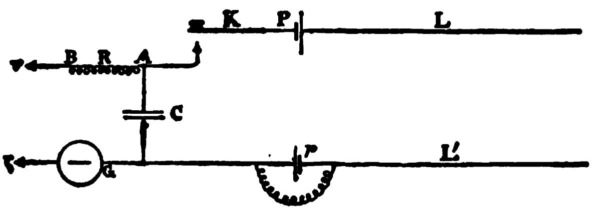


Fig. 267. Circuit for Measuring Mutual Inductance on Aerial Lines.

be measured. By means of the key K the primary line L is earthed through the resistance R; an electrical impulse is sent through this line from the battery P that produces inductive effects on the other line L′. The grounds rr_1, as well as the earths at the remote ends of the lines, must be entirely separate from each other. The deflection produced on the galvanometer by the mutual inductance of the lines, and the charge of the condenser C, is proportional to $M - CRR'$, C being the capacity of the condenser, R the resistance of the rheostat, and R' that of the line L′. By adjusting the rheostat and condenser till no deflection is observed,

$$M = CRR'. \qquad (93)$$

As earth currents are likely to give much trouble in obtaining the final balance, a small battery p, with an adjustable shunt, may be placed in L′, and arranged to neutralize such disturbances.

CHAPTER XI.

CONTINUOUS CURRENT CONDUCTORS.

THE CAPACITY OF CONDUCTORS.

Art. 342. Conductors. — When a quantity of positive electricity is placed upon any perfectly insulated body, it occupies for the first infinitesimal period of time, a small surface immediately surrounding the point of contact, and raises the potential of this surface. Very rapidly, however, the charge distributes itself over the entire surface; bringing every point thereof to the same potential. As, by hypothesis, the body is perfectly insulated, this distribution of the charge can only take place by a passage of *Electric Energy*. The property thus possessed by all substances to permit with varying degrees of rapidity the transfer of electrical energy is called conductivity.

343. In some materials the distribution of the charge takes place almost instantaneously, while for others an extremely long time is required. Good conductors are those which permit the distribution of the charge to take place with great rapidity, while those requiring a greater length of time are called poor conductors, or insulators. If the substance under consideration is in the form of a wire, one end of which is maintained at a higher potential than the other, a continual passage of electrical energy will take place from the end having the higher potential to that which is lower. This condition once established, the quantity of electricity stored on the surface of the wire remains uniformly distributed, and evidently a steady flow or current takes place.

344. From experiment it is ascertained that wires of different material, of the same geometrical dimensions, submitted to the same differences of potential, transmit very different quantities of electricity during the same interval of time. The quantity, therefore, of electricity which one substance, under precisely similar conditions, is able to transmit, compared with that of another, is a measure of its conducting power.

345. Resistance: Ohm's Law. — Let E be the difference of potential maintained between the extremities of a conductor, and I the intensity of the current; that is to say, the quantity of electricity that passes any given cross-section in successive equal intervals of time; if R is the resistance of the conductor, then —

$$I = \frac{E}{R}, \qquad R = \frac{E}{I}, \qquad RI = E. \tag{94}$$

With a given difference of potential E, I decreases directly in proportion as R increases, and also with a definite resistance R, I is directly proportioned to E. This is the famous law of Dr. Ohm, that thus unites by an algebraic equation the three most important electric quantities. Be it noted, however, that Ohm's formula, in this form, applies to *steady and continuous currents only.*

346. The resistance R of a conductor depends not only upon the material used, but also upon its geometrical dimensions. It is, therefore, possible to express the resistance of any conductor as a function of its geometrical magnitudes, and of a coefficient depending upon the physical constitution of the material employed. Dr. Ohm, further, established the proposition that the resistance of any conductor is inversely proportioned to the cross section S, measured normal to the direction of the current, and directly proportioned to the length l of the conductor, and to a coefficient ρ, which he denominated the *specific resistance* of the material. Thus, algebraically,

$$R = \frac{\rho l}{S}. \tag{95}$$

The geometrical dimensions being easily ascertained, it is sufficient for the purposes of calculation to know ρ for the materials under consideration. Substituting in (94), —

$$\frac{\rho l}{S} = \frac{E}{I}. \tag{96}$$

If ρ, l, S, and E are constant, S / I gives the current density per unit of area of the cross-section of the conductor.

347. Specific Resistance. — The resistance offered by a unit volume of any substance, as compared with the resistance of a unit volume of any other substance, selected as a standard, is termed "Specific Resistance." As the metal silver has the least resistance of any now known, or in other words is the best conductor, it is

usually selected as the standard. In English measures the cubic inch is the volume adopted for comparison, while in the C. G. S. system the cubic centimeter is used. Thus the absolute resistances of various substances would be the opposition offered per cubic inch or cubic centimeter, while the specific resistance is the ratio of the absolute resistance to the absolute resistance of silver. The value of the absolute and specific resistances of the chief metals will be found in TABLE No. 45. For wire work the resistance of a grain

TABLE No. 45.

Chemically pure Metals arranged in Order of Increasing Resistance for the Same Length and Sectional Area.

NAME OF METAL.	RESISTANCE IN MICROHMS AT 0° CENTIGRADE.		RELATIVE RESISTANCE.
	Cubic Centimeter.	Cubic Inch.	
Silver, annealed	1.504	0.5921	1.
Copper, annealed	1.598	0.6292	1.063
Silver, hard-drawn	1.634	0.6433	1.086
Copper, hard-drawn	1.634	0.6433	1.086
Gold, annealed	2.058	0.8102	1.369
Gold, hard-drawn	2.094	0.8247	1.393
Aluminum, annealed	2.912	1.147	1.935
Zinc, pressed	5.626	2.215	3.741
Platinum, annealed	9.057	3.565	6.022
Iron, annealed	9.716	3.825	6.460
Gold-silver alloy (2 oz. gold, 1 oz. silver), hard or annealed	10.87	4.281	7.228
Nickel, annealed	12.47	4.907	8.285
Tin, pressed	13.21	5.202	8.784
Lead, pressed	19.63	7.728	13.05
German silver, hard or annealed	20.93	8.240	13.92
Platinum-silver alloy (1 oz. platinum, 2 oz. silver), hard or annealed	24.39	9.603	16.21
Antimony, pressed	35.50	13.98	23.60
Mercury	94.32	37.15	62.73
Bismuth, pressed	131.2	51.65	87.23

foot, or the resistance of a wire weighing one grain, and one foot in length, and also the resistance of a mil foot, or the resistance of a wire one mil ($\frac{1}{1000}$ of an inch) in diameter, and one foot long, are convenient working quantities. TABLE No. 46 supplies this data for the most common metals and alloys, giving also the values of the gramme-meter, and millimeter-meter.

348. It should be carefully noted that different specimens of apparently chemically pure metal give different resistance, that can only be accounted for on the supposition that the varying processes

TABLE No. 46.

Resistances of Metals for Grain-foot, Mil-foot, Gramme-meter, and Millimeter-meter. (Legal Ohms.)

Name of metals arranged in order of increasing resistance for the same length and weight.	Resistance of a wire 1 foot long, weighing 1 grain.	Resistance of a wire 1 foot long, $\frac{1}{1000}$ of an inch in diameter.	Resistance of a wire 1 meter long, weighing 1 gramme.	Resistance of a wire 1 meter long, 1 millimeter in diameter.
	Ohms 0° C.	Ohms 0° C.	Ohms 0° C.	Ohms 0° C.
Aluminum, annealed	0.1074	17.53	0.0749	0.03710
Copper, annealed	0.2041	9.612	0.1420	0.02034
Copper, hard-drawn	0.2083	9.83	0.1453	0.02081
Silver, annealed	0.2190	9.048	0.1527	0.01916
Silver, hard-drawn	0.2389	9.826	0.1662	0.02080
Zinc, pressed	0.5766	33.85	0.4023	0.07163
Gold, annealed	0.5785	12.38	0.4035	0.02620
Gold, hard-drawn	0.5884	12.60	0.4104	0.02668
Iron, annealed	1.085	58.45	0.7570	0.1237
Tin, pressed	1.380	79.47	0.9632	0.1682
Gold-silver alloy (2 oz. gold, 1 oz. silver), hard or annealed	2.364	65.37	1.650	0.1384
German silver, hard or annealed	2.622	125.91	1.830	0.2666
Platinum, annealed	2.779	54.49	1.938	0.1153
Lead, pressed	3.200	. . . .	2.232	0.2498
Antimony, pressed	3.418	213.6	2.384	0.4521
Platinum-silver (1 oz. platinum, 2 oz. silver), hard or annealed	4.197	146.70	2.924	0.3106
Bismuth, pressed	18.44	789.3	12.88	1.670
Mercury	18.51	572.3	12.91	1.211

The following specific data relative to the resistances of copper in various forms has received the sanction of the report of a committee upon wiring, appointed by the American Society of Electrical Engineers.

Table of Values based upon Matthiessen's Correct Standard.

	B. A. UNITS. 0° C.	LEGAL OHMS. 0° C.
Matthiessen's Standard Meter-gramme, hard	.1469	.1453
Meter-gramme, soft	.1436	.1420
Meter-millimeter, hard	.02104	.02080
Meter-millimeter, soft	.02057	.02034
Cubic-centimeter, hard	.000001652	.000001634
Cubic-centimeter, soft	.000001616	.000001598
Mil-foot, hard	9.940	9.829
Mil-foot, soft	9.720	9.612

Specific resistance of hard copper (1 cub. cent.) = 1634 (C. G. S. units).
Specific resistance of soft copper (1 cub. cent.) = 1598 (C. G. S. units).
Matthiessen's Standard specific gravity of hard copper, 8.89.
Resistance of hard copper is 1.0226 times that of soft copper.
Resistance of soft copper is .9779 times that of hard copper.
Legal ohm is equal to 1.0112 B. A. units.
B. A. unit is equal to .9889 Legal ohms.

of manufacture produce corresponding inequality in molecular structure, sufficient to account for these discrepancies. Even with pure specimens, in special cases, a variation of 16 per cent in the same

metal tested at the same temperature has been noted. In ordinary commercial products the range of variation may naturally be still greater.

349. Effect of Temperature. — Specific resistances, also, are functions of temperature. For pure metals, the resistance increases as the temperature is augmented. The formula representing the effect of temperature may be written —

$$R_t = R_o (1 + \alpha t + \beta t^2), \tag{97}$$

in which R_t is the final temperature, R_o the temperature at which the specific resistance is originally measured, and α and β are coefficients denoting the function of the specific resistance and temperature. For any purpose but the most exact calculation, the approximation, —

$$R_t = R_o (1 + \alpha t), \tag{98}$$

is amply sufficient. The values of these coefficients are shown in Tables Nos. 47 and 48. For copper, formula (97) becomes (approximately) —

$$R_t = R_o (1 + .00387\, t \ + \ .00000597\, t^2). \tag{99}$$

Table No. 47.

Value of α and β in Formula $R_t = R_o (1 + \alpha t + \beta t^2)$.

Description of Metals.	α	β
Pure Metals	+ 0.003824	+ 0.00000126
Mercury	+ 0.0007485	− 0.000000398
Platinum-silver	+ 0.00031	− 0.000000398
German silver	+ 0.0004433	+ 0.000000152

Table No. 48.

Value of α in Formula $R_t = R_o (1 + \alpha t)$.

Description of Metals.	α	Description of Metals.	α
Silver	0.377×10^{-2}	Antimony	0.389×10^{-2}
Copper	0.388×10^{-2}	Bismuth	0.354×10^{-2}
Gold	0.365×10^{-2}	Mercury	0.088×10^{-2}
Aluminum . . .	0.390×10^{-2}	Alloy 2 Pt. + 1 Ag.	0.022 to 0.031×10^{-2}
Platinum	0.247×10^{-2}	2 Au. + 1 Ag.	0.065×10^{-2}
Iron	0.453×10^{-2}	8 Pt. + 1 Ir.	0.133×10^{-2}
Tin	0.365×10^{-2}	German silver . .	0.028 to 0.044×10^{-2}
Lead	0.387×10^{-2}		

350. It is often desirable to know the temperature variation of copper over comparatively wide ranges. TABLE 49 shows the resistance of copper per mil-foot as calculated by Dr. F. A. C. Perrine.

TABLE No. 49.

Resistance of 1 Mil-foot of Copper Wire at Different Temperatures Fahrenheit.

Temp. in Deg.	B. A. Units.	Legal Ohms.	Temp. in Deg.	B. A. Units.	Legal Ohms.	Temp. in Deg.	B. A. Units.	Legal Ohms.
0	9.06773	8.96707	34	9.75985	9.65152	68	10.49337	10.37689
1	9.08749	8.98662	35	9.78084	9.67227	69	10.51557	10.39884
2	9.10729	9.00620	36	9.80186	9.69306	70	10.53780	10.42083
3	9.12713	9.02582	37	9.82291	9.71388	71	10.56008	10.44286
4	9.14701	9.04547	38	9.84400	9.73473	72	10.58238	10.46492
5	9.16691	9.06516	39	9.86513	9.75563	73	10.60473	10.48702
6	9.18686	9.08489	40	9.88629	9.77655	74	10.62711	10.50915
7	9.20684	9.10464	41	9.90749	9.79752	75	10.64952	10.53131
8	9.22686	9.12444	42	9.92872	9.81851	76	10.67197	10.55351
9	9.24691	9.14427	43	9.94999	9.83955	77	10.69446	10.57575
10	9.26700	9.16413	44	9.97130	9.86062	78	10.71698	10.59802
11	9.28712	9.18403	45	9.99264	9.88172	79	10.73954	10.62033
12	9.30728	9.20397	46	10.01402	9.90286	80	10.76214	10.64268
13	9.32748	9.22394	47	10.03543	9.92403	81	10.78477	10.66505
14	9.34771	9.24395	48	10.05688	9.94525	82	10.80743	10.68747
15	9.36798	9.26399	49	10.07836	9.96649	83	10.83013	10.70992
16	9.38828	9.28407	50	10.09988	9.98777	84	10.85287	10.73240
17	9.40862	9.30418	51	10.12144	10.00909	85	10.87564	10.75492
18	9.42899	9.32433	52	10.14303	10.03044	86	10.89845	10.77748
19	9.44940	9.34451	53	10.16466	10.05183	87	10.92130	10.80007
20	9.46985	9.36473	54	10.18632	10.07325	88	10.94418	10.82270
21	9.49033	9.38499	55	10.20802	10.09471	89	10.96709	10.84536
22	9.51085	9.40528	56	10.22975	10.11620	90	10.99005	10.86806
23	9.53140	9.42560	57	10.25153	10.13773	91	11.01304	10.89079
24	9.55199	9.44596	58	10.27333	10.15930	92	11.03606	10.91356
25	9.57262	9.46636	59	10.29517	10.18090	93	11.05912	10.93636
26	9.59328	9.48679	60	10.31705	10.20253	94	11.08221	10.95920
27	9.61397	9.50726	61	10.33897	10.22420	95	11.10535	10.98208
28	9.63471	9.52776	62	10.36092	10.24591	96	11.12851	11.00499
29	9.65548	9.54830	63	10.38290	10.26765	97	11.15172	11.02793
30	9.67628	9.56887	64	10.40492	10.28943	98	11.17495	11.05091
31	9.69712	9.58948	65	10.42698	10.31124	99	11.19823	11.07393
32	9.71800	9.61013	66	10.44907	10.33309	100	11.22154	11.09698
33	9.73891	9.63080	67	10.47120	10.35497			

TABLE No. 49 affords convenient data to calculate the rise in temperature of dynamo machinery. Approximately 10 per cent change in resistance is accompanied by 45° rise in temperature, so by measuring the resistance before a trial run, and immediately after, the increase in temperature can be closely calculated.

351. Resistance of Dielectrics. — Experiment shows that there is a large class of bodies which permit of the transmission of electrical energy so slowly that they may be termed *non*-conductors, insulators, or dielectrics. Generally speaking, the metals and solutions of the metallic salts may be classed as conductors, while all other substances fall in the category of insulators. There is, however, much variation in the relative value of non-conductors as insulators. In electrical construction the property of high resistance is employed entirely to isolate, or insulate, electrical currents in such a manner as to confine the transfer of energy along the paths which it is desired to have it take. Insulators may be applied to conduct the circuits in one of two ways.

352. FIRST. — The insulating substance may be arranged as a series of supports to which the circuit is attached from point to point, in order to separate it entirely from electrical communication with other bodies. For this purpose dielectrics, such as wood, glass, porcelain, india-rubber, and their various compounds, are molded into appropriate forms, mechanically arranged to permit of the attachment of the conductor circuit to the insulator, and then the attachment of the insulator to the support designed to carry the circuit, in such a manner as to electrically isolate the circuit by means of the insulators from the supports. The various forms of insulators for this purpose have been already treated in Chapter III.

353. SECOND. — The insulating substance may be arranged as a uniform and continuous coating, so applied as to surround and envelop the circuit from end to end, thus rendering additional support unnecessary, and allowing the circuit to be placed in proximity to the ground or other bodies, at the same time preserving the electrical isolation. For this purpose the various forms of india-rubber are the basis of nearly all insulating materials. It is usual to secure sufficient mechanical strength by covering the conductor with one or more layers of fibrous material, such as braid composed of hemp, cotton, or silk, or by wrapping it with sheets of paper or jute, or similar material. These fibrous coverings may be impregnated with insulating compound, either previous or subsequent to their application to the conductor. As an example, the cables manufactured by Siemens Bros. are covered with jute impregnated with ozokerite. The Farranti mains are separated by a number of layers of paper

impregnated with a compound of black wax. The Edison conductors are embedded in their tubes in a special mixture of india-rubber and resins. The various forms of okonite, ozokerite, and india-rubber covered wires all depend upon protection consisting of various india-rubber compounds, each applied in a manner peculiar to the particular manufacturer.

354. The various forms of india-rubber, under the names of caoutchouc and gutta-percha, are most extensively used for cable insulation, although the melting-point of the latter is so low as to prevent its wide adoption.

Gutta-percha is a material of varying composition, depending upon its mode of manufacture; and, consequently, having a specific resistance varying between 25×10^{12} and 500×10^{12} ohms-centimeter. By fairly good methods of manufacture and the employment of pure materials, a resistance of 200×10^{12} ohms-centimeter, at a temperature of 24° C., may be obtained. Caoutchouc has also a varying composition and resistance. It is valuable, however, in its ability to resist heat. By submitting the substance to the process of vulcanization at 130° C., a temperature much higher than should ever be attained by the passage of the current, a valuable and durable insulator is obtained, having a resistance of 7500×10^{12} ohms-centimeter.

355. The variation in specific resistance of dielectrics under changes in temperature is very much more rapid and much larger in amount than those of metals. This variation can only be expressed by an exponential equation, $R_o = R_t \mathbf{a}^t (100)$, $\mathbf{a}$ being a coefficient that, owing to the process of manufacture, has to be determined separately for each specimen of insulating compound. Experiments upon gutta-percha used in submarine cable work assign a value to it between the limits of 0.876 and 0.894. For caoutchouc, the value is less carefully established, but probably lies between $\mathbf{a} = 0.941$ and $\mathbf{a} = 0.955$. For a variation of between 12° and 15° C., on either side of a temperature of 24° C., the specific resistance is approximately halved or doubled. For other insulating materials, the processes of manufacture vary too widely to permit the establishment of temperature coefficients. TABLE No. 50 gives the specific resistance of some of the more common insulators.

356. Line Leakage. — Where transmission lines are supported

Table No. 50.

Specific Resistance of Insulators.

Name.	Resistance in Megohms per Cubic Centimeter.	Name.	Resistance in Megohms per Cubic Centimeter.
Mica	84×10^{-6}	Olive oil	1×10^{-6}
Gutta-Percha	450×10^{-6}	Lard oil	$.35 \times 10^{-6}$
Shellac	9000×10^{-6}	Stearic acid	350×10^{-6}
Ebonite	28000×10^{-6}	Benzine	14×10^{-6}
Hooper's compound	15000×10^{-6}	Wood tar	1670×10^{-6}
Paraffine	34000×10^{-6}	Ozokerite (crude)	450×10^{-6}
Paraffine oil	8×10^{-6}		

upon molded insulators, as in the ordinary forms of telegraph lines and other bare wire installations, the resistance of the line insulation varies from time to time, depending upon the state of the weather, the cleanliness of the insulating surfaces, and the number of points of attachment of the conductor to the insulators. Owing to these indeterminate factors, it is impossible to predict or calculate, excepting within very wide limits, the insulation resistance of such lines. Data for the probable resistance value to be expected from molded insulators will be found in Chapter III. For conductors which are entirely covered by insulating material, such as underground and submarine cables, the insulation resistance is much more exactly known, and usually operates under very much narrower variations. As there is no known substance that forms a perfect insulator, there is found, in the most carefully constructed lines, surrounded with the greatest amount of protection, a constant and quite perceptible electrical leakage taking place through the dielectric substances employed for insulation. Knowing the specific resistance of the dielectric and the geometrical relations of the conductor and insulator, the probable insulation resistance may be quite closely calculated. Thus, in Fig. 268, consider the case of a cable having a central conductor of the radius R, surrounded by a layer of insulating material having a radius R_2 and of a specific resistance ρ, and let L be the length of the cable. The resistance of an infinitely thin layer of a thickness dR_2 of the insulator at a distance R_1 from the center of the cable will be

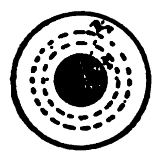


Fig. 268.
Section of Insulated Conductor.

$$\frac{\rho dR_2}{2\pi R_1 L},$$

and the resistance of the entire coating is obtained by integrating the previous expression between the limits R and R_2, obtaining the value

$$\frac{\rho}{2\pi L}\int_{R}^{R_2}\frac{dR_2}{R_1} = \frac{\rho}{2\pi L} \times \log_e \frac{R_2}{R}. \qquad (101)$$

The portion $\rho/2\pi$ is a constant factor for any given dielectric, so if A represents this factor, and L be made unity, and the diameter of the core and cable be substituted for R and R_2, the insulation resistance is given by the expression $A \log D / d$.

357. Distribution of Potential in a Conducting Circuit. — In the preceding paragraphs the relation expressed by Ohm's formula has been considered as applied to a circuit having a uniform resistance, and subjected to the effect of a single unvarying electro-motive force. Such a simple state rarely exists in practice, thus making it necessary to now investigate the conditions which obtain under more complex relationships. Electrical circuits usually consist of a generator of some description, the office of which is to impress upon the circuit an electro-motive force of sufficient amount to perform the work demanded; a line of conducting material serving to connect the generator with the various receiving mechanisms employed to utilize the energy produced; and lastly, the receivers of various kinds, in which the transmitted energy is applied to useful work. An analysis, therefore, of the entire circuit separates it into three parts deserving of consideration.

First. The generator, or source of electro-motive force.

Second. The line, or conducting system.

Third. The receivers.

In each of these divisions a certain amount of electro-motive force is expended, being employed either to overcome the resistance of the separate divisions, or expended in the receivers.

358. From a commercial standpoint, the expenditure of the electro-motive force may be separated into two parts: —

First. The amount necessary to overcome the resistance of the various parts of the circuit, in order to convey from point to point the necessary quantity of electricity.

Second. The electro-motive force usefully expended in producing mechanical work, or the evolution of energy in such a form as to be commercially valuable in the receivers.

359. That portion of the electro-motive force expended in overcoming the resistance of the various parts of the circuit is, as will be subsequently shown, transformed into heat, which by radiation is dissipated and lost, so far as its commercial value in the receiver is considered, excepting in so far as its employment for the purpose of transporting the current from point to point of the circuit be embraced in the term of commercial use. This energy used in overcoming the resistance of the circuit is frequently, though erroneously, termed "wasted energy;" for it is solely by virtue of the expenditure of this portion of the total energy of the circuit that the remainder of the energy is transferred from the point of production to the point of consumption. Inasmuch as there is no known substance possessing no resistance, every part of the circuit involves the expenditure and dissipation as heat of a greater or less quantity of electro-motive force, in order to transmit the necessary current.

360. Transforming Ohm's formula, $IR = E$ is obtained, indicating that the quantity of electro-motive force expended in the circuit is equivalent to the current, multiplied by the resistance of the conductor.

If i, i', i'', etc., be the currents in various portions of a circuit, and r, r', r'', etc., be the resistance of the corresponding parts of the circuit, and e, e', e'', etc., be the expenditures of electro-motive force in each of these corresponding parts of the circuit, the following relations hold: —

$$ir + i'r' + i''r'' +, \text{etc.} = e + e' + e'' +, \text{etc.} = \Sigma ir = \Sigma e = IR = E, \qquad (102)$$

the capital letters standing for the sum of the quantities represented by small type.

In the above equations, great care must be taken to apply to each of the electro-motive forces its appropriate sign, in order that the summation may give the algebraic sum of the various electro-motive forces. Consider the example of a dynamo employed to charge a storage battery. Suppose that the dynamo furnishes a potential of 12 volts, and is employed in charging 5 cells of storage battery, the total potential of which, when fully charged at 2 volts per cell, would amount to 10 volts.

Suppose the resistance of the generator to be $\frac{2}{10}$ of an ohm, and that of the leads to be $\frac{1}{10}$ of an ohm, and the resistance of the storage

battery $\frac{2}{10}$ of an ohm. The total resistance of the circuit will then be $\frac{5}{10}$ of an ohm, and through this the generator will be capable of transmitting a current of 24 amperes. The resistance of $\frac{2}{10}$ of an ohm for the storage battery is made upon the assumption that the charging is commenced when the battery is entirely discharged, and that the cells only oppose to the passage of the current their ohmic resistance. As the charging proceeds, an electro-motive force is developed in the storage battery, which opposes that of the generator, tending constantly, as it increases, to cut down the effective electro-motive force, thus reducing the amount of current flowing. When the batteries are charged to their normal rating, each one would furnish a *counter* electro-motive force of 2 volts, the 5 cells aggregating a total counter electro-motive force of 10 volts. Under these circumstances, the total effective resistance of the circuit would be $\frac{1}{2}$ ohm for the ohmic resistance of the generator, leads, and battery, with a counter electro-motive force of 10 volts developed by the cells. Assuming the previous notation, $e=2$, $\Sigma e=10$, $E=12$, $R=\frac{1}{2}$, the value of the current then becomes —

$$I = \frac{E - \Sigma e}{R} = \frac{12 - (2 \times 5)}{\frac{1}{2}} = 4.$$

Thus, when the batteries are sufficiently charged to give a counter electro-motive force of 2 volts each, the current in the circuit would be reduced from 24 amperes to 4 amperes. If the charging be continued, the electro-motive force of the cells gradually rises more and more, until finally the opposing electro-motive force exactly balances that of the generator, and the charging automatically ceases.

361. The most convenient way to represent the potential distribution in a complicated circuit is the graphical method, in which values along the axis of Y be taken to represent the electro-motive force, and those along the axis of X represent the relative lengths and resistances of the different parts of the circuit.

Thus, in diagram, Fig. 269 represents the previously cited example of a dynamo machine charging 5 cells of storage battery. On the right hand of the illustration the general circuit is indicated, AB being the lead from the positive pole of the dynamo to the storage battery, BC the battery, CD the negative lead returning from the battery to the generator, and DA the circuit in the dynamo machine. On the left hand of the figure, assume OY is the potential axis, single

volts being represented to scale, as shown in the diagram. Assume OX as the axis of resistance, in a similar manner. Under the supposition that the battery is charged to a potential 2 volts per cell, the total current flowing in the circuit will be 4 amperes, as previously shown. Assume, also, that the resistances in the various parts of the circuit, named AB, BC, CD, and DA, are uniform throughout each of the separate parts, the current, of course, being constant throughout the entire circuit. The fall of potential in AB will then be $ri = e = .05 \times 4 = .2$ volts. Along OY lay off OY positively upward, to represent 12 volts, the total potential of the generator. From A lay off AB horizontally, equal to .05 of an ohm, the resistance of AB in the other diagram. Lay off BB′ vertically, negatively

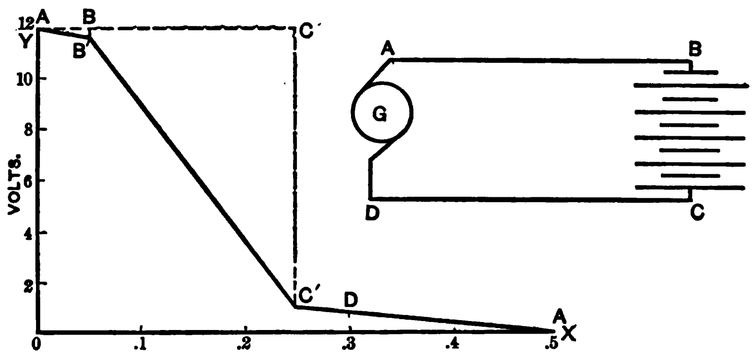


Fig. 269. Diagram of Potential Distribution in a Simple Circuit.

downward equal to .2 volts, and draw AB′; then AB′ represents the distribution of the potential throughout the lead AB. Between B and C there is an ohmic resistance of .2 ohm, and a counter electromotive force of 2 volts per cell, or 10 volts. The fall of potential in this part of the circuit will then, evidently, be $r'i + \Sigma e = .2 \times 4 + 2 \times 5 = 10.8$ volts. From B lay off $\overline{BC}$, horizontally, equal to .2 ohms, and CC′, vertically downward, equal to 10.8+.2 volts. Join $\overline{B'C'}$, the line $\overline{B'C'}$ representing the distribution of potential throughout the battery. The fall of potential in the lead CD is calculated in a manner similar to that indicated for AB, and the fall of potential in the generator DA in the same way; and thus, in the left hand illustration, the irregular line AB′C′DA indicates, for each of the various parts of the circuit, the distribution of the generator potential.

362. The Effect of Leakage. — In the preceding section the lines $\overline{AB'}$ and $\overline{C'D}$ in the left diagram represent the distribution of potential along the conductors $\overline{AB}$ and $\overline{CD}$ that unite the generator to the receivers. It is evident, from the reasoning and construction employed, that these are straight lines, having equations of the form —

$$y = ax + b \tag{103}$$

and that throughout the entire length of each conductor a constant and uniform current existed. This condition can only be fulfilled by assuming that the conductors are perfectly insulated; for, if the insulation is defective in any way, some electricity will escape sidewise between the conductors, and the current will be less at the point B than it is at A by this amount of leakage. Consider two points in the conductor, the first one at a distance x from the origin, and the second at a distance $x + dx$. The electro-motive force acting between these points is dE, while the resistance of the conductor between them is rdx, when r is the resistance of the lead for unit of length. The current flowing between these points is then —

$$I = -\frac{dE}{rdx}. \tag{104}$$

If the conductors were perfectly insulated, this value would be constant throughout the entire length, and equation would be that of a straight line. If the conductor leaks, then *more* electricity enters every element at the point x than leaves the element at the point $x + dx$, the difference in the quantity which enters the element and that which leaves it going to supply the leakage. If r_1 be the insulation resistance per unit of length, r_1/dx is the insulation resistance of the element dx, and the flow of electricity sidewise from this element is —

$$-dI = \frac{Edx}{r_1}. \tag{105}$$

Eliminating I by differentiating equations (104) and (105), and putting

$$m^2 = \frac{r_1}{r},$$

$$\frac{E}{m^2} = \frac{d^2E}{dx^2}. \tag{106}$$

But this is the differential equation of the arc of a catenary,[1] which when integrated gives rise to the equation —

$$E = Ae^{\frac{x}{m}} + Be^{-\frac{x}{m}}. \tag{107}$$

[1] See Rankins's *Applied Mechanics*, p. 175.

For ordinary transmission, lines of moderate length, well built, carefully insulated and maintained, the leakage is so small that without sensible error it may be neglected. For very long lines, such as submarine cables or overland telegraph or telephone lines, the straight line assumption is not sufficient, and the catenary equation should be used.

363. Conductance. — From Ohm's formula, it appears that the resistance of any circuit is proportional to the geometrical dimensions of the conductor, and to its specific resistance. If R be the resistance of any conductor, the reciprocal of R, or $1/R$, gives a quantity which is appropriately denominated, "The *Conductance* of the Circuit," or a quantity to which the ability to transmit electrical energy is proportional. If, between the terminals of any generator, a number of conducting circuits be extended, having the resistance of r, r', r'', etc., the conductance of each branch will be $1/r$, $1/r'$, $1/r''$, etc. It is evident that the total conductance is equal to the sum of the conductances of the individual parts. Thus, representing by c, c', c'', etc., the individual conductances, and by C the total conducting power of the circuit, $C=c+c'+c''$, etc. But c, c', c'', etc.; are respectively equal to $1/r$, $1/r'$, $1/r''$, etc., or the conductance is equal to the sum of the reciprocals of the individual resistances. As resistance is the reciprocal of conductance, the total resistance, therefore, of a number of branch circuits is equal to the reciprocal of the sum of the reciprocals of the individual resistances, or symbolically: —

$$R = \frac{1}{\frac{1}{r} + \frac{1}{r'} + \frac{1}{r''} + \text{etc.}}. \tag{108}$$

364. A graphic method of quickly determining the resistance of two branch circuits is given by Mr. Preece.[1]

Assume in the diagram, Fig. 270, the line $\overline{AB}$ drawn horizontally to represent the resistance of one of the branch circuits; lay off $\overline{BC}$ to the same scale equal to the other resistance, and at C erect a perpendicular $\overline{CD}$ equivalent to $\overline{BC}$. Join A and D, and at B erect a perpendicular $\overline{BE}$, which will, to the same scale, represent the joint resistance of the two resistances, $\overline{AB}$ and $\overline{BC}$. By drawing the line $\overline{BD}$, and dividing $\overline{AB}$, $\overline{BD}$, and $\overline{BE}$, according to the proper proportional scale of each line, joint resistances may be easily found in the

[1] See *Manual of Telephony*, by Preece and Stubbs, p. 164.

following manner: Select upon $\overline{AB}$ a number representing one of the resistances; in a similar manner, select upon $\overline{BD}$ a number representing the other resistance. Lay a straight edge across these two points, and the number given upon $\overline{BE}$ at the intersection of the straight edge with $\overline{BE}$ is the joint resistance of the two circuits. The scale on $\overline{AB}$ and $\overline{BE}$ is a simple decimal scale of equal parts. The scale on $\overline{BD}$ is also a decimal scale of equal parts, and is related to the scales on $\overline{AB}$ and $\overline{BE}$ in the proportion of $1 : \sqrt{2}$. As all three scales are decimal, the significant figures on all of them may be multiplied or divided at pleasure by any power of 10, and thus the diagram extended to cover any desired range of resistance. The geometrical relation of the three scales is self-evident. In the case of several branch circuits, the joint resistances of any two may be found by the diagram, and then this value combined with the resistance of any one of the remaining branches. By a continuance of this process any number of circuits may, by the diagram, be calculated.

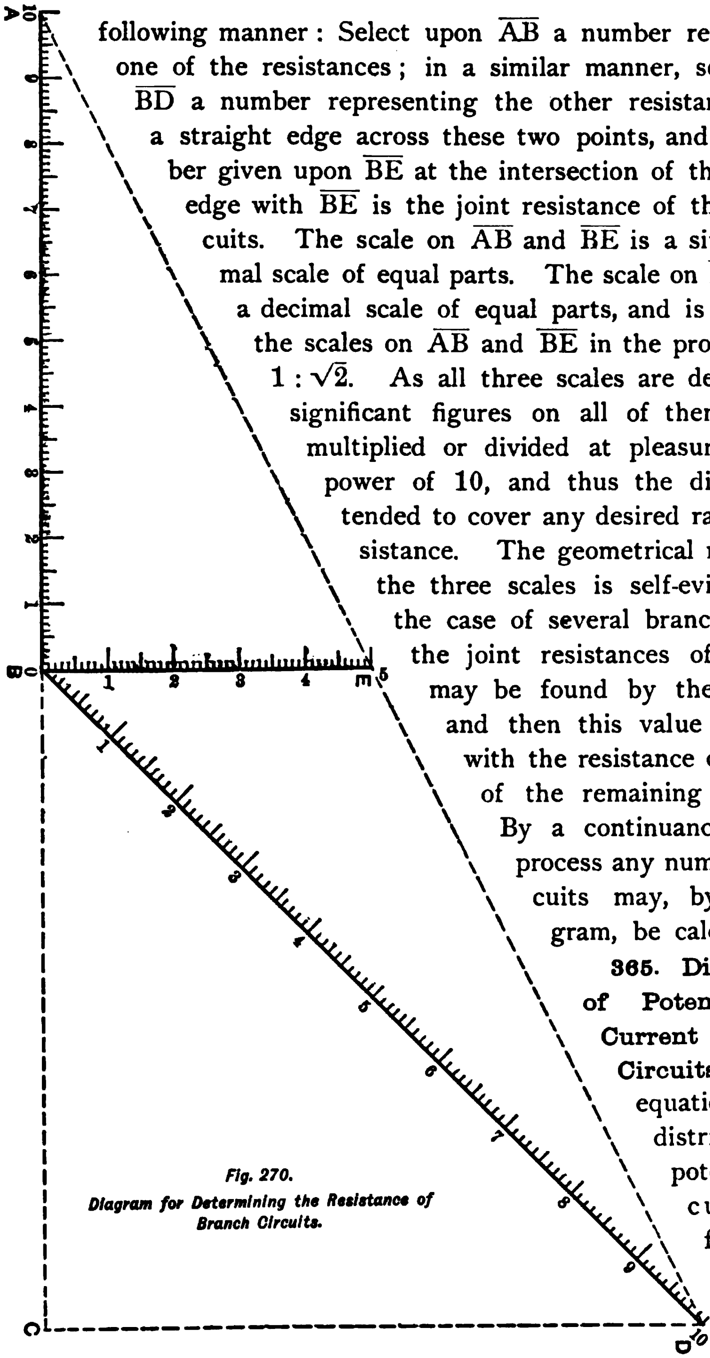


Fig. 270.
Diagram for Determining the Resistance of Branch Circuits.

365. Distribution of Potential and Current in Branch Circuits. — The equations for distribution of potential and current so far given apply simply to a

circuit consisting of a single source of electro-motive force, introduced in a circuit consisting of a single conductor extending from pole to pole of the generator. In practice, however, actual installations are usually very much more complex, frequently consisting of a number of generators placed at different points of a complex network of conductors, which ramify in all directions over the territory to be supplied. To determine accurately the description of potential and current in a complicated network, is a matter of exceeding importance to the electrical engineer. While calculations of this kind are based on simple algebraic applications of the laws of Ohm and Kirchhoff, a complete solution of the distributing problem is difficult of successful solution, owing to the fact that, while the principles are simple, the application of them leads, usually, to exceedingly complicated and intricate equations. Any network of circuits may always be resolved into one of four elementary cases.

366. CASE 1. — Is that of a simple circuit embracing a generator placed in a continuous, straight conductor, extending from one pole to the other of the generator without branches, and may be treated directly by Ohm's law.

367. CASE 2. — This consists of a generator supplied with a circuit consisting of one or more branches, as shown in Fig. 271, in which E is the generator or source of electromotive force, *ab* and *ac* the conductors from the generator to the points *b* and *c*, at which points the circuit branches or divides into two parts of varying resistance. Let E denote the *E.M.F.* of the generator, and r_e its resistance. Let r_1 be the resistance of *ab* and *ac*, and r_2 and r_3 the respective resistances of the two branches from *b* to *c*, then the combined resistance between *bc* is —

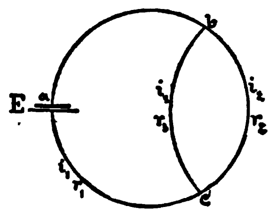


Fig. 271. Branch Circuits.

$$[r_2 ; r_3] = \frac{r_2 \times r_3}{r_2 + r_3}. \tag{109}$$

The total resistance of the entire circuit R is,

$$R = r_e + r_1 + \frac{r_2 \times r_3}{r_2 + r_3}. \tag{110}$$

Denote the respective currents in the various parts of the circuit by i_1, i_2, and i_3, as indicated in the diagram, then,—

(112)

(112)

(113)

(114)

... similar equations

... in Fig. 272, ... E$_1$ and E$_2$. ... respectively ab, ... resistance of ... i_1, and i_2,

... acting E.M.F.s, ... the following

(115)

(116)

(117)

When the double ... to be taken ... where the ... the E.M.F.s are ... that, excepting ...

the constants of the circuits, there will be current in all of the branches.

To make $i=0$, —

$$\frac{E_1}{E_2}=\frac{r_1}{r_2}, \qquad \text{or } E_1=E_2\frac{r_1}{r_2}. \tag{118}$$

To make $i_1=0$, —

$$\frac{E_1}{E_2}=\frac{r}{r+r_2}, \text{ or } E_1=E_2\frac{r}{r+r_2}. \tag{119}$$

To make $i_2=0$, —

$$\frac{E_1}{E_2}=\frac{r+r_1}{r}, \text{ or } E_1=E_2\frac{r+r_1}{r}. \tag{120}$$

As there are several quantities (*E.M.F.s* and resistances) in these expressions, parallel results may be attained by changing either one,

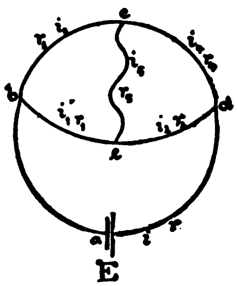


Fig. 273.

or any desired combination, of the variables to attain the desired ratios. When there are n branches in the network and n *E.M.F.s* acting, similar equations may be deduced.

369. CASE 4. — The last elementary combination is illustrated in Fig. 273, and consists of an *E.M.F.* acting in a circuit of seven branches, $\overline{ab}$, $\overline{ad}$, $\overline{bc}$, $\overline{be}$, $\overline{dc}$, $\overline{de}$, and $\overline{ec}$ as shown. As above, let E represent the *E.M.F.*, i, i_1, etc., and r, r_1, etc., the respective currents and resistances in the several branches as shown in the diagram, then

$$\frac{+ r_1r_4 + r_1r_5 + r_2r_3 + r_2r_5 + r_3r_4 + r_3r_5 + r_4r_5}{N} \times E; \tag{121}$$

$$\frac{+ r_3r_4 + r_3r_5 + r_4r_5}{N} \times E; \tag{122}$$

$$i_1 = \frac{E\,(r_2 + r_3)}{r'_1 r_2 + r'_1 r_3 + r_2 r_3}; \tag{111}$$

$$i_2 = \frac{E r_3}{r'_1 r_2 + r'_1 r_3 + r_2 r_3}; \tag{112}$$

$$i_3 = \frac{E r_2}{r'_1 r_2 + r'_1 r_3 + r_2 r_3}; \tag{113}$$

$$\frac{i_2}{i_3} = \frac{r_3}{r_2}. \tag{114}$$

Here $r'_1 = r_e + r_1$. If there are n branches, n similar equations may be formed.

368. CASE 3.—This is indicated diagrammatically in Fig. 272, in which there are two sources of electro-motive force, E_1 and E_2. In a circuit consisting of three branches that are respectively *ab*, *ad*, *cb*, *cd*, and *db*, let r, r_1, and r_2, be the respective resistance of the several branches as indicated in the diagram, and i, i_1, and i_2,

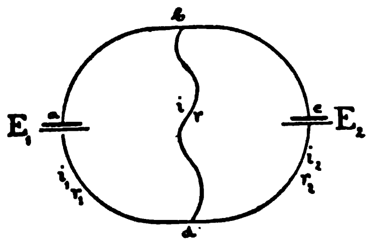


Fig. 272.

the corresponding currents. Let E_1 and E_2 be the acting *E.M.F.s*, then the applications of Kirchhoff's laws give rise to the following equations for the current values:—

$$i = \frac{E_1 r_2 \pm E_2 r_1}{r r_1 + r r_2 + r_1 r_2}; \tag{115}$$

$$i_1 = \frac{E_1\,(r + r_2) \mp E_2 r}{r r_1 + r r_2 + r_1 r_2}; \tag{116}$$

$$i_2 = \frac{E_2\,(r + r_1) \mp E_1 r}{r r_1 + r r_2 + r_1 r_2}. \tag{117}$$

When the double sign ($\pm$ or $\mp$) is used, the upper sign is to be taken in cases where the *E.M.F.s oppose* each other, and the lower one where the *E.M.F.s* act together. Considering these equations, it is evident that, excepting when particular values are assigned to

the constants of the circuits, there will be current in all of the branches.

To make $i=0$, —

$$\frac{E_1}{E_2}=\frac{r_1}{r_2}, \qquad \text{or } E_1=E_2\frac{r_1}{r_2}. \tag{118}$$

To make $i_1=0$, —

$$\frac{E_1}{E_2}=\frac{r}{r+r_2}, \text{ or } E_1=E_2\frac{r}{r+r_2}. \tag{119}$$

To make $i_2=0$, —

$$\frac{E_1}{E_2}=\frac{r+r_1}{r}, \text{ or } E_1=E_2\frac{r+r_1}{r}. \tag{120}$$

As there are several quantities (*E.M.F.s* and resistances) in these expressions, parallel results may be attained by changing either one,

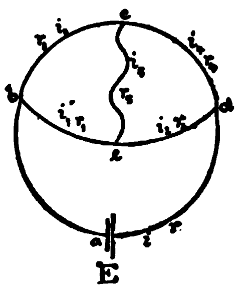


Fig. 273.

or any desired combination, of the variables to attain the desired ratios. When there are n branches in the network and n *E.M.F.s* acting, similar equations may be deduced.

369. Case 4. — The last elementary combination is illustrated in Fig. 273, and consists of an *E.M.F.* acting in a circuit of seven branches, $\overline{ab}$, $\overline{ad}$, $\overline{bc}$, $\overline{be}$, $\overline{dc}$, $\overline{de}$, and $\overline{ec}$ as shown. As above, let E represent the *E.M.F.*, i, i_1, etc., and r, r_1, etc., the respective currents and resistances in the several branches as shown in the diagram, then, —

$$i) \quad i=\frac{r_1r_2+r_1r_4+r_1r_5+r_2r_3+r_2r_5+r_3r_4+r_3r_5+r_4r_5}{N}\times E; \tag{121}$$

$$k) \quad i_1=\frac{r_2r_3+r_3r_4+r_3r_5+r_4r_5}{N}\times E; \tag{122}$$

l) $$i_2 = \frac{r_1 r_4 + r_3 r_4 + r_3 r_5 + r_4 r_5}{N} \times E; \tag{123}$$

m) $$i_3 = \frac{r_1 r_2 + r_1 r_4 + r_1 r_5 + r_2 r_5}{N} \times E; \tag{124}$$

n) $$i_4 = \frac{r_1 r_2 + r_1 r_5 + r_2 r_3 + r_2 r_5}{N} \times E; \tag{125}$$

o) $$i_5 = \frac{r_1 r_4 - r_2 r_3}{N} \times E. \tag{126}$$

In the above expressions, —

$$\begin{aligned} N = {} & r r_1 r_2 + r r_1 r_4 + r r_1 r_5 + r r_2 r_3 + r r_2 r_5 + r r_3 r_4 + r r_3 r_5 + r r_4 r_5 + r_1 r_2 r_3 \\ & + r_1 r_2 r_4 + r_1 r_3 r_4 + r_1 r_3 r_5 + r_1 r_4 r_5 + r_2 r_3 r_4 + r_2 r_3 r_5 + r_2 r_4 r_5. \end{aligned}$$

Equation (126) shows that in the branch *ec*, the current i_5 becomes zero, when, —

$$r_1 r_4 = r_2 r_3, \text{ or } \frac{r_1}{r_2} = \frac{r_3}{r_4}, \text{ or } \frac{r_1}{r_3} = \frac{r_2}{r_4}. \tag{127}$$

370. All networks, no matter how complicated, may be analyzed by resolving them into combinations of the foregoing elementary forms. By then successively applying the equations given for each of the elementary forms and summing the results, the distribution of current and potential, no matter how complicated, may be finally arrived at. It should be noted, however, that algebraic processes of this kind are exceedingly complicated, and are particularly liable to lead to error, owing to the multiplicity and complexity of the symbols, and, therefore, great care must be taken to avoid numerical mistakes in attaining the final result.

CHAPTER XII.

CONTINUOUS CURRENT CONDUCTORS.

THE HEATING OF CONDUCTORS.

Art. 371. Joule's Law. — A portion of the electrical energy delivered to any conductor is found to be expended in the conductor itself, and by some mysterious process, sometimes termed by investigators "molecular friction," is transformed into heat, and serves to raise the temperature of the material forming the conductor. Experiment has shown that the quantity of heat thus produced is proportional to the square of the current, the resistance of the circuit, and the time during which the current flows.

To Doctor Joule is due the mathematical expression for the amount of heat thus developed.

Let I be the current in amperes,
R the resistance of the circuit,
T the time in seconds during which the current flows,
H the heat developed in calories (gramme degree),
s the cross-section of the conductor,
d the diameter of the conductor,
l its length,
ρ the specific resistance of the material forming the conductor.

Joule's Law indicates, for the amount of heat developed in any circuit, —

$$H = .24\,I^2RT. \qquad (128)$$

The resistance R of the circuit is $\rho l/s$, which, for a cylindrical conductor, becomes $4\rho l/\pi d^2$. Substituting in the preceding formula, —

$$H = \frac{.30557\,\rho l I^2 T}{d^2}. \qquad (129)$$

If l and T are written as units of length and time respectively, then the above expression, formula (129) gives the amount of heat evolved per unit of length of the conductor per unit of time. The

heat thus generated augments the temperature of the conductor; and were it not for radiation and convection, this elevation of the conductor temperature would increase until the fusing-point of the circuit was reached, and the current interrupted by the melting of the conductor. When the cooling of the circuit equals the heat evolution, equilibrium is obtained, the conductor remaining at a constant temperature above its surroundings, as long as the current remains constant. To safely design electrical circuits, in order that their carrying capacity may be, on the one hand, such as to exempt them from becoming sources of danger, and on the other hand, to attain an economical disposition of the conducting material, is a matter of supreme importance.

372. Location of Circuit. — It is necessary to consider conductors under the various aspects in which electrical circuits may be placed.

First. Bare wires may be freely suspended in the atmosphere.

Second. Bare wires may be inclosed in panel moldings, or other forms of interior conduit.

Third. Insulated wires may be freely suspended in the air.

Fourth. Insulated cables may be buried in underground conduits, or extended under water; and, as a corollary, adjacent underground conductors may exercise a mutual influence on each other, the passage of the current in one cable being sufficient to cause the temperature of the conductor to seriously influence that of a neighboring cable. Each of these cases will be considered successively.

373. First, Bare Wires Freely Suspended. — The resulting temperature to be attained by electrical conductors has been studied in England by Professor Forbes, and investigated in this country by Mr. A. E. Kennelly. Both of these investigators have based their researches upon the laws for radiation and convection established by Dulong and Petit. Mr. Kennelly's experiments have been the more complete and exhaustive, and, forming a classic paper presented to the Edison Convention in 1889, are usually assumed as indicating the best present knowledge on the subject.

374. Radiation and Convection. — Two causes are, manifestly, operative to reduce the temperature of a conductor.

First. Heat may be lost by direct radiation from the surface of the conductor.

Second. Heat may be lost by convection.

The quantity of heat radiated by a conductor is proportional to the amount of radiating surface, the difference in the temperature between the conductor and that of its surroundings, the time during which radiation takes place, and to an arbitrary coefficient depending upon the nature of the radiating surface.

Thus, if κ be the coefficient of radiation per unit of area,

θ the temperature of the surrounding air,

t the temperature attained by the conductor,

the radiating power of length l and diameter d is $\kappa\pi dl(t-\theta)T$.

If l and T are respectively units of length and time, the expression per unit of length per unit of time becomes $3.1416\ \kappa d(t-\theta)$.

Mr. Kennelly's experiments, confirming the investigations of Dulong and Petit, indicate that for radiation the quantity of heat dissipated per square centimeter of surface is given by the expression $(1.0077^{\theta})\ (1.007^{t}-1)C$, in which C is a constant depending upon the physical character of the radiating surface.

375. For highly polished metals (the poorest radiators), C is equal to one; while for roughened and blackened surfaces C has a greater value, being usually assumed as two, but sometimes rising to a higher value. For electrical calculation, it is more convenient to express the energy lost in the conductor in watts, instead of thermal units or calories.

Denoting then the total energy transformed in the conductor into heat by W watts, and that portion of W lost by radiation by W_r, and the portion lost by convection by W_c, —

$$W = W_r + W_c.$$

Under these circumstances, the expression for the radiation per square centimeter is —

$$W_r = .05625\ (1.0077^{\theta})\ (1.007^{t}-1)\ C. \tag{130}$$

For a polished wire of diameter d and any surface, the radiation becomes —

$$W_r = .05625\ [(1.0077^{\theta})\ (1.007^{t}-1)\ \pi dC] \tag{131}$$

per unit of length and time.

376. Cooling is also aided by convection. The amount of heat lost from this cause, as determined by Mr. Kennelly, is —

$$W_c = .00175\ (t-\theta); \tag{132}$$

and the investigation indicated that this relation was independent of the amount of surface, holding true for a wire of any diameter per unit of length.

This relation is found strictly applicable for still air in an inclosed location. But Mr. Kennelly's experiments show that, for ordinary aerial lines, even under the most unfavorable assumption of calm weather, the above quantity can be increased by an amount equal to —

$$.013\,d\,(t-\theta).$$

Therefore the complete expression for W_c becomes —

$$W_c = (.00175 + .013\,d)\,(t-\theta). \tag{133}$$

377. The amount of energy in watts W developed in the conductors, per unit of length and time, is I^2R. As soon as the temperature of the conductor ceases to rise, there must evidently be equilibrium between the heat evolution in the conductor and the amount lost by radiation and convection. Then, —

$$I^2R = .05625\,[(1.0077^\theta)\,(1.007^t - 1)]\,C\pi d + [(.00175 + .013\,d)\,(t-\theta)]. \tag{134}$$

To simplify, let $[(.00175 + .013\,d)\,(t-\theta)] = a,$

and $.05625\,\pi\,[(1.0077^\theta)\,(1.007^t - 1)] = b;$

then, $I^2R = bdC + a;$

but $R = \dfrac{4\,\rho_0\,(1 + \alpha t + \beta t^2)}{\pi d^2},$

at any temperature above 0° C.; hence, as the temperature attained by the conductor is a function of the resistance, this quantity must be substituted for R.

$$I^2 = \frac{\pi d^2\,(bdC + a)}{4\,\rho_0\,(1 + \alpha t + \beta t^2)} = \frac{\pi d^2}{4} \times \frac{bdC + a}{\rho_0\,(1 + \alpha t + \beta t^2)},$$

$$I = .8862\,d\sqrt{\frac{bdC + a}{\rho_0\,(1 + \alpha t + \beta t^2)}}. \tag{135}$$

As both θ and t enter into the quantity under the radical sign, I can only be obtained by successive approximations.

378. In TABLES Nos. 51 and 52 will be found the values of —

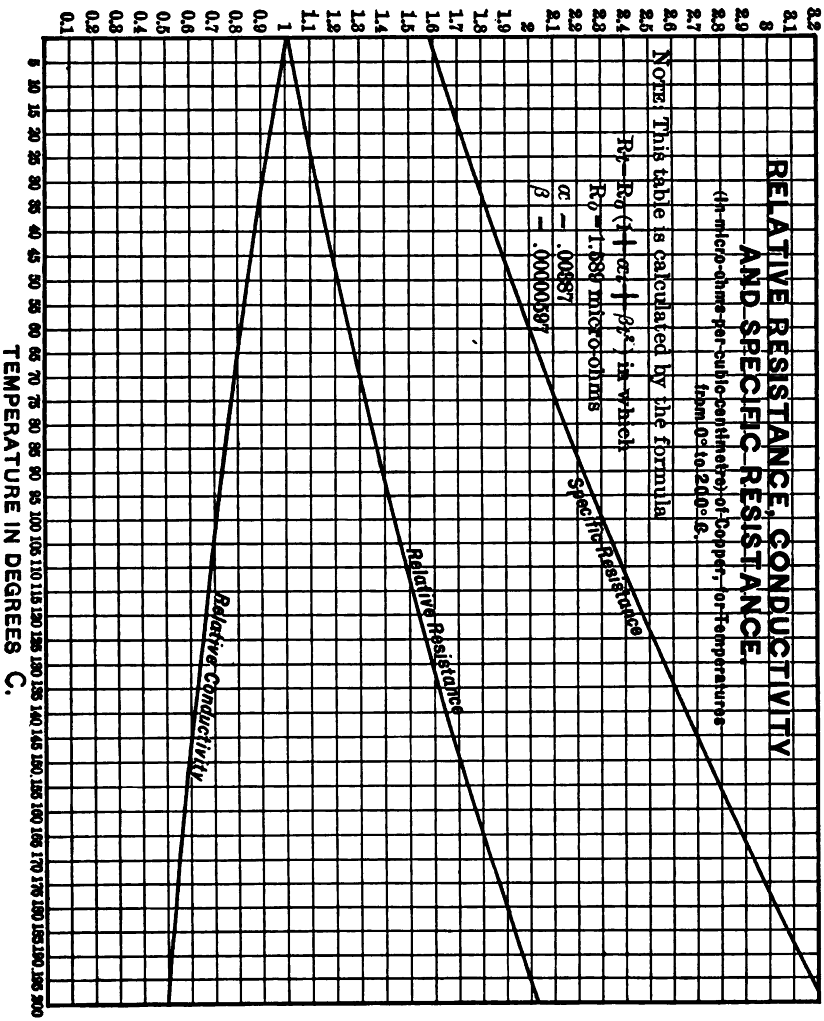
$$R_t = \rho_0\,(1 + \alpha t + \beta t^2),$$
$$.05625\,\pi\,(1.0077^\theta),$$
$$1.007^t - 1,$$

and $0.00175 + 0.013\,d;$

by means of which, by simple substitution, the carrying capacity of any wire in amperes may be found for any surrounding temperature, and any determined rise of temperature, between 0° and 200° C.

TABLE No. 51.

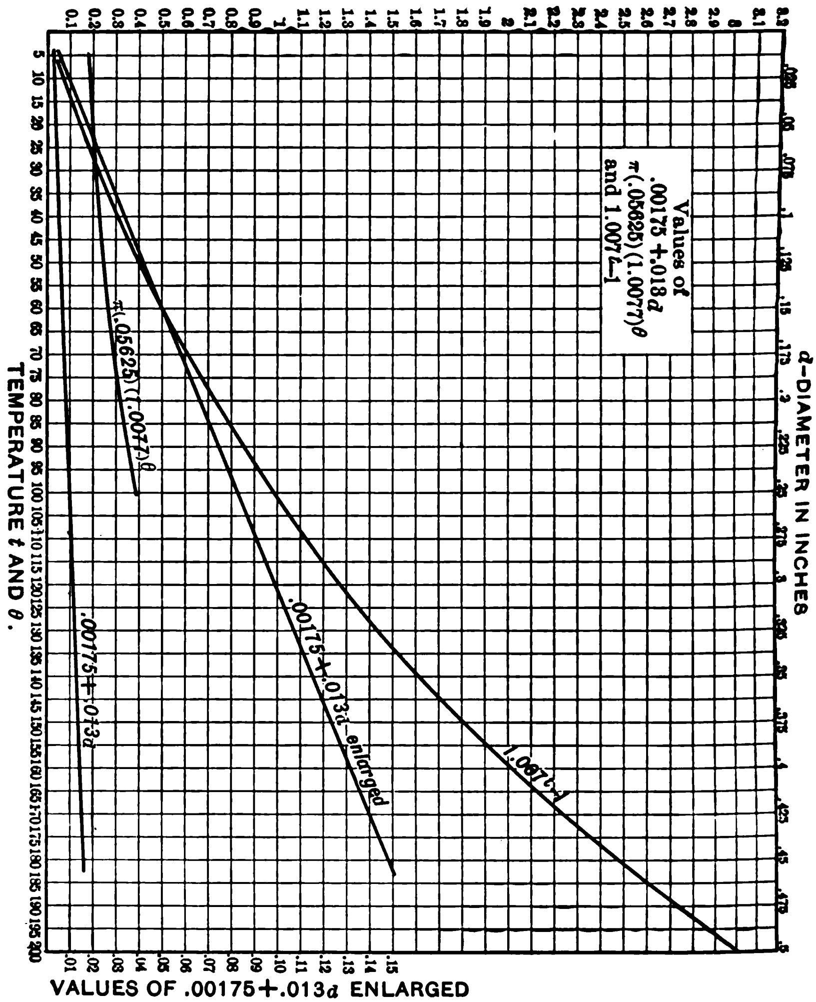
Copper Resistance.



379. In TABLE No. 51 the base line is assumed horizontally opposite 1 on the left-hand vertical axis, the relative resistances being

reckoned positively upward, and the relative conductivity negatively downward. The specific resistance is a positive curve, running upward from the point 1.589 on the left-hand axis. The temperature

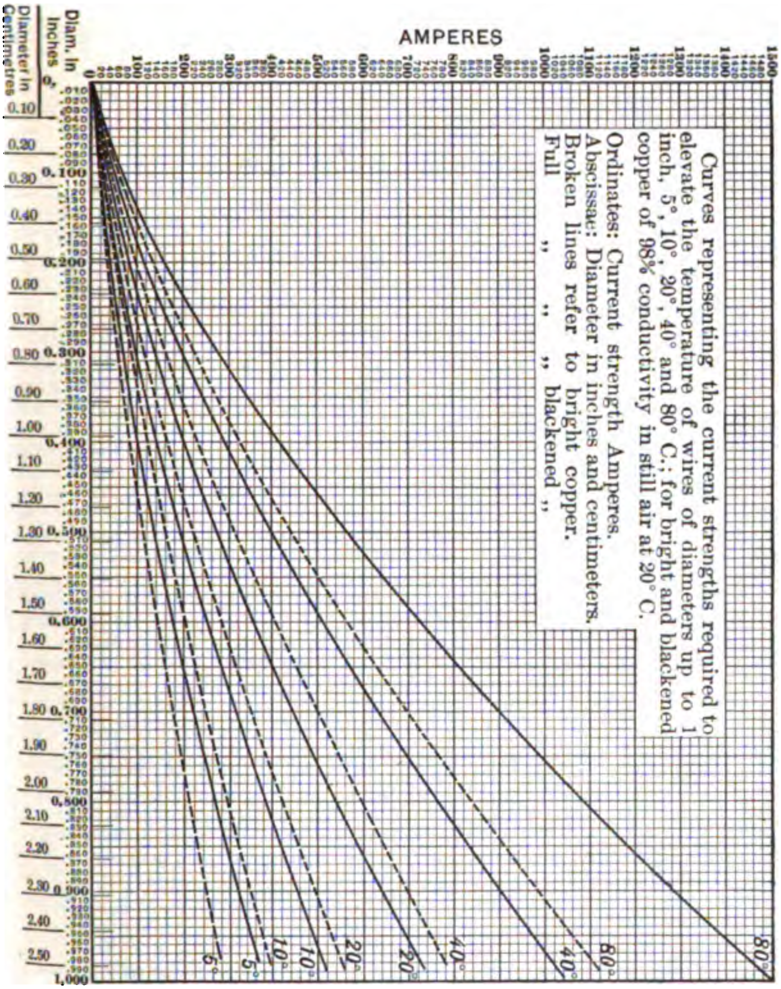
TABLE No. 52.



scale will be found horizontally along the bottom, and the resistance scale on the left hand. In TABLE No. 52, for the curve giving values of .00175+.013d, the values of d are horizontally along the top of the

sheet, the temperature scale horizontally along the bottom. Two curves are given for this expression; one plotted to the natural scale of the sheet, as indicated on the left-hand axis, and the other exagger-

TABLE No. 54.



ated ten times, the axis being on the right hand. The other two curves, for values of $.05625\pi\,(1.0077^{\theta})$ and $1.007^{t}-1$, hardly need explanation, being referred to the lower and left-hand axes.

380. By means of the preceding formulæ and tables, quite accu-

rate determinations may be made of the probable temperature to be obtained in any conductor by the passage of any current. For aerial lines, the free circulation of air and effect of wind is usually to reduce the temperature below that indicated by the formula. For a good approximation, the TABLES Nos. 53 (in pocket) and 54, deduced from Kennelly's experiments, may be employed for determining the probable temperature of the conductors.

TABLE No. 55.

Safe Currents for Paneled Wires.

Amperes.	Minimum Safe Diameter of Copper Wire. Inches.	Minimum Safe Diameter of Copper Wire. Cms.	Circular Mils per Ampere.	Fall of Potential in Wire at Full Load. Volts per Foot.	Fall of Potential in Wire at Full Load. Volts per Meter.	Amperes.	Minimum Safe Diameter of Copper Wire. Inches.	Minimum Safe Diameter of Copper Wire. Cms.	Circular Mils per Ampere.	Fall of Potential in Wire at Full Load. Volts per Foot.	Fall of Potential in Wire at Full Load. Volts per Meter.
1	0.015	0.038	225	0.0503	0.165	130	0.377	0.958	1090	0.0103	0.0337
5	0.043	0.109	370	0.0306	0.100	140	0.396	1.01	1120	0.0099	0.0327
10	0.069	0.175	480	0.0237	0.0777	150	0.415	1.05	1150	0.00975	0.0320
15	0.090	0.229	540	0.0208	0.0681	175	0.461	1.17	1210	0.00929	0.0305
20	0.109	0.277	590	0.0189	0.0621	200	0.504	1.28	1270	0.00887	0.0291
25	0.126	0.320	640	0.0177	0.0581	225	0.545	1.38	1320	0.00853	0.0280
30	0.142	0.361	670	0.0167	0.0548	250	0.585	1.49	1370	0.00817	0.0268
35	0.158	0.401	710	0.0158	0.0518	275	0.623	1.58	1410	0.00793	0.0262
40	0.172	0.437	740	0.0152	0.0499	300	0.660	1.68	1450	0.00771	0.0253
45	0.186	0.472	770	0.0147	0.0481	325	0.697	1.77	1490	0.00753	0.0247
50	0.200	0.508	800	0.0141	0.0461	350	0.732	1.86	1530	0.00734	0.0241
55	0.213	0.541	825	0.0136	0.0447	375	0.766	1.95	1570	0.00716	0.0235
60	0.225	0.572	845	0.0133	0.0437	400	0.800	2.03	1600	0.00714	0.0231
65	0.238	0.605	870	0.0129	0.0423	425	0.832	2.11	1630	0.00692	0.0227
70	0.250	0.635	890	0.0126	0.0413	450	0.865	2.20	1660	0.00674	0.0221
75	0.262	0.665	915	0.0123	0.0403	475	0.897	2.28	1690	0.00665	0.0218
80	0.274	0.696	940	0.0120	0.0393	500	0.928	2.36	1720	0.00652	0.0214
85	0.285	0.724	960	0.0118	0.0386	550	0.988	2.51	1775	0.00634	0.0208
90	0.296	0.752	970	0.0116	0.0379	600	1.049	2.66	1840	0.00616	0.0202
95	0.307	0.780	990	0.0113	0.0372	700	1.16	2.95	1920	0.00585	0.0192
100	0.318	0.808	1010	0.0111	0.0365	800	1.27	3.23	2020	0.00558	0.0133
110	0.339	0.861	1040	0.0108	0.0353	900	1.37	3.48	2080	0.00539	0.0177
120	0.358	0.909	1070	0.0105	0.0346	1000	1.47	3.73	2160	0.00521	0.0171

DATA.—Insulated house wires carrying continuous currents, and incased in wooden paneling. Copper resistivity, 1.650 microhms @ 0° C. = 1.870 microhms @ 34° C. assumed temperature of full load; conductivity allowed, 98 per cent.

381. It should be carefully noted that the rise of temperature of the conductor increases its resistance a very notable amount, and should not be forgotten in the design of the circuit. To compensate for this extra resistance, either additional conductor section must be provided, or a greater pressure at the terminals of the generator. See TABLE No. 55 for full data for proportioning circuits.

382. Second, Paneled Wire. — Interior wiring is usually either protected by ornamental moldings, or run in interior conduits of some description. Being thus in a confined location, the effects of radiation and convection are reduced to a minimum. The circuits may also be surrounded by inflammable material, so that particular care must be exercised to secure safety. The consensus of opinion of the American and Foreign Underwriters' Associations limits the allowed elevation of temperature in paneled conductors to a rise not to exceed 10° C. For this amount, Mr. Kennelly's experiments indicate the safe current in amperes to be expressed by the relations, —

$I = 560d^{\frac{3}{2}}$ . . . . if d is in inches.
$I = 0.01775d^{\frac{3}{2}}$ . . . . if d is in mils.
$I = 138d^{\frac{3}{2}}$ . . . . if d is in centimeters.
$I = 4.375d^{\frac{3}{2}}$ . . . . if d is in millimeters.

Reciprocally —

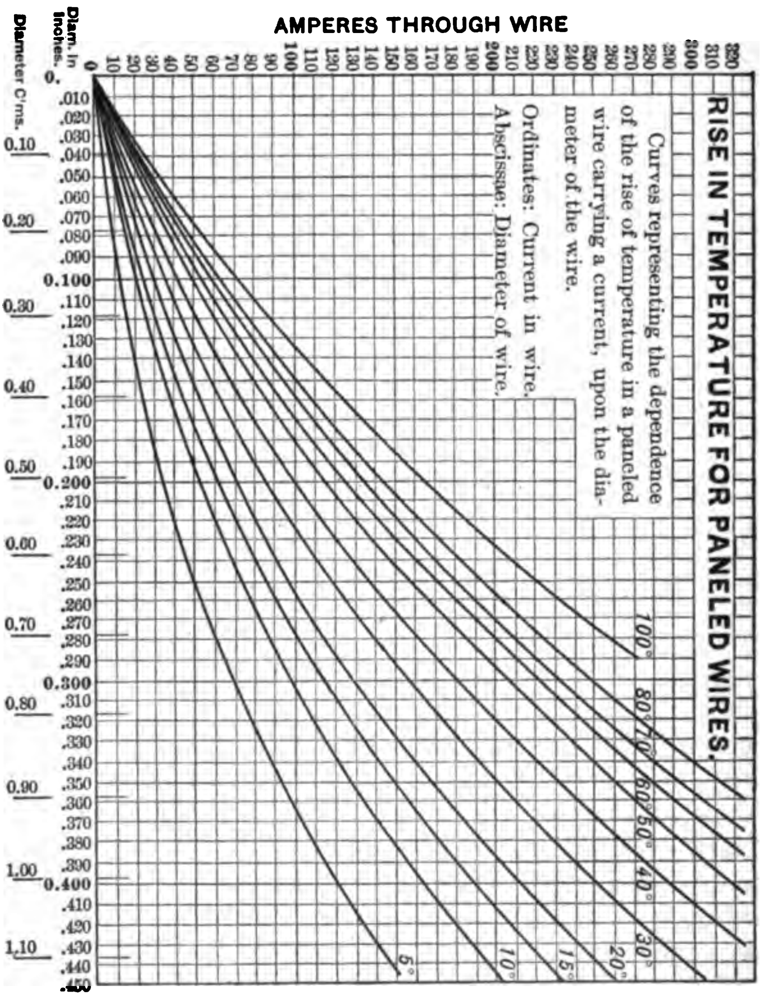
$d = 0.0147I^{\frac{2}{3}}$ . . . . if d is in inches.
$d = 14.7I^{\frac{2}{3}}$ . . . . if d is in mils.
$d = 0.0374I^{\frac{2}{3}}$ . . . . if d is in centimeters.
$d = 0.374I^{\frac{2}{3}}$ . . . . if d is in millimeters.

From these data, and as a result of his experiments, Mr. Kennelly gives Tables Nos. 55 and 56; Table No. 56 indicating the current and resulting temperature, in paneled wire, up to 300 amperes, and to a diameter of .450″; Table No. 55 giving the minimum safe diameter and fall of potential, up to 1,000 amperes, for a rise of temperature of 10° C.

383. Third, Insulated Wire Freely Suspended. — Apparently, insulated wires, with their non-conducting coatings, would be subjected to a greater elevation of temperature than bare wire. The insulation, however, increases the amount of radiating surface of the conductor, and also provides a surface which, from its physical characteristics, is a much more efficient radiator than the polished metal. It seems probable that this increased efficiency and size of the surface fully counterpoise, at least in most cases, the non-conducting effect of the insulating covering. From Professor Forbes's experiments, this relation would seem to be substantiated, and therefore ordinary insulated wires may usually be treated as if they were uninsulated. This branch of the subject, however, is worthy of more extended investigation.

384. Rheostats and Heaters. — In many electrical appliances it is customary to control the amount of energy delivered to the translating device by the aid of a resistance capable of being varied to suit

TABLE NO. 56.



the demands upon the receiver, which acts as a dam, or valve, interposed in the circuit to control the amount of current. Such resistances are termed rheostats, and dissipate a certain amount of energy as electricity, transforming same into heat. The determination of

the size of wire to be used for such purposes may be made by the use of the preceding formulæ. Usually, however, rheostats are made either of German silver or iron wire; and for either of these materials, TABLES Nos. 57, 58, and 59 may be used to obviate calculation. A number of devices have recently made their appearance for heating by electricity, such as car-heaters, flat-irons, cooking-utensils, and the like. Nearly all of them are based upon the transformation of electrical energy into heat energy by the interposition of the resistance of a bare conductor, usually consisting of a small metallic wire embedded in a vitreous enamel of high melting-point.

TABLE No. 57.

Safe Current for Galvanized Iron Wire. Rheostats.

D in Circular Mils.	Maximum Safe Current in Amperes in Wood Frames.	Maximum Safe Current in Amperes in Iron Frames.	Maximum Safe Current for One Minute.	Feet per Ohm.
59536	55	63.8	125	645
50625	48	55.6	110	549
42849	41	47.5	90	463
36864	30	34.8	78	398
31329	26	30.1	67	337
26244	23	26.6	56	283
21904	20	23.2	46	236
18225	17	19.7	36	196
14400	14.5	16.2	32	155
11025	12	13.9	22	119
8464	10	11.6	17	91.4
6400	8	9.28	13	69.1
5184	6	6.96	11	56.0
3969	5	5.8	8.9	42.8
2916	3.7	4.29	8	31.4

The energy supplied by the device is, by the resistance of the wire, transformed into heat, and serves to raise the temperature of the entire apparatus to a useful limit. All such devices may be calculated by the methods already indicated, due consideration being given to the conducting power of the enamel, in which the heating part of the circuit is embedded. Some heating contrivances, however, designed to operate upon alternating current circuits, take advantage of the work done by hysteresis in rapidly alternating magnetic cycles. For devices of this kind, it is hardly necessary to state that the preceding calculations do not apply.

385. Cost of Electrical Heating.— The cost of electrical heating in its various forms is, probably, the most important factor in the

development of this branch of industry. Probably electrical heating has its widest development in the warming of street-cars during the cold season of the year. Car-heaters operated by coal cost from $15.00 to $25.00, with an installation expense of $1.50. With fuel at $4.50 per ton, and labor $1.50 per day, the cost of car-heating by fuel stoves is about 16 cents per day of 24 hours. Per contra, electric heaters cost from $35.00 to $40.00, and the expense for current amounts to from 30 to 40 cents per day in moderate weather, and from 60 to 80 cents per day in the coldest weather. These figures are based on fuel at $3.00 per ton at the generating-station. For

TABLE No. 58.

Safe Current for Tinned Iron Wire. Rheostats.

D in Circular Mils.	Maximum Safe Current Capacity. Wood Frame.	Maximum Safe Current Capacity. Iron Frame.	Maximum Safe Current Capacity for One Minute.	Feet per Ohm.
16509	17.4	20.3	43.6	205
13094	14.6	17.1	36.6	173
10381	12.3	14.3	30.8	137
8234	10.3	12	25.7	108
6529	8.7	10.1	21.8	86.4
5178	7.3	8.5	18.3	68.5
4106	6.1	7.1	15.3	54.3
3256	5.1	6	12.9	43.1
2582	4.3	5	10.8	34.1
2048	3.6	4.2	9.1	27.1
1624	3.0	3.5	7.6	21.4
1252	2.52	2.9	6.3	16.5
1021	2.17	2.5	5.4	13.5
810	1.82	2.1	4.5	10.7
642	1.53	1.77	3.8	8.49
509	1.28	1.49	3.2	6.73
404	1.08	1.2	2.3	5.34

cooking by electricity, it is found that an ordinary oven requires about 25 amperes at 110 volts, a frying-pan 2½ amperes, a flat-iron from 2 to 3 amperes, and a soldering-iron from 2 to 3 amperes. It is claimed that ordinary meat can be roasted in an electric oven, supplied with 25 amperes, in from 7 to 8 minutes per pound of meat introduced. For heating water, the cost under the present rates for current, averages about 2 to 5 cents per gallon of water heated. An ordinary oven is entailed with an expense of from 8 to 6 cents per hour. Under these circumstances, if the electrical current be estimated at an expense of $60.00 per H.P. annum of 400 hours, it

would correspond to coal at $6.00 a ton, which is not very different from the actual expense to small consumers.

386. Fuse Wires. — Electrical circuits are protected against overloading, in the majority of cases, by the interposition at various points of short pieces of fusible metal so designed that a slight access of current above the normal amount, for which the circuit is

TABLE No. 59.

Safe Current in German Silver Wire. Rheostats.

D in Circular Mils.	Maximum Safe Current in Amperes.	Feet per Ohm.	D in Circular Mils.	Maximum Safe Current in Amperes.	Feet per Ohm.
10381	8.5	60.9	1252.4	1.21	7.25
8234	5.4	47.6	1021.5	.99	5.91
6529.9	4.6	37.8	810.1	.88	4.69
5178.4	3.8	29.9	642.7	.66	3.72
4106.8	3.2	23.7	509.45	.55	2.95
3256.7	2.7	18.8	404.01	.488	2.33
2582.9	2.3	14.9	320.04	.434	1.85
2048.2	1.9	11.8	254.01	.385	1.47
1624.3	1.65	9.40	201.5	.343	1.16

calculated, will melt the fuse, and afford protection from further injury by opening the leads. The first experimental determination of the constants for fuse wires was made by Mr. Preece in 1884; the investigation showed that the relation between the diameter of the wire and the fusing current is expressed by the equation, —

$$I = ad^{\frac{3}{2}}, \tag{136}$$

in which a is a constant for a given metal or alloy. In 1890 continued investigation by Mr. Preece (published in the *Electrical Engineer*) showed that the following values for a should be assumed: —

Copper	2530	Platinoid	1173
Silver	1900	Iron	777.4
Aluminum	1873	Tin	405.5
Platinum	1277	Alloy (Tin 1, Lead 2) .	325.5
German Silver	1292	Lead	340.6

These values indicate the current in amperes required for fusing a cylindrical conductor of one centimeter in diameter, of each of the materials named. Still more recent data by the same author are com-

TABLE No. 60.

Giving the Sizes of Various Wires which will be Fused by a Given Current.

BY MR. W. H. PREECE.

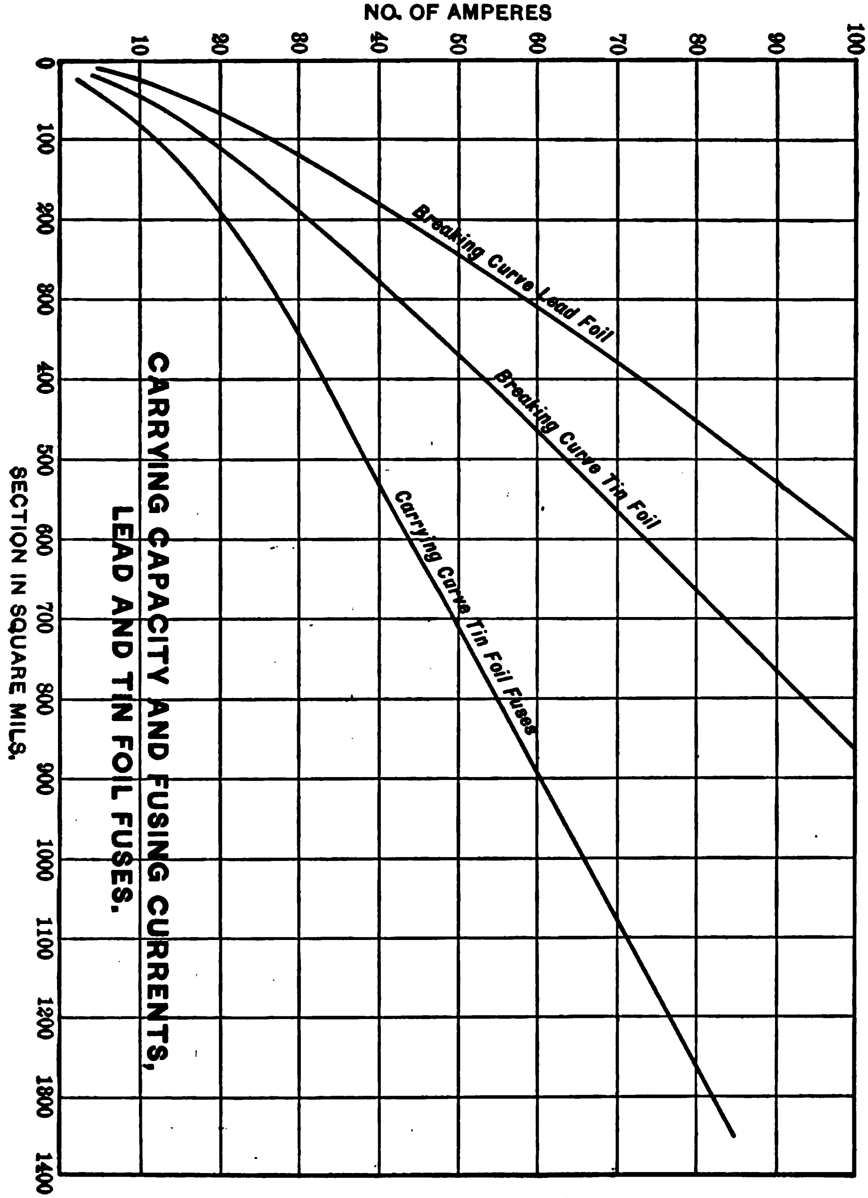
Current in Amperes.	Tin Wire.		Lead Wire.		Copper Wire.		Iron Wire.	
	Diameter Inches.	Approx. S. W. G.	Diameter Inches.	Approx. S. W. G.	Diameter Inches.	Approx. S. W. G.	Diameter Inches.	Approx. S. W. G.
1	0.0072	36	0.0081	35	0.0021	47	0.0047	40
2	0.0113	31	0.0128	30	0.0034	43	0.0074	36
3	0.0149	28	0.0168	27	0.0044	41	0.0097	33
4	0.0181	26	0.0203	25	0.0053	39	0.0117	31
5	0.0210	25	0.0236	23	0.0062	38	0.0136	29
10	0.0334	21	0.0375	20	0.0098	33	0.0216	24
15	0.0437	19	0.0491	18	0.0129	30	0.0283	22
20	0.0529	17	0.0595	17	0.0156	28	0.0343	20.5
25	0.0614	16	0.0690	15	0.0181	26	0.0398	19
30	0.0694	15	0.0779	14	0.0205	25	0.0450	18.5
35	0.0769	14.5	0.0864	13.5	0.0227	24	0.0498	18
40	0.0840	13.5	0.0944	13	0.0248	23	0.0545	17
45	0.0909	13	0.1021	12	0.0268	22	0.0589	16.5
50	0.0975	12.5	0.1095	11.5	0.0288	22	0.0632	16
60	0.1101	11	0.1237	10	0.0325	21	0.0714	15
70	0.1220	10	0.1371	9.5	0.0360	20	0.0791	14
80	0.1334	9.5	0.1499	8.5	0.0394	19	0.0864	13.5
90	0.1443	9	0.1621	8	0.0426	18.5	0.0935	13
100	0.1548	8.5	0.1739	7	0.0457	18	0.1003	12
120	0.1748	7	0.1964	6	0.0516	17.5	0.1133	11
140	0.1937	6	0.2176	5	0.0572	17	0.1255	10
160	0.2118	5	0.2379	4	0.0625	16	0.1372	9.5
180	0.2291	4	0.2573	3	0.0676	16	0.1484	9
200	0.2457	3.5	0.2760	2	0.0725	15	0.1592	8
250	0.2851	1.5	0.3203	0	0.0841	13.5	0.1848	6.5

TABLE No. 61.

Data Commercial Fuse Wire.

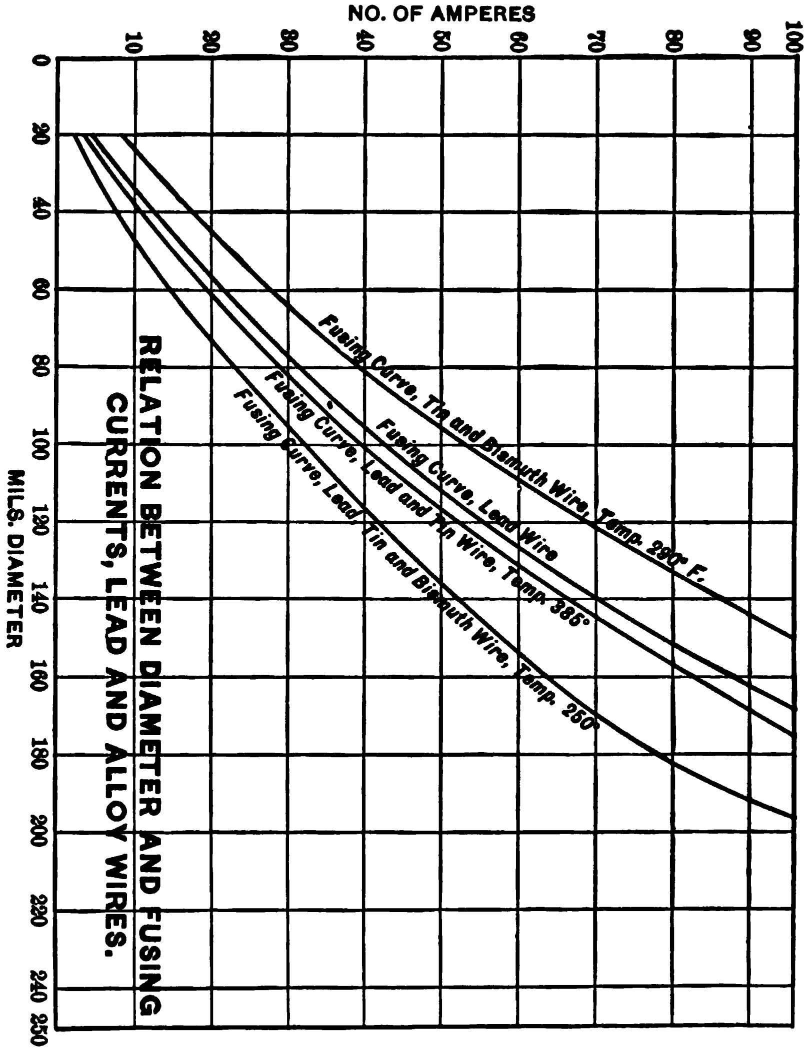
Rated Capacity. Amperes.	Fusing Current. Amperes.	Diameter in Thousandths of an Inch.	Sectional Area of Wire. Fractional Parts of Square Inch.	B. and S. Gauge. Nearest Number.	Rated Capacity. Amperes.	Fusing Current. Amperes.	Diameter in Thousandths of an Inch.	Sectional Area of Wire. Fractional Parts of Square Inch.	B. and S. Gauge. Nearest Number.
1	1.730	.010	.00007	30	40	54.10	.100	.00785	10
3	4.892	.020	.00031	24	50	63.11	.110	.00950	9
5	8.988	.030	.00070	20	60	81.08	.130	.01327	8
7	11.32	.035	.00096	19	70	90.61	.140	.01539	7
10	13.84	.040	.00125	18	80	100.50	.150	.01767	6½
15	19.34	.050	.00196	16	90	110.70	.160	.02010	6
20	25.42	.060	.00294	14	100	132.10	.180	.02544	5
25	32.04	.070	.00384	13	125	154.70	200	.03141	4
30	39.14	.080	.00502	12					

TABLE No. 62.


NO. OF AMPERES
10
20
30
40
50
60
70
80
90
100
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
SECTION IN SQUARE MILS.
CARRYING CAPACITY AND FUSING CURRENTS,
LEAD AND TIN FOIL FUSES.
Breaking Curve Lead Foil
Breaking Curve Tin Foil
Carrying Curve Tin Foil Fuses

piled in TABLE No. 60. The more fusible metals, such as lead, tin, or bismuth, or alloys of various proportions of them, are chiefly used for fuse wires; and great difficulty has been experienced in obtaining

TABLE No. 63.



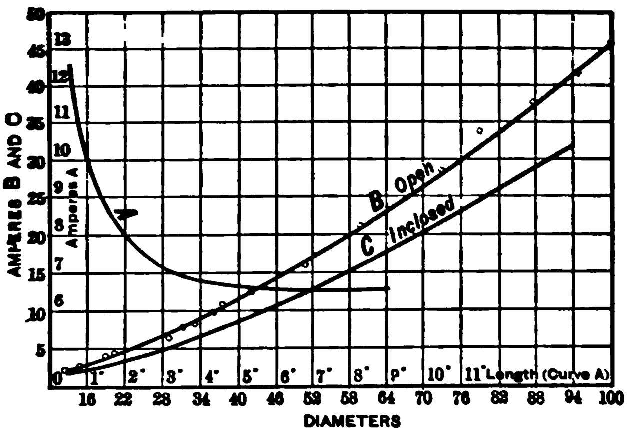
veritable ratings. As each manufacturer used different proportions of the alloying ingredients, designating them simply by trade numbers, and as usually different batches from the same maker possess

varying melting-points, owing to differences in composition, no rules could be given for fuse wires, beyond the arbitrary directions of the maker. Mr. Bathurst, in the *Electrical World*, has given the results of a recent investigation of the subject, deducing the results given in TABLES Nos. 61, 62, and 63.

387. The terminals to which the fuse is connected exercise a very marked effect upon the fusing-point, especially when the fuse is of short length. Some experiments made by Mr. C. P. Matthews of Cornell University indicate the relation which exists between the length of a fuse and the amount of current required to melt it.

TABLE No. 64.

Relation between Length and Carrying Capacity of Fuse Wires.



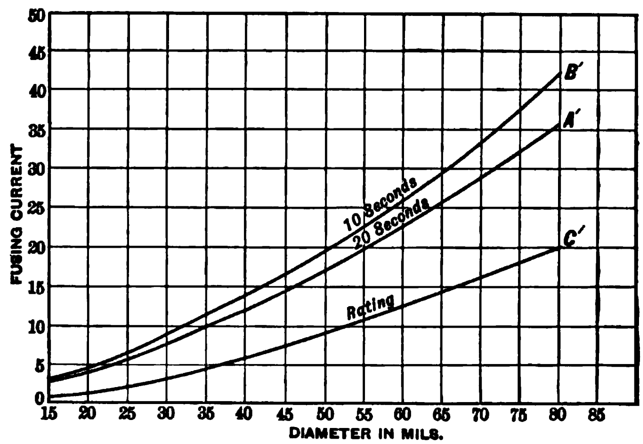
These relations are plotted in TABLE No. 64. For the curve A in this table the axis of X indicates the length of the fuse between terminals, while the axis of Y gives the fusing current in amperes for each length. For example, it will be noted that a fuse eight inches in length requires a current of 6.6 amperes, while a half-inch fuse of the same material, tin, carries a current of 12.5 amperes; thus showing a variation of more than 100 per cent in the carrying capacity of the fuse, produced simply by the effect of its length. The terminals act to conduct away the heat developed in the fuse by the passage of the current, and to dissipate the same, so that the heat energy developed is not allowed to act upon the safety device.

The curves B and C in TABLE No. 64 indicate the effect of inclosing the fuse wire in a glass tube, in order to concentrate the heat action. The inclosure acts to check radiation and convection, and so depresses the carrying capacity, causing the fuse to fail with a smaller current. Manifestly, fuses should be selected with reference to the kind of fuse block in which they are to be used, as a closed block would evidently require a larger fuse than one freely exposed to the atmosphere.

The effect of time during which the current acts, also exercises an important factor in the behavior of fuse wires. TABLE No. 64

TABLE No. 65.

Effect on Carrying Capacity of Fuse Wires of the Duration of the Current.



indicates the importance of this factor. Curve A′ indicates the behavior of a fuse under a current causing failure in twenty seconds, B′ under a current causing failure in ten seconds, while curve C′ is the rating of the fuse. The results of Mr. Matthews's experiments on tin-lead alloys are embodied in TABLES Nos. 66 and 67. The tests on the actual compositions given in TABLE No. 66 indicate that the mixture used quite closely followed the formula, — $I = ad^{\frac{3}{2}}$, from which, for the alloys given, the breaking current for various sizes of wire may be found. The fusing constant for unit wires of lead-tin alloys was determined and plotted in TABLE No. 67. From this curve, the current to fuse any alloy of lead and tin may be ascertained, and

by substituting in Formula (136) the limiting current deduced for any wire.

388. It has also been ascertained that the behavior of the fuse

TABLE NO. 66.

Carrying Capacity, Lead and Tin Fuse Wires.

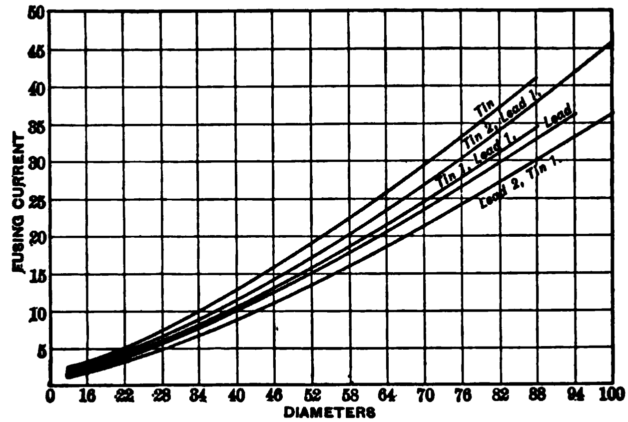
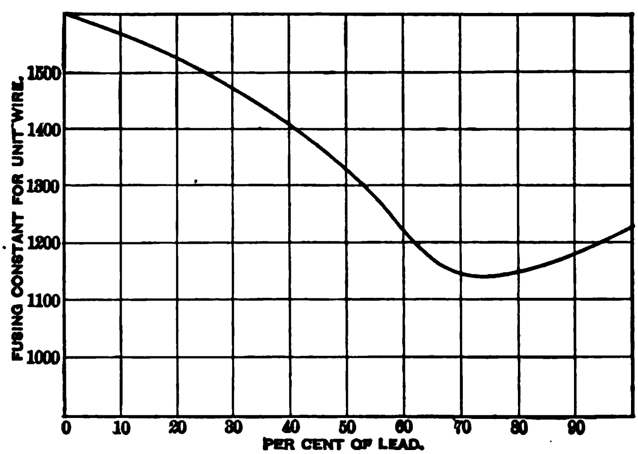


TABLE NO. 67.

Fuse Wire Curves, Lead and Tin Alloys.



wires under alternating currents appears to be different from that under a continuous current.

Experience has shown that the action of the alternating current seems to have a disintegrating effect upon the fuses, which causes

them to blow more readily after use, thus necessitating a constant renewal of the fuses, and supervision over the circuits. From investigation by Mr. Sturtevant of Cornell University, it would seem that the alternating current exercises an action upon the fuse wire, which causes some molecular change, probably making the wire crystallize, and causing the fuse to become brittle. As a result of this change, the fuses, after a short time, were found to fail with a lower current on an alternating circuit than caused them to yield when they were first introduced. These experiments indicate a grave objection to the use of fuse wires as protectors for alternating circuits, from the fact that if when first introduced they have only such a reasonable margin of safety as will save the circuit from injury from overloading, they will inevitably fail after a short use, thus adding largely to maintenance expense, and to the interruptions to the service. On the contrary, if the fuses are introduced of sufficient size in the beginning to have a long life, they will not protect the circuits from overloading in the early days of their introduction. Other investigators question these conclusions, and it is probable that additional experiment is needed to settle the question.

389. Fourth, The Heating of Insulated Cables. — Mr. Kennelley's[1] investigations, as given in a recent paper to the Association of Edison Illuminating Companies, have also extended to the calculation of the temperature that insulated, sheathed, or armored cables will probably attain when subjected to the passage of a current. The simplest case is that of a solid, cylindrical conductor having a radius r, surrounded by an insulated covering of radius r', over which a lead sheath or armor wire is laid, the whole cable being placed in such a location that the sheath is maintained at the constant temperature of the surrounding medium, as, for example, in the case of a submarine cable. The temperature attained by the conductor will depend upon the amount of energy transformed into heat in the core, and on the resistance offered to diffusion of this heat by the surrounding envelope of insulating covering. If the insulator had no thermal resistance, the heat would evidently be diffused and carried away as fast as it was produced. The transference of heat energy taking place between any two planes, separated by a uniform medium, is

[1] Paper before the Edison Illuminating Companies, August, 1893.

governed by laws similar to those that apply to electrical circuits. So the amount of heat passing depends on the difference of thermal potential between the planes, the geometrical form of the medium separating the planes, the specific thermal resistance of the medium, and the time during which the thermal potential acts. If θ is the temperature of the coolest plane, and t that of the warmest one, then $t - \theta$ is the thermal potential tending to cause heat energy to pass from one to the other, which, for strict accuracy, should be referred to the Centigrade scale of the air thermometer. If l be the distance between the planes, S the cross-section of the separating medium, and ρ' the specific thermal resistance, then,—

$$H = S\frac{(t-\theta)\,T}{\rho' l},$$

H being expressed in gramme calories. If l and T are units of length and time, then,—

$$H = S\frac{(t-\theta)}{\rho'}, \qquad \text{and} \qquad (t-\theta) = H\frac{\rho'}{S}. \tag{137}$$

The similarity between this formula and that for the current in an electrical circuit is evident. In fact, H can be termed the *Heat* current. For a cable having a conducting core of radius r and a coating of insulation r', Mr. Kennelly shows that the resistance to the heat current will be —

$$\frac{\rho'}{2\pi}\log_e\frac{r'}{r} = .159\,\rho'\log_e\frac{r'}{r}; \tag{138}$$

therefore, the heat current will be —

$$H = \frac{t-\theta}{.159\,\rho'\log_e\frac{r'}{r}} \qquad t-\theta = H\left(.159\,\rho'\log_e\frac{r'}{r}\right); \tag{139}$$

but $H = .24\, I^2R$, whence,—

$$I = \sqrt{\frac{t-\theta}{R\left(.159\,\rho'\log_e\frac{r'}{r}\right)}}. \tag{140}$$

To simplify, let —

$$A = t-\theta, \qquad \text{and} \qquad B = .159\,\rho'\log_e\frac{r'}{r};$$

then,

$$I = \sqrt{\frac{A}{RB}}. \tag{141}$$

390. From this formula, the carrying capacity of a submarine cable may be calculated when the geometrical dimensions, thermal resistance, and permissible core temperature are known. Unfortunately, data on specific thermal resistance are very meager; tests on a Siemens cable indicated the thermal resistance to be 750 units, while tests on cables buried in sandy soil give 50 as a mean for specific resistance for the earth. Additional data, so far as can be ascertained, are given, TABLE No. 68. With due regard to the preservation of insulation, the core temperature should never be allowed to rise over 60° to 65° C.; and as the temperature of the cable environment may reach 30° to 35° C., there remains a possible difference in temperature of 35° C.

TABLE No. 68.

Giving Specific Thermal Conductivity in C. G. S. Units.

NAME OF SUBSTANCE.	SPECIFIC CONDUCTIVITY.	NAME OF SUBSTANCE.	SPECIFIC CONDUCTIVITY.
Vulcanized Rubber . . .	.000098	Glass	.0005
Beeswax	.000087	Wood	.0003
Felt	.000087	Caoutchouc	.00041
Vulcanite	.000083	Gutta-percha	.00048
Cotton Wool	.000043	Sandy Loam	.008
Sawdust	.000123	Bricks and Cement . . .	.003
Sand	.000131	India Rubber	.0004
Paraffine	.000113	Sand with Air Spaces . .	.09

391. Conduit Cables. — Conduit cables differ from submarine cables to the extent that the sheath is not exposed to the cooling of a water circulation, and therefore the sheath more nearly approximates to the core temperature. In other words, the ground or conduit interposes an additional thermal resistance to the diffusion of heat. Calling the ground thermal resistance B', —

$$H = \frac{A}{B + B'}. \tag{142}$$

The value of B' is given by the formula, —

$$B' = .159\, \rho'' \log_e \sqrt{n^4 - n^2}, \tag{143}$$

in which ρ'' is the ground thermal resistance, and n the ratio of the depth of the center of the cable below the top of the soil, to the radius of the outside of the cable. For a conduit cable, then, formula (141) must include B', and will stand, —

$$I = \sqrt{\frac{A}{R\,(B + B')}}. \tag{144}$$

392. The Effect of Adjacent Cables. — When several cables occupy adjacent ducts of a conduit, or are buried in the same trench, the temperature attained by each will depend in part on the amount of heat it receives from its neighbors. Each cable may be regarded as a center of radiation, the surrounding soil being imagined as divided into a number of cylindrical layers of increasing diameter and decreasing temperature. The effect on neighboring cables may be ascertained by determining the temperature of the cylinder in which the affected cable rests. For this purpose, Mr. Kennelly gives the following TABLE, No. 69, the left-hand column giving the probable temperature to be found in the successive cylindrical layers in per cent of the *sheath* temperature of the cable at the center; while the right-hand column indicates the distance in centimeters of the layers from the center.

TABLE No. 69.

Temperature Relations of Neighboring Underground Cables.

Percentage of Sheath Elevation.	Horizontal Distance between Axes. Cm.	Percentage of Sheath Elevation.	Horizontal Distance between Axes. Cm.	Percentage of Sheath Elevation.	Horizontal Distance between Axes. Cm.
95	3.15	60	14.58	30	56.0
90	3.93	55	18.16	25	71.2
85	4.88	50	22.65	20	91.8
80	6.07	45	28.88	15	121.5
75	7.55	40	35.38	10	169.1
70	9.40	35	44.38	5	269.6
65	11.71				

393. Suspended Cables. — There only remains to consider heavily insulated sheathed cables suspended in air. Such cases are presented by cables that are in part on pole lines, or that are compelled to pass for some distance through a large vault or subway. The limiting current for an aerial cable is obtained from the formula, —

$$I = \sqrt{\frac{A}{R\left(B + \frac{1}{e}\right)}}, \tag{145}$$

in which e is the total heat emission per unit of area of the *sheath.* With due regard to the safety of the cable, the temperature of the sheath should not rise more than 20° or 30° above the atmosphere; for that range e may be assumed as sensibly constant, and its value deter-

mined from equations on page 287, and I determined by substitution in (145).

394. Concentric Cables.—The case of concentric cables, where one conductor is entirely surrounded by another, heat being evolved in both conductors, may be treated as an aggravated case of "*Adjacent Cables,*" and calculated accordingly. Large factors of safety should, however, be allowed, as many factors enter into the heating problem that are as yet not completely determined.

CHAPTER XIII.

CONDUCTORS FOR ALTERNATING CURRENTS.

Art. 395. General Considerations. — When alternating current circuits commenced to attain a commercial importance, they were proportioned according to Ohm's formula and Joule's formula, as given in the preceding chapter; but results experimentally determined indicated wide departures from these laws. Sometimes the current in the conductors was much less than that which would be expected from the electro-motive force and resistance of the circuit. In other cases, electro-motive forces absorbed by the circuits were far in excess of the product of the current and the resistance of the conductor; and the product of the current in amperes, and the electro-motive force in volts, failed to give the amount of energy in watts. Where more than one electro-motive force operated upon a circuit, the resulting electro-motive force was sometimes found to be greater than the algebraic sum of the electro-motive forces, and in other cases to be less. When branch circuits were used, the currents in the respective branches did not divide proportionally to the resistance, thus causing the anomaly of the sum of the currents in the branch circuits to be sometimes greater than the total current in the main circuit, and sometimes less. In other peculiar instances it was found that the electro-motive force at the terminals of long lines of mains was greater than the impressed electro-motive force given by the generator at the beginning of the circuit. It became, therefore, essential to more carefully study the distribution of current and potential in alternating circuits, in an endeavor to reconcile these anomalies with the laws of the conservation of energy.

396. In the constant current circuit, the office of the generator is to produce at one point in the conductor a constant and steady elevation of electrical potential. A familiar comparison may be made to the case of an air-blower, or fan. The office of the fan is to create a certain elevation of air pressure; and the quantity of the resulting current of air depends entirely upon the friction, or resist-

ance, of the pipes or ducts through which the air flows. If the pipe leading from the fan be entirely closed, the revolution of the fan-wheel simply increases the air pressure inside of the fan-casing; and as the inclosed air revolves with the moving wheel, little or no energy is consumed beyond that required to overcome the friction of the bearings, and no current of air is transmitted. By opening the air-pipe a current of air is immediately established, the quantity of which depends directly upon the resistance of the pipe. The energy of this air-current is directly proportional to the product of the pressure given by the fan and the quantity of the air flowing; and, consequently, the energy absorbed by the fan is in a like manner proportional to the aforementioned product. A dynamo operating upon an open circuit is occupied only in raising the electrical potential between the brushes; and, as the circuit is open, no energy is expended in the circuit, and no energy is absorbed by the dynamo, excepting that necessary to overcome the frictional resistances of the machine itself. On closing the circuit through a varying resistance, the energy delivered by the generator to the circuit is directly proportional to the product of the current and the electro-motive force, while the current is inversely proportional to the resistance; also the energy absorbed by the generator is correspondingly proportional to the same quantities. In the continuous current generator, the electro-motive force produced is a constant and unvarying quantity. With the alternating current generator, the electro-motive force is a constantly varying quantity, causing the current to vary in a like manner, in which variation may be found the origin of the previously mentioned anomalies.

397. Classification. — The apparent discrepancy between the distribution of current and potential in an alternating current circuit, and the apportionment as indicated by the laws of Ohm and Joule, may be conveniently, for investigation, divided into three parts: —

CASE 1. *Skin Effect.*

CASE 2. *Inductance.*

Sec. *a.* Effect of Inductance.

Sec. *b.* Effect of Mutual Inductance.

CASE 3. *Capacity.*

These divisions will now be separately treated.

398. CASE 1: **Skin Effect — Current Density.** — According to

the latest theories regarding the nature of electricity, it is believed that electrical disturbance is due to elastic reactions set up in the ether. In dielectric bodies the ether is, as it were, confined and prevented from moving. In conductors, on the contrary, the ether particles find themselves at liberty, and are more free to move under the stresses set up by electrical action. A conductor, therefore, may be simply regarded as a hole in the dielectric, through which the stresses set up between the ether particles can relieve themselves more freely. Assuming the truth of this supposition, it is evident that the dielectric, rather than the conductor, is worthy of interest and investigation. Thus, in the case of an alternating current, the dielectric is stressed first in one direction and then in the other. With a continuous current, however, the stress would always be in the same direction. In the latter case, the ether particles, finding themselves in the neighborhood of a conductor, would, so to speak, soak into the wire, and there relieved from the confining action of the dielectric, would be enabled to move and adjust themselves to the stress imposed by the generator. Evidently some time must be required for this action to take place. With an alternating current, however, the ether particles are alternately stressed first in one direction and then in the other; and if the reversals occur so frequently as to prevent the ether particles from penetrating the conductor, it is obvious that the interior of the wire will not be subjected to electric action, and will be of little or no service as a conductor. An analogy to this phenomenon may be obtained in alternately heating and cooling a body. Consider a round copper rod (one of the best conductors of heat) to be alternately plunged with great rapidity first into a furnace and then into a freezing mixture. The first effect of the furnace is to heat intensely the exterior of the rod, and by conduction all portions of the metal tend to assume the same temperature. This, however, requires time; and if, before the heat energy can proceed to the central portion of the rod, it be plunged into the freezing mixture, the effect of the furnace is annulled, and the conductor tends to become chilled. Thus, it is conceivable that, if the alternations be sufficiently rapid in proportion to the speed of conduction, the interior of the body could never be affected, no matter how intense the source of either heat or cold. So, with an alternating current, if the reversals are extremely rapid, the outer

layers of the conductor only are affected, and the electrical action commonly known as "A Current" is confined to the surface of the conductor. If the conductor be large and the reversals quite rapid, the exterior only plays a part in the transference of energy. Owing to this restriction of the current to the surface of the conductor, it is plain that the interior becomes of no value for the purpose of power transmission, and that calculations as to the resistance of the circuit must be based solely upon that portion of the cross-section that is affected at each reversal. Evidently the resistance is increased in proportion to the restriction of the conductor, and this in turn is proportional to the frequency of the reversals. The extra resistance entailed by lack of penetration of the energy into the body of the conductor is chiefly noticeable with wires or cables of large size, and may readily be practically obviated by using a stranded conductor, in which the interior strands are frequently brought to the surface, or one formed of strips, in which the various component parts of the conductor are brought into close proximity with the dielectric, in order that they may be more fully exposed to the penetrating influence of the energy. This phenomenon of increased resistance has been treated from a mathematical standpoint by Lord Kelvin, Lord Rayleigh, and Mr. Heaviside; but the mathematical discussion transcends the scope of this volume. Professor Gray,[1] in his *Absolute Measurements in Electricity and Magnetism*, shows that the effective resistance to rapidly alternating currents may, without sensible error, be represented by the ohmic resistance of a cylindrical shell of certain thickness, on the outside of the conductor, throughout which the current density is sensibly constant. The following, TABLE No. 70, gives the thickness of copper and iron shells to be assumed in calculating the apparent resistance of alternating circuits:—

TABLE No. 70.

Thickness of Shell on Cylindrical Conductors Affected by the Current in an Alternating Circuit.

FREQUENCY.	THICKNESS OF SHELL IN CM.	
	Copper.	Iron.
80	.719	.0976
120	.587	.0789
160	.509	.0691
200	.455	.0671

1 *Absolute Measurements in Electricity and Magnetism*, vol. ii., part i., p. 338.

A consideration of these data indicates the futility of employing thick wire or cable for high frequency currents, as the center of the conductor is valueless. Lord Kelvin shows that the increase in apparent resistance due to unequal current density, for the same wires, varies as the square root of the frequency, that is as $\sqrt{n}$. The following, TABLE No. 71, shows the factor for virtual or effective resistance for various periods:

TABLE No. 71.

Skin Effect Factors for Conductors Carrying Alternating Currents.

Diam. and B. & S. Gauge.	Frequencies.									
	15	20	25	33	40	50	60	80	100	130
2"	1.111	1.160	1.265	1.405	1.531	1.682	1.826	2.074	2.290	2.560
1¾"	1.072	1.114	1.170	1.270	1.366	1.495	1.622	1.841	2.030	2.272
1½"	1.042	1.064	1.098	1.161	1.223	1.321	1.420	1.610	1.765	1.983
1¼"	1.019	1.030	1.053	1.084	1.118	1.176	1.239	1.374	1.506	1.694
1⅛"	1.010	1.019	1.035	1.059	1.080	1.124	1.168	1.270	1.382	1.545
1"	1.005	1.010	1.020	1.038	1.052	1.080	1.111	1.181	1.263	1.397
¾"	1.002	1.002	1.007	1.014	1.016	1.028	1.040	1.066	1.100	1.156
½"	1.001	1.001	1.002	1.005	1.006	1.007	1.008	1.011	1.022	1.039
0000	......	......	1.001	1.003	1.005	1.005	1.006	1.010	1.015	1.027
000	......	......	......	1.001	1.002	1.002	1.005	1.007	1.010	1.017
00	......	......	......	......	1.001	1.001	1.002	1.004	1.006	1.010
0	......	......	......	......	......	......	1.001	1.002	1.005	1.008
1	......	......	......	......	......	......	......	1.001	1.002	1.005
2	......	......	......	......	......	......	......	......	1.001	1.002
3	......	......	......	......	......	......	......	......	......	1.001
4	......	......	......	......	......	......	......	......	......	1.000

399. CASE 2: **Inductance.**

SEC. *a.* — EFFECT OF INDUCTANCE.

Magnetic Field Due to Current. — If some iron filings be sprinkled on a glass plate placed over a small magnet, and the plate gently tapped to overcome the frictional resistance between the surface and the filings, the particles of iron are seen to arrange themselves along a series of lines that form closed curves extending from pole to pole of the magnet.

400. To Faraday is due the conception that the entire space surrounding any magnet is thus filled with "Lines of Magnetic Force," the filings on the plate merely serving to render the state of space adjacent to the magnet visible to the eye. In the C. G. S. system of units, a magnetic pole is defined as a magnet of such

strength as to repel an equal and similar magnet with a force of one dyne, when the two poles are placed one centimeter apart. Coulomb expressed the law of magnetic action by the equation —

$$F = \frac{mm'}{d^2}, \tag{146}$$

in which F is the mutual attraction or repulsion, m and m' the strengths of the two poles, and d the distance between them. This expression is equivalent to asserting that the force between the poles varies as the product of the pole strengths, divided by the square of the distance separating them. If both poles have the same sign, the product is positive; and, as repulsion exists, it is termed a positive quantity. Conversely, a negative sign would be applied to the force of attraction between two opposite poles. The strength or intensity of any magnetic field, at any point, is estimated by the effect which the field produces upon a unit positive magnetic pole placed at the point in question. Imagine a unit magnetic pole placed at any point in a magnetic field, and so disassociated from all matter as to be perfectly free to move. The direction in the field in which this hypothetical pole would then travel is termed the "Positive direction" of the lines of force; and the effort which the unit pole would exert in its motion is the measure of the magnetic strength, and is usually expressed by H. H varies from point to point in all magnetic fields, though in large dynamos the field is so strong that H is sensibly uniform through the space under consideration, both in direction and magnitude. If, in every square centimeter of a field, the unit pole be acted upon with a force of one dyne, there is said to be "one line of Magnetic Force per centimeter," and the field is said to have "one unit of Intensity." When the intensity is H, there are H lines of force to each square centimeter; thus the intensity of the magnetic field is conceived of as proportional to the number of lines of magnetic force passing through each square centimeter of surface perpendicular to the direction of the lines of force.

401. Suppose a sphere to be described about a unit magnetic pole, having a radius of 1 cm. The surface of this sphere contains 4π sq. cm. As, by definition, the unit pole emits one line of force through each square centimeter, 4π lines of force will emanate from a unit pole; and from a pole with the strength m, there will be $4\pi m$

lines of force. In air the number of lines is the same as the number of lines of magnetizing force, and for many other substances this proposition holds true.

402. Some elements possess the property of greatly augmenting the number of lines, which are then termed "Lines of Induction." This is notably the case with iron. The sum of all lines per square centimeter, normal to the direction of the lines, is termed "The Total Induction," and is denoted by B. In a non-magnetic medium $B=H$; but in one that is magnetic, $B > H$, and the ratio of B/H is called the permeability, and is symbolized by μ.

403. Faraday showed that every circuit carrying a current excites a magnetic field surrounding the conductor, in which the lines are closed circles inclosing the conductor. The sum of all the lines of force passing through the area inclosed by any electrical circuit is termed "The Total Induction of the Circuit," and, when the circuit is placed away from magnetic media, is directly proportional to the current.

According to the C. G. S. system, a unit current is one which, flowing in a circuit of 1 cm. radius, acts on a unit magnetic pole placed at the center of the circuit with a force of one dyne per square centimeter of length of the circuit. The ampere is one-tenth of the C. G. S. unit. If F be the number of lines threading a circuit, and I be the current, then F varies as I, or $F = LI$, in which L is termed the coefficient of inductance, and may be defined as the ratio of the total inductance to the current producing it. Thus, in a circuit of 2 cm. radius, carrying two units of current, there will be 25.12 lines of force linked with the circuit; for, by definition, each unit length of the conductor acts on a unit pole at the center with one unit or one line of force per unit of current. As the conductor is 4 cm. in diameter, the length of the circuit will be 12.56 cm.; and as there are two units of current flowing, there will be a total of 25.12 lines of force linked in the circuit.

In this case $F = 25.12$; $I = 2$; and hence, L will equal $\frac{25.12}{2}$ or 12.56. Thus, if the geometrical dimensions of the circuit be known, the current flowing, and the permeability of the surrounding medium, it is possible to calculate the coefficient L. For ordinary cases, L is constant, and will be so considered. If the current is a variable one, changing from time to time, the relation of the current and

the induction during any small interval of time is given by the equation —

$$\frac{dF}{dt} = L\frac{di}{dt}.$$

404. *Electro-Motive Force due to Varying Field.* — Faraday proved that, when a conductor is so moved in a magnetic field as to cut the lines of force, the conductor becomes the seat of an electromotive force directly proportional to the rate at which the lines are cut, and acting at right angles to the direction in which the conductor moves, so as to oppose, or obstruct, the motion of the conductor. From this, it is evident that to move a closed conductor in any magnetic field requires the expenditure of energy. Faraday further showed that, if the circuit be maintained stationary, and the magnetic field varied so as to increase or diminish the number of lines of induction, a similar result is obtained. Thus, at any instant, $e = -dF/dt$. In this equation, e symbols the electro-motive force developed at any instant, while the negative sign indicates that the direction of this electro-motive force is such as to oppose a change in the number of lines of induction that thread the circuit. A C. G. S. unit of *E. M. F.* is developed when there is a change in induction of one line per second; and as this quantity is too small for convenient use, the volt is 10^8 times the C. G. S. unit.

405. *Equation of Energy.* — In Chapter XI. it has been demonstrated by Ohm's law that, for circuits acted on by constant *E. M. Fs*, the current is

$$I = \frac{E - e}{R},$$

in which E is the electro-motive force of the generator, R the resistance of the circuit, and e any opposing *E. M. F.s.* When a variable current exists in a circuit, the preceding paragraphs show that there is *always* an *E. M. F.* set up, due to the inductance of the circuit, that *opposes* the *E. M. F.* of the generator, having a value numerically equal to —

$$e = \frac{dF}{dt}; \tag{147}$$

but,

$$\frac{dF}{dt} = \frac{Ldi}{dt};$$

hence,

$$e = L\frac{di}{dt}. \tag{148}$$

The total energy supplied in a time T to a circuit is EIT watts. By Joule's law, it is shown that I^2RT watts are transformed into heat and dissipated by radiation. Throughout an infinitesimal of time, any *E. M. F.* and any current may be considered constant. Also, by the principle of conservation of energy, the total energy expended in a circuit in any time must be equal to the total energy delivered to it. The energy equation is, therefore, for the time dt—

$$eidt = Ri^2dt + Li\frac{di}{dt}dt; \qquad (149)$$

dividing by idt,

$$e = Ri + L\frac{di}{dt}. \qquad (150)$$

In this equation, e is any instantaneous value of the *E. M. F.* impressed by the generator on the circuit; Ri is the *E. M. F.* expended in overcoming the ohmic resistance of the conductor; $L\frac{di}{dt}$ is the *E. M. F.* required to balance the counter *E. M. F.* set up in the circuit by the change initiated in the magnetic field by a constantly varying current.

406. *Expenditure of Energy.* — As the current variations are a consequence of the varying impressed *E. M. F.*, the counter *E. M. F.* of inductance is directly connected with, and is a function of, the impressed *E. M. F.* As the current varies from zero to a maximum I, the energy expended in heat during any complete current cycle is $\frac{1}{2}RI^2$, while that expended in the magnetic field is—

$$\int_0^I Lidi = \frac{1}{2}LI^2. \qquad (151)$$

As that portion of the energy represented by $\frac{1}{2}LI^2$ is intimately connected with the nature of the variations of the impressed *E. M. F.*, it is now necessary to closely study their phenomena.

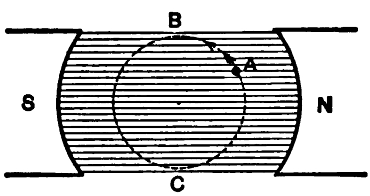


Fig. 274. Diagram of the Motion of a Conductor in a Magnetic Field.

Let the diagram, Fig. 274, represent a uniform magnetic field, in which the lines of force are indicated by straight lines extending between the poles N and S. Suppose A to be the cross-section of a closed conductor revolving uniformly in the direction of the dotted circle. The rate at which the conductor will cut the force

lines is seen by inspection to vary as the sine of the angle of rotation, being a maximum when the conductor is moving across the diameter NS, and a minimum when it is at right angles to this line. As the *E. M. F.* initiated is, at any instant, numerically equal to the rate of cutting, the *E. M. F.* is a sine function of the angle of rotation, passing, in every revolution, through two zero points at B and C, and then through a positive and negative maximum at the intersections of the diameter NS. As the rotation is uniform, the *E. M. F.* is a sine function of the time of rotation. In practice, the curves of alternating electro-motive forces are found to closely approach the preceding proposition; and even when the departure from a simple sine curve is considerable, by Fourier's theorem, it may be demonstrated that any *E. M. F.* curve may be expressed as

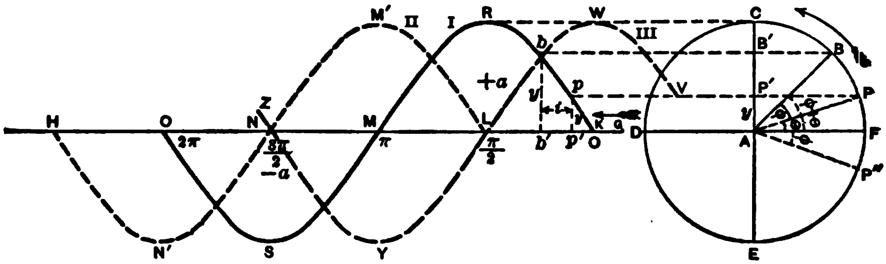


Fig. 275. Diagram of Harmonic Motion.

the sum of a series of terms, each of which is a simple sine function of the time of rotation. Consider now the curve of a sine function.

407. *Harmonic Motion.* — In the diagram of "Harmonic Motion," Fig. 275, suppose the line $\overline{AB}$ to be pivoted at the point A, and to revolve about, in the plane of the paper, this point as a center. The end of the line B will trace the circumference FCDE. Assuming F as the starting-point, the projection of $\overline{AB}$ at any time, on the diameter $\overline{CE}$, is $\overline{AB}$ sin BAF. When B is at F, the projection is zero. When B is at C, $\overline{AB}$ coincides with $\overline{AC}$, the projection being a maximum equal to the radius of the circle. If the diagram be viewed edgewise, by placing the eye in the plane of the paper somewhere in the prolongation of $\overline{AF}$, the point B will appear to travel uniformly backward and forward along the line $\overline{EC}$ from E to C. Such motion is termed "Harmonic." The radius of the circle is the amplitude, designated by a, while the time T required to make a

complete revolution is termed "The Period." Positive rotation is reckoned as counter-clockwise, in the direction of the arrow. The crank of a steam-engine, when viewed from a point in the line prolonging the piston-rod, or the motion of the bob of a clock pendulum, when seen from the point of suspension, are familiar examples of harmonic motion. If $\overline{AB}$ describes any angle ϕ in a time t seconds, the angular velocity denoted by ω is —

$$\frac{\phi}{t}, \qquad \text{or} \qquad \phi = \omega t,$$

where ω is the angle described in a unit of time; also, as the entire circumference is described in the time T, —

$$\omega = \frac{2\pi}{T}, \qquad \text{and} \qquad \phi = \frac{2\pi t}{T}.$$

The number of revolutions in one second is $1/T$, and is designated as the "Periodicity, or Frequency," usually denoted by n. The angular velocity may be expressed in terms of the frequency; that is to say, $\omega = 2\pi n$, and $\phi = 2\pi nt$. Reckoning time from F, when the projection of $\overline{AB}$ on $\overline{AC}$ is zero, and denoting the projection on $\overline{AC}$ by y, —

$$y = a \sin \phi = a \sin \omega t.$$

and y is a sine, or harmonic function of the time. If the time interval be reckoned from some other point, say P, then there is an angle θ between this point and the point of zero projection F. This angle is termed "The Angle of Epoch," and the angle $\phi + \theta$ is called the phase. In this case

$$y = a \sin (\phi + \theta);$$

or,

$$y = a \sin (\omega t + \theta). \tag{152}$$

408. When θ is positive, measured from F in the direction of the arrow, it is often called the "Angle of Advance." When it is negative, measured clockwise from F to some point P″, it is termed "The Angle of Lag." It is readily seen that when the angle of phase is zero, $\overline{AB}$ coincides with $\overline{AF}$, and the projection on y is zero. When the phase is 90°, and $\overline{AB}$ falls along $\overline{AC}$, the projection is a positive maximum, and $y = + a$. At 180° y is again zero, and $\overline{AB}$ coincides with $\overline{AD}$, and lastly at 270°, $y = - a$, a negative maximum $\overline{AB}$ coinciding with $\overline{AE}$. With every revolution this cycle is repeated. Conceive now that while $\overline{AB}$ is revolving about A, the paper be moved steadily and uniformly in a direction contrary to the arrow mark, while A remains stationary. The point A will trace a line

$\overline{GH}$, while the point B will trace a sinuous line KRMSO. As the motion of the paper is uniform, the distances along $\overline{GH}$ will represent intervals of time, while the vertical distances between $\overline{GH}$ and the points of the curve will represent successive projections of $\overline{AB}$ on $\overline{AC}$. In the diagram, the various elements of the curve are represented, as follows: —

Generating point B.
Circle of revolution FCDE.
Axis of time $\overline{GH}$ or axis of x.
Amplitude $a = \overline{AB} = \overline{AF} = \overline{KL} = \overline{LR}$.
Angle of advance θ = PAF = Angle of epoch.
Angle of lag $-\theta$ = FAP″.
Angle described in time $t = \phi$ = PAB.
Angle of phase $\theta + \phi$ = FAB.
Time of epoch Kp'.
Time of phase i + K$p' = Kb'$.

409. The point B in the diagram may represent the cross-section of any one of the armature conductors of the common dynamo, and so the path described by this and all other armature conductors will coincide with that in which B moves. As the *E.M.F.* initiated in each conductor is proportional to the rate at which the lines of the magnetic field are cut, and as this rate varies according to the sine of the angle of rotation, the sine curve KRMS is representative of the periodic variations of the *E.M.F.* set up in the conductor. As the *E.M.F.* is constantly varying, the current will be correspondingly periodic, and the locus of its curve will be a line similar to the curve KRMS.

410. *Average Values.* — If, in the equation —

$$y = a \sin (\omega t + \theta),$$

E or I be substituted for a, y becomes the value of the *E.M.F.* or current at any given instant.

$$e = E \sin (\omega t + \theta); \tag{153}$$
$$i = I \sin (\omega t + \theta). \tag{154}$$

In practical work the average values of these quantities are much more in demand than the above instantaneous values. As a sine curve is a succession of similar and equal positive and negative cycles, the average ordinate for any period, or succession of periods,

is algebraically zero; but as the latter half of each period is the same as the first with its sign reversed, the average ordinate for any half period will be the arithmetical mean ordinate. During a half period, indicated by $T/2$, a certain quantity Q of electricity will flow through the circuit, and the alternating current may be compared to a steady current which would deliver the same *quantity* of electricity throughout the same circuit, in the same time. If $\mathfrak{I}$ is this current. then —

$$Q = \mathfrak{I}\frac{T}{2} = \int_o^{\frac{T}{2}} idt; \qquad (155)$$

but,

$$i = I \sin(\omega t + \theta);$$

whence

$$\mathfrak{I}\frac{T}{2} = \int_o^{\frac{T}{2}} I \sin(\omega t + \theta)\, dt = \frac{IT}{\pi};$$

and

$$\mathfrak{I} = \frac{2\,I}{\pi} = .6369\,I. \qquad (156)$$

Thus it appears that a continuous current necessary to deliver in the same time the same amount of electricity as an alternating one, will have .6369 of the value of the maximum ordinate of the alternating current.

411. Alternating currents may also be compared to continuous currents by noting the relative thermal or chemical effects produced. Any current, whether alternating or continuous, in traversing a conductor evolves heat at a rate which is measured by $\mathfrak{I}^2RT$. The readings of a Cardew voltmeter are obtained by noting the heating effect produced in a long, even, high resistance wire. As the thermal effect is proportional to the square of the current, and as the current is proportional to the voltage, it is evident that the instrument really measures the mean square of all the instantaneous current values that occur while the measurement is being made; and if the relation of the mean square to the maximum value be known, the readings of the voltmeter will furnish the necessary data to calculate the *E.M.F.* curve. By Joule's law, the heat evolved is $\mathfrak{I}^2RT$; hence, during half a period, —

$$\mathfrak{I}^2R\frac{T}{2} = \int_o^{\frac{T}{2}} i^2Rdt; \qquad (157)$$

replacing i by its value from equation (154), —

$$\mathfrak{I}^2R\frac{T}{2} = RI^2\int_o^{\frac{T}{2}} \sin^2(\omega t + \theta)\, dt.$$

$$\mathfrak{I} = \frac{I}{\sqrt{2}} = .707\,I. \qquad (158)$$

Thus it appears that the arithmetical mean electro-motive force or current is less than that indicated by the Cardew voltmeter, or other similar instrument; for, from equation (156), the arithmetical mean is .6369; and from the preceding equation (158), the square root of the mean squares is .707, and the difference, .071, is about ten per cent. Having the voltmeter readings, the maximum electro-motive force may be obtained by multiplying the voltmeter value by 1.415.

412. *The Solution of the Energy Equation.*

A. CIRCUITS CONTAINING RESISTANCE AND INDUCTANCE.

Having thus considered the elementary properties of the curve of harmonic motion, the way is prepared for a general solution of the equation representing the balance of energy in an alternating current circuit. Referring to Fig. 207, assume t to be reckoned from F, and $\mathrm{E} = \overline{\mathrm{AB}}$, then, $e = E \sin \omega t$; also, as has been proved,—

$$e = Ri + L\frac{di}{dt};$$

hence,

$$Ri + L\frac{di}{dt} = E \sin \omega t; \tag{159}$$

transposing, and dividing by L,—

$$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}\sin \omega t. \tag{160}$$

This is a linear differential equation of the first order, of which the general type is,—

$$\frac{dy}{dx} + Py - Q = 0.$$

The solution[1] of such an equation is,—

$$y = e^{-\int P dx}\left[\int Qe^{\int P dx}dx + C\right].$$

Substituting the values of the coefficients derived from equation (159), and performing the integration indicated in the exponents of e,—

$$i = \frac{E}{L}e^{-\frac{Rt}{L}}\int e^{\frac{Rt}{L}}\sin \omega t\, dt + Ce^{-\frac{Rt}{L}}.$$

Integrating by the rules for exponential functions,[2] and reducing to simplest form,—

$$i = \frac{E}{\sqrt{R^2 + L^2\omega^2}}\sin\left(\omega t - \tan^{-1}\frac{L\omega}{R}\right) + Ce^{-\frac{Rt}{L}}.$$

[1] See Carr's *Synopsis of Pure Mathematics*, p. 472, art. 3110.

[2] Carr, *Synopsis of Pure Mathematics*, p. 325, art. 1998.

It can be shown[1] that the constant of integration which contains the exponential term applies to the circuit only for a minute period of time immediately succeeding the first application of the *E.M.F.* This term may therefore be disregarded in a consideration of the constant *régime* of an alternating current circuit; the equation therefore reduces to —

$$i = \frac{E}{\sqrt{R^2 + L^2\omega^2}} \sin\left(\omega t - \tan^{-1}\frac{L\omega}{R}\right). \qquad (161)$$

413. This equation indicates, —

First. An harmonic impressed *E.M.F.* in a circuit containing resistance and inductance produces a current that is a sine function of the periodic time.

414. *Second.* The current lags behind the *E.M.F.* by an angle of which the tangent is $L\omega/R$.

415. *Third.* When $\sin\left(\omega t - \tan^{-1}\frac{L\omega}{R}\right) = 1$, the current attains the maximum value, and—

$$I = \frac{E}{\sqrt{R^2 + L^2\omega^2}}. \qquad (162)$$

The quantity $L\omega$ is called "Reactance," and the quantity $\sqrt{R^2 + L^2\omega^2}$, the apparent resistance of the circuit, is denominated "Impedance," often symbolized by J, and will be more fully treated under the sections on "Graphical Methods." If this quantity be substituted for R in Ohm's formula when applied to alternating circuits, his equation will hold true.

416. *Fourth.* If $L = 0$, the equation reduces to $i = E \sin \omega t / R$ which accords directly with Ohm's law. Impedance, as deduced from this expression, causes the current to lag behind the impressed *E.M.F.*, and reduces its successive values.

417. *Fifth.* If $R = 0$, $i = \frac{E}{L\omega} \sin(\omega t - 90°)$, indicating that when the resistance is so small as to be negligible, the current cannot exceed $E/L\omega$, and then lags 90° behind the impressed *E.M.F.*

418. *Sixth.* If either R or L become indefinitely large, the current reduces to zero.

[1] Fleming, *Alternate Current Transformer*, p. 102.

SEC. *b*. — THE EFFECT OF MUTUAL INDUCTANCE.

419. If a circuit A is placed in such a manner as to be in close proximity to a second circuit B, that carries an alternating current, the varying magnetic field initiated by the B circuit will react on the A circuit and set up therein an *E.M.F.* If it be imagined that two circuits are so close together as to occupy the same space, it is evident the total induction of the B circuit will pass through the A circuit, and the *E.M.F.* set up in the A circuit will be equal to the inductance of the circuit, and may be represented by L_B. If the conditions be reversed, and the current be assumed in the A circuit, its inductance will act in a similar manner on the B circuit, with an effect to be measured by L_A. Now, if current flows in both circuits, each will react upon the other, proportionally to the current in each. In this hypothesis, the two circuits are assumed to be so close together that all the lines of force generated by each will be linked with the other, and the coefficient M of the mutual inductance may be defined as "The total induction, linked with both the circuits, divided by the sum of the currents in both circuits." If, as in the previous supposition, two circuits are supposed to coincide in space, it is evident that —

$$M < \text{ or } = L_A,$$
$$M < \text{ or } = L_B;$$

therefore,
$$M^2 < \text{ or } = L_A L_B.$$

and the maximum possible value of M is the square root of the product of the inductances.

B. THE GENERAL EQUATION OF ENERGY FOR MUTUALLY INDUCTIVE CIRCUITS.

420. For a simple circuit having resistance and inductance in series, the energy equation has two terms, Ri^2 denoting the portion transformed into heat, and $Li\frac{di}{dt}$ measuring the amount expended in the magnetic field. When there are two or more circuits in close proximity, a portion of the magnetic field created by each will be employed in inducing an *E.M.F.* in the other circuits, and may be measured by the product of the coefficient of mutual inductance and the current. Consider the case of two circuits A and B with impressed *E.M.F.s* E_A and E_B, with resistances R_A R_B, inductances

L_A L_B, currents I_A I_B, and a mutual inductance M, then for circuit A the energy equation is —

$$E_A dt = R_A I_A^2 dt + L_A I_A dI_A + M I_A dI_B; \tag{163}$$

and for B,

$$E_B dt = R_B I_B^2 dt + L_B I_B dI_B + M I_B dI_A; \tag{164}$$

adding,

$$(E_A + E_B)\,dt = (R_A I_A^2 + R_B I_B^2)\,dt + L_A I_A dI_A + L_B I_B dI_B + M\,(I_A dI_B + I_B dI_A). \tag{165}$$

The first term on the right-hand side of equation (165) is the heating effect, the second and third are the energies expended in inductance, while the fourth is that due to mutual reaction; but the second, third, and fourth terms form the exact differential of —

$$\frac{L_A I_A^2}{2} + \frac{L_B I_B^2}{2} + M I_A I_B,$$

and the equation for the circuits reduces to —

$$E_A + E_B = \tfrac{1}{2}\,(L_A I_A + L_B I_B) + M I_A I_B + R_A I_A^2 + R_B I_B^2. \tag{166}$$

An extension of the same process may be used when there are more than two circuits. Usually alternating circuits can be so designed and erected as to reduce mutually inductive effects to so small values that they may be neglected. Exceptions occur in the construction of dynamo machinery. Rarely more than two circuits are involved, and usually only one of the two is subjected to an impressed *E.M.F.* In such cases the presence of iron renders the introduction of the permeability factor μ essential. This subject will be further expanded in the sections on Graphical Methods.

421. *Coefficients of Inductance.* — Conformably with the C. G. S. system, inductances are lengths, and theoretically can be computed from the geometrical relations of the circuit. As the process of calculation is sometimes tedious, the most useful values are here appended. Several units have been in vogue for inductance. English and Continental electricians have been in favor of adopting the term "secohm" or "quad," being the equivalent of 1,000,000,000 cm. of length or an earth quadrant. In this country the term "Henry" is authorized for the same value. In many respects, inductances behave like resistances, absorbing from the circuit a certain amount of energy. Unlike resistance, this energy is not transformed into heat; but in some, at present unknown, manner, is stored in the mag-

netic field, and when the circuit is interrupted, appears in the form of the well-known extra current. In the following values, the circuits are assumed to be of non-magnetic material, and to be immersed in a non-magnetic medium, in which the permeability $\mu = 1$; when the composition of the circuit, or when the circuits, are adjacent to materials in which the permeability differs from the above values, the necessary permeability factor must be introduced in all of the formulæ. In the expressions, C. G. S. units are employed, l (length), d (distance), r (radius), all in centimeters; and the values obtained for the coefficients are likewise in the same unit. To reduce these values to "henrys," the value of L should be divided by 1,000,000,000. The currents also are in C. G. S. units, and must be changed where amperes are used, by multiplying by the proper value. Mutual inductance is symbolized by M, and self inductance by L (see p. 612).

422. *First.* **Inductance of a circuit of two long parallel copper wires of radius r, and interaxial distance d per unit of length**

$$L = l\left(.5 + 2 \log_e \frac{d}{r}\right).$$

When the length of the circuit is l, or half the length of the conductor measured around the loop, the total inductance is —

$$L = 2\,l\left(.5 + 2 \log_e \frac{d}{r}\right).$$

423. *Second.* Coil of a single layer; l = length of coil, n = number of turns, r = radius of coil.

$$L = \frac{4\pi^2 r^2 n^2}{l}.$$

424. *Third.* Coil of several layers; l = length of coil, R = radius of outer layer, r = radius of inner layer, n = number of turns.

$$L = \frac{4\pi^2 n^4}{3l^3}(R-r)(R^3-r^3).$$

425. *Fourth.* Coil in the form of a ring; A = radius of the ring, a = radius of the coil, n = number of turns.

$$L = 2\pi n^2(A - \sqrt{A^2 - a^2}).$$

426. *Fifth.* If a second coil be formed on the first, having m turns of wire, the mutual inductance is—

$$M = 2\pi nm(A - \sqrt{A^2 - a^2}).$$

427. *Sixth.* Two wires wound parallel to each other, having a length l, and radii r and r', and having a distance between centers of b.

$$L = 2l\left(.5 + \log_e \frac{b^2}{rr'}\right).$$

428. *Seventh.* Mutual inductance of two concentric coils; l = length, the outer one having m turns, the inner one n, and a radius of r.

$$M = \frac{4\pi^2 r^2 nm}{l}.$$

429. *Eighth.* Two circles each of a radius r, parallel to each other, and separated by a distance d, have a mutual inductance of—

$$M = 4\pi r\left[1 + \frac{3d^2}{16r^2} + \ldots\right]\log_e \frac{8r}{d} - 4\pi r\left[2 + \frac{d^2}{16r^2} + \ldots\right].$$

430. CASE 3: **Effect of Capacity.** — If any very carefully insulated conductor be connected to any source of electrical energy, a uniform distribution of potential rapidly takes place, all parts of the conductor presently arriving at that of the source of supply. Experiment and theory indicate that the change in the potential of the different parts of the conductor, necessary to accomplish the condition of potential equality with the source, is accompanied by the transfer of a certain quantity of electricity. If the potential of the source is greater than that of the conductor, the transfer is from the source to the conductor, while if the converse be true, the electrical movement is in the inverse direction. The amount of electricity which, under these circumstances, can be received by a body measures its "Capacity." In the C. G. S. system the capacity of a body is measured by the quantity of electricity that is absorbed during an increase of potential of one C. G. S. unit, between the body receiving the charge and that of its surroundings. The unit of capacity is the Farad, and is that amount of capacity which, when charged with one coulomb of electricity, will exhibit a difference of potential of one volt. So large a capacity as a farad exists only in imagination; for the capacity of the earth is only about .0007 farad, and that of the sun .076 farad. Practically, the Microfarad, or the one-millionth of a farad, is usually employed.

The capacity of any conductor depends upon its size and shape; and upon the size, shape, proximity, and state of insulation of neighboring bodies; and the nature of the dielectric that separates them.

Also, as electricity behaves as if it possessed elasticity, capacity is not a constant quantity, but depends upon the potential acting in the circuit. By increasing the surface of the conductor, and decreasing the distance separating it from neighboring conductors, capacity is augmented. Arrangements of conductors to attain the greatest possible capacity are called "Condensers," of which the common Leyden Jar, or a collection of metal plates separated by thin sheets of dielectric, are familiar examples. The capacity of a condenser may be determined from the formula[1] —

$$C = \frac{S}{4\pi d},$$

in which S is the area of the plates, d the distance between them, C being the capacity in electrostatic units.

431. Suppose a simple circuit containing a condenser in series with resistance. During the first few infinitesimals of time succeeding the establishment of difference of potential at any part of the circuit, electricity flows into the line and is absorbed by the condenser. During the time of charge, the difference of potential at the terminals of the condenser gradually rises until it equals that of the generator, and then the flow of current stops. A parallel example could be drawn by imagining a steam-boiler to be connected by a long pipe to a tank, or reservoir. On opening the valve the steam will flow into the tank until the pressure in the tank equals that of the boiler, when the flow will cease. Also, the higher the boiler pressure, the greater the quantity of steam that will be forced into the tank. When the pressure is equalized no further flow takes place, excepting a slight transfer necessary to compensate for condensation or leakage. As no electric circuit is perfectly insulated, the parallel is still closer, as after the condenser is charged there still remains a slight flow to compensate for poor insulation and dielectric leakage.

432. Now, consider the case of an alternating current circuit, under an harmonic *E.M.F.*, that with every cycle constantly varies between a plus and minus maximum. Throughout one part of the cycle the current is gaining strength from the minus maximum to the plus maximum, and the difference of potential at the terminals of the condenser is thereby being constantly increased. Throughout the

[1] See Barker's *Physics*, p. 560.

other half, as the impressed *E.M.F.* is decreasing, the condenser is discharging itself into the circuit, and returning some of the energy previously absorbed. . If, in the steam-boiler example, the pressure in the boiler be imagined to undergo a periodic variation, there would be backward and forward flow between the boiler and the tank, and the tank and boiler, in each cycle.

Throughout every period of any alternating circuit, a certain quantity of electricity is set in motion by the impressed *E.M.F.* Manifestly, if a condenser of sufficient size to absorb, under the potential of the impressed *E.M.F.*, this quantity of electricity, the presence of such a condenser in the circuit will not effect the apparent quantity of the current. By definition, capacity is a function of the acting *E.M.F.*, or in other words, the potential at the terminals of the condenser is proportional to the charge it contains. Hence, the potential at the condenser terminals in any such circuit is an harmonic sine function of the period of the circuit; and the condenser acts to introduce into the circuit an additional *E.M.F.*, of which the account must be taken in a consideration of current and potential distribution.

433. The Solution of the Energy Equation for Circuit with Capacity.

C. CIRCUITS CONTAINING RESISTANCE AND CAPACITY.

As the capacity of a condenser is the amount of electricity in one conductor, when there is a unit difference of potential between the pair of conductors forming the combination, the charge q at any other potential E will be —

$$q = CE, \tag{167}$$

C being the capacity of the condenser. The energy W expended in charging a condenser can be shown[1] to be $W = \frac{1}{2} q^2 / C$; differentiating, —

$$dW = \frac{qdq}{C}. \tag{168}$$

The total energy delivered to a circuit in a time dt is $eidt$. When there exist resistance and capacity but no inductance, this quantity must be spent in heat and in charging the capacity. The heat expenditure is measured by Ri^2dt. That stored in the condenser is plainly $\frac{qdq}{Cdt}dt$; hence, the energy equation is —

$$eidt = Ri^2dt + \frac{qdq}{Cdt}dt. \tag{169}$$

[1] Barker's *Physics*, p. 566.

With a current i flowing for a time dt, a quantity of electricity idt will pass into the condenser, hence —

$$idt = dq, \qquad \text{or} \qquad q = \int idt; \tag{170}$$

therefore,

$$eidt = Ri^2 dt + \frac{1}{C} idt \int idt;$$

dividing by idt, —

$$e = Ri + \frac{1}{C}\int idt, \tag{171}$$

an equation resembling that applying to circuits containing resistance and inductance. Differentiating to get rid of the sign of integration, and transposing, —

$$\frac{di}{dt} - \frac{1}{R}\frac{de}{dt} + \frac{i}{RC} = 0.$$

Remembering that $e = E \sin \omega t$, and employing a similar method of solution to that indicated on p. 324, and neglecting the constant of integration, —

$$i = \frac{E}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}} \sin\left(\omega t + \tan^{-1}\frac{1}{RC\omega}\right). \tag{172}$$

434. This equation indicates —

First. That an harmonic impressed *E.M.F.* in a circuit containing resistance and capacity gives rise to a current that is an harmonic sine function of the periodic time.

435. *Second.* The current is in *advance* of the *E.M.F.* by an angle of which the tangent is $1 / RC\omega$.

436. *Third.* When $\sin(\omega t + \tan^{-1} 1 / RC\omega)$ becomes unity, the current attains its maximum value, and —

$$I = \frac{E}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}}. \tag{173}$$

The quantity $\sqrt{R^2 + 1 / C^2\omega^2}$ is the apparent resistance of the circuit, termed "Impedance," often symbolized by J, and will be more fully treated in the sections on "Graphical Methods." If this quantity be substituted for R in Ohm's formula, the equation will hold true when applied to alternating currents. $1 / C\omega$ is the "Reactance of the Circuit."

437. *Fourth.* When C diminishes to 0, i becomes 0. Such a condition obtains when the size of the conductor is indefinitely decreased, or the thickness of the dielectric indefinitely increased.

438. *Fifth.* If $R = 0$, the equation reduces to

$$i = \frac{E}{1 / C\omega} \sin(\omega t + 90^\circ) = C\omega E \sin(\omega t + 90^\circ), \tag{174}$$

showing a current 90° in advance of the impressed *E.M.F.*, with a maximum value numerically equal to $C\omega E$. If the resistance of the circuit be so small as to be negligible, the above condition is attained when the condenser is short-circuited.

439. *Sixth.* If R increases to infinity, i reduces to 0. but if C increases, $\tan^{-1}\frac{1}{CR\omega}$ decreases, and at $C = \infty$ the formula becomes $i = E / R \sin \omega t$, thus reducing to Ohm's equation, with the current in phase with the impressed *E.M.F.* Such a condition occurs when the thickness of the dielectric is reduced to zero, and the condenser plates touch each other. The interpretation of this result is found in the statement that under such circumstances no charge, no matter how large, can produce any difference of potential between the plates. Making the condenser infinitely large is equivalent to removing it from the circuit. Compare these deductions with those derived from equation (161).

The symbol ω has been employed as an abbreviation of $2\pi n$; but this quantity is the distance traveled by the generator point B (see Fig. 275) in one second of time, when the radius of the generating circle, or amplitude, is unity. When the amplitude has any other value, it must be introduced into the above expression. As both inductance and capacity are expressed in units of length, the expressions for the reactance $L\omega$, or $2\pi nL$, and $1 / C\omega$, or $1 / 2\pi nC$, are distances. If L and C be thought of as the radii of the generating circle, $L\omega$ and $1 / C\omega$ are values of the speed at which the generating point is traveling. Mr. Kennelly has very aptly termed these values the "Inductance Speed" and the "Reciprocal of the Capacity Speed."

D. THE ENERGY EQUATION FOR CIRCUITS CONTAINING RESISTANCE, INDUCTANCE, AND CAPACITY.

440. In the energy equation for a circuit with resistance and inductance, —

$$i = \frac{E}{\sqrt{R^2 + L^2\omega^2}} \sin\left(\omega t - \tan^{-1}\frac{L\omega}{R}\right).$$

Suppose $L\omega$ to be equal to $L'\omega + L''\omega$ then —

$$i = \frac{E}{\sqrt{R^2 + (L'\omega + L''\omega)^2}} \sin\left(\omega t - \tan^{-1}\frac{L'\omega + L''\omega}{R}\right);$$

now, for $L''\omega$ substitute $- 1 / C\omega$, and —

$$i = \frac{E}{\sqrt{R^2 + \left(L'\omega - \frac{1}{C\omega}\right)^2}} \sin\left[\omega t + \tan^{-1}\left(\frac{1}{CR\omega} - \frac{L'\omega}{R}\right)\right]$$

441. This equation indicates —

First. That both current and charge are simple harmonic functions, and may either lag behind the impressed *E.M.F.* or be in advance of it, depending as to whether $L'\omega >$ or $< 1 / C\omega$, the algebraic sum of these quantities determining the tangent of the angular relation of the current and impressed *E.M.F.*

Second. If $L'\omega = 1/C\omega$, the two quantities neutralize each other, and the current is in phase with the *E.M.F.*; the equation then reduces to —

$$i = \frac{E}{R} \sin \omega t.$$

It is thus evident that a judicious relation of inductances and capacities may be employed to adjust the angular relation of current and *E.M.F.* in any desired fashion.

Third. When the sine becomes unity, the maximum value of the current is reached, as —

$$I = \frac{E}{\sqrt{R^2 + \left(L'\omega - \frac{1}{C\omega}\right)^2}}.$$

The quantity —

$$\sqrt{R^2 + \left(L'\omega - \frac{1}{C\omega}\right)^2}$$

is the "Impedance of the Current," and behaves like a resistance; for when this quantity is substituted for R in Ohm's formula, it applies perfectly to alternating circuits. The quantity $L'\omega - \frac{1}{C\omega}$ is the "Reactance," to be more fully treated in the paragraphs on "Graphical Methods."

Fourth. Either R, L, or C may vary from 0 to ∞, and resulting current determined by the principles already indicated as applying to the limits of these quantities.

Fifth. In a circuit containing resistance, inductance, and capacity, the impressed *E.M.F.* is expended in balancing three quantities: the heat losses, measured by RI^2; the reactance due to inductance, estimated by $L\omega I$; and that due to capacity, equal to $I/C\omega$. It is easy to see that $I/C\omega$ can have such a value as to cause this component of the *E.M.F.* to exceed, numerically, the impressed *E.M.F.*

442. While the preceding equations demonstrate mathematically the behavior of a circuit under the action of a periodically varying *E.M.F.*, it may aid the mental picture one forms of what is believed to take place to reason something as follows:

To produce a current an *E.M.F.* is necessary, so the first thing that happens is the application of the *E.M.F.* As the current is the result, the *E.M.F.* must antedate it, though the time interval between them may easily be believed to be less than any assignable quantity, or zero.

The establishment of a current is accompanied by the creation of a magnetic field in the space about the conductor, whose lines of force as they come into being cut the conductor aud set up another *E.M.F.* By Art. 404 this *E.M.F.* of self-induction, the inductive reactance, or the inductance, as it is variously called, is opposed to the original *E.M.F.*; hence the applied *E M.F.* must do two things: it must drive the current against the ohmic resistance and it must overbalance the counter *E.M.F.* set up by magnetic field that accompanies the current; so at each instant the impressed *E.M.F.* must be as much greater than that demanded by Ohm's Law as may be needed to overbalance the *E.M.F.* of self-induction. Numerically the latter is $2\pi n \,.\, LI$.

Fig. 275 shows the impressed *E.M.F.* to vary along the line *ORO.* If the *E.M.F.* varies, the current will vary; if part of the applied *E.M.F.* is expended in balancing the counter *E.M.F.*, the maximum current cannot occur at exactly the same time as the maximum *E.M.F.* ordinate, but will be enough later to allow the impressed *E.M.F.* to grow sufficiently to overcome both the ohmic resistance and inductance. Therefore the current is said to lag behind the *E.M.F.*

As the number of lines of force which appear or disappear in any interval of time depends upon the time rate of change of the current during that interval, the intensity of the magnetic field created by the current will be greatest when the current is changing most rapidly, or at points *O*, *M* and *O* when the curve *OMO* represents current. Hence a curve which represents the *E.M.F.* of self-induction will have its maximum ordinate at points *O*, *M* and *O* when current ordinates are zero, and the inductive *E.M.F.* is said to lag 90 degrees behind the curreut, or to be in quadrature with it. As the inductance is due to the magnetic field created by the current, the ordinates of the *E.M.F.* curve will, at every point, be proportional to the current curve but in opposition, and the inductance will tend to retard the growth of the current when the latter is increasing, and augment it when it is decreasing.

Consider a circuit containing capacity. Under an impressed *E.M.F.* current will flow into the condenser until (by Arts. 431 and 432) the accumulating charge causes the *E.M.F.* across the terminals of the condenser to balance the *E.M.F.* applied to the circuit that tends to charge the condenser. Then the current will cease to flow. When the ordinate representing the *E.M.F.* over the terminals of the condenser is a maximum, the current ordinate is zero, and vice versa; so the current and condenser *E.M.F.* ordinates are 90 degrees apart.

Thus it has been shown that the maximum *E.M.F.* of inductive reactance and the maximum *E.M.F.* over the capacity occur when the current ordinate is zero, and also when the *E.M.F.* of inductive reactance is increasing the current is decreasing and conversely; obviously these two *E.M.F.*'s are 180° apart. The effect of capacity is therefore to introduce a negative reactance. This is often called "*Condensance.*"

443. **Growth of Current.**—There are some noteworthy consequences of the preceding relationships: Suppose an *E.M.F.* be applied to circuit containing both resistance and inductance, the full flow of current cannot instantly take place, but a sensible interval of time must be allowed for the current to grow. At the moment of the application of the *E.M.F.* there is no current. Assume time to be measured from this instant; required the amount of current at any time, *t* seconds thereafter.

From Art. 405,

$$e = Ri + \frac{Ldi}{dt};$$

$$(e - Ri)dt = Ldi;$$

$$dt = \frac{L}{e - Ri}di.$$

Integrate to any value of t and solve for I, which is then the current value t seconds after application of the $E.M.F.$,

$$I = \frac{E}{R}(1 - E^{-\frac{Rt}{L}}).$$

This equation shows that the current gradually rises along a logarithmic curve after the $E.M.F.$ is applied. Theoretically it will never reach its ultimate value; practically $E^{-\frac{Rt}{L}}$ becomes negligible after a very small interval of time. The quantity $\frac{L}{R}$ is known as the time constant of the circuit, and the larger this ratio the greater the time consumed by the current in reaching its (approximate) full value. As a numerical illustration, assume a circuit in which

$$E = 100 \text{ volts},$$
$$R = 10 \text{ ohms},$$
$$L = .2 \text{ henrys},$$

to find the curve representing current growth. Substituting these values the preceding formula becomes

$$I = 10(1 - E^{-50t}),$$

and the relation between t and I is as follows:

t in seconds.	I in amperes.	t in seconds.	I in amperes.
.002	1.00	.037	8.00
.004	2.00	.052	9.00
.007	3.00	.060	9.25
.010	4.00	.070	9.40
.014	5.00	.080	9.65
.020	6.00	.090	9.85
.027	7.00	.100	9.90

By the same process of reasoning it can be shown that when the $E.M.F.$ is interrupted the current does not immediately cease, but

continues to flow for a perceptible interval of time, gradually decaying along a logarithmic curve whose ordinates are complementary to those shown above. Similarly the growth and decay of the current in circuit containing capacity can be deduced, and are found to consist of a corresponding logarithmic curve.

444. Power in A.C. Circuits.—It has been shown that the power transmitted by a *D.C.* circuit is measured by the product of the applied *E.M.F.* and the current or *E.I.*, but in an alternating current circuit containing either inductance or capacity, or both, such a rule needs qualification. As a portion of the applied *E.M.F.* is occupied in counterbalancing the reactive *E.M.F.* (that of inductance or condensance), the product of the applied *E.M.F.* and current gives results which are *too large* by such a proportion of the impressed *E.M.F.* as is occupied in overcoming the reactive *E.M.F.* An inspection of Fig. 275 shows that that fraction of the impressed *E.M.F.* which accompanies the current and is employed in overcoming the ohmic resistance, corresponding to the *E.M.F.* of a *D.C.* circuit, is proportional to the cosine of the angle of lag or lead. This is often called the *Power Electro-Motive-Force;* so if E' symbolizes the power electro-motive-force,

$$E' = E \cos \theta,$$

and by using this value the power conveyed by an alternating current can at once be found, and is $E'I \cos \theta$.

$$\text{The ratio } \frac{EI}{E'I}$$

is the relation between the apparent watts (the volt-amperes) and the true watts or actual power delivered, and is usually called the *Power Factor.* This leads to an easy method of measuring the angle between the current and applied *E.M.F.* for

$$\frac{EI}{E'I} = \cos \theta,$$

and all that is necessary is to measure the volt-amperes (with a voltmeter and an ammeter) and simultaneously the true watts (with a watt-meter), take the quotient, and look up the angle in a table of natural cosines. (see page 610).

445. Resonance.—The important lesson is that the relative amounts of inductance and capacity exercise a most powerful influence on an alternating-current circuit, and the engineer has it in his power to balance one against the other to almost any desired extent. It is impossible theoretically to do more than neutralize an inductance by a capacity, or conversely, thus leaving the circuit free from all impedance save that of ohmic resistance. Practically even this result cannot be fully achieved, for no inductance is completely free from ohmic resistance, and there is no known dielectric for a condenser that is exempt from some waste of energy in an at present mysterious molecular action, so the *E.M.F.s* of inductance and condensance can never be exactly in quadrature with the current, but lag or lead by angles which are a little less than 90 degrees, so all the energy stored in the magnetic field and the condenser is never fully returned to the circuit. But when inductance and capacity approximately neutralize each other some very startling results may take place, which from analogy to a similar acoustic phenomenon are termed *Electrical Resonance.*

Electric transmission is believed to take place by a series of waves; hence every circuit can have a natural period of vibration of its own. If an *E.M.F.* be applied to a circuit for an instant, a wave moves along it, is reflected from the ends, and travels backward and forward until the impressed energy is finally frittered away as heat. An analogous phenomenon can be observed by striking a long wooden rod with a hammer; a pulse will travel backward and forward along the rod, being successively reflected from each end until the energy delivered by the blow is expended in sound and heat. The period of vibration of the rod depends upon its elasticity and density. The electrical analogues of friction, inertia, and elasticity are resistance, inductance, and capacity. Now if an *E.M.F.* is applied to a circuit, a wave is started which will travel backwards and forwards in such a time as is determined by the natural period of vibration. If the impressed *E.M.F.* is an alternating one, and if its periodicity is the same as the natural period of the circuit, every wave of the applied *E.M.F.* will coincide with a natural wave and tend to augment it. The operation is exactly the same as that of a swing. By means of a single push the swinger attains but a small

amplitude, while a series of properly timed repeated impulses will soon carry him through a long arc.

In circuits having inductance and capacity in series the natural period of vibration is closely given by the formula $T = \frac{2}{1000}\sqrt{LC}$, in which T is the time of one vibration in seconds, L is in henrys, and C in microfarads. If in any circuit the frequency of the generator is such as to closely agree with the natural period, there may be an enormous rise in voltage, for if it were not for resistance, which dissipates some energy in heat during each swing, there would be no limit to this cumulative effect. The relation between initial voltage and that due to resonant effect can be approximately found by the expression $E' = \frac{nLE}{R}$, in which E' is *E.M.F.* of resonance and E the applied *E.M.F.*, while n, L, and R are respectively the frequency, inductance, and resistance. In series circuits there may also be localized elevations of voltage, say over the terminals of the inductance, or the condenser. Such perturbations are easily calculated, for the drop in voltage over any part of the circuit is equal to the reactance multiplied by the current, and must always be reckoned with, and insulation designed not merely to resist the normal difference of potential, but sufficient to withstand the localized increase in voltage.

446. A numerical may illustrate more concretely. Assume a circuit in which

$E = 1000$ volts. $\qquad 2\pi nL = L\omega = 2\times 3.1415\times 60\times .2 = 75.4$ ohms.

$n = 60$ cycles $\frac{1}{2\pi nC} = \frac{1}{C\omega} = \frac{1}{2\times 3.1415\times 60\times .000033} = 78.125$ ohms.

$R = 30$ ohms.

$L = 200$ millihenrys $= .2$ henrys.

$C = 33$ mf. $\qquad = .000035$ farads.

Suppose R, L, and C to be capable of any desired arrangement.

1. *Resistance only.*—If $E = 1000$ is the effective *E.M.F.* (as by

voltmeter), the maximum *E.M.F.* by which the insulation is stressed is, by Art. 411.

$$1000 \times \sqrt{2} = 1414 \text{ volts.}$$

The average *E.M.F.* is, by Art. 411,

$$1414 \times .6369 = 900 \text{ volts.}$$

The current will be $I = \dfrac{1000}{30} = 33.33$ amperes, and as L and C are zero, the current is in phase with the *E.M.F.*, and the power transmitted is $EI = 33300$ watts.

2. *Inductance only.*—The current will be

$$I = \frac{E}{2\pi nL} = \frac{1000}{75.4} = 13.26 \text{ amperes.}$$

3. *Capacity only.*—The current will be

$$I = \frac{E}{\dfrac{1}{2\pi nC}} = E \times 2\pi nC = 1000 \times .0128 = 12.8 \text{ amperes.}$$

In Nos. 2 and 3 $R = 0$, hence

$$\tan\theta = \frac{\text{reactance}}{\text{resistance}} = \frac{\text{reactance}}{0} = \infty,$$

and the current is 90° away from the *E.M.F.* In No. 2 the reactance is positive, hence the current lags; in No. 3 it is negative and the current leads. In both $\cos\theta$ is 0 and the power delivered is 0.

4. *Resistance and Inductance.*—The current will be

$$I = \frac{E}{\sqrt{R^2 + (L\omega)^2}} = \frac{1000}{\sqrt{30^2 + 75.4^2}} = 12.28 \text{ amperes.}$$

Tan θ (angle between *E.M.F.* and current)

$$= \frac{\text{reactance}}{\text{resistance}} = \frac{75.4}{30} = 2.513 = 68°\ 18',$$

and current lags.

Cos $\theta = .3697$, and the power transmitted is

$$EI \cos \theta = 1000 \times 12.28. \times .3697 = 4540 \text{ watts.}$$

The drop over the resistance is $12.28 \times 30 = 368.4$ volts.

" " " " inductance is $12.28 \times 75.4 = 925.9$ volts.

5. *Resistance and Capacity.*—The current will be

$$I = \frac{E}{\sqrt{R^2 + \left(-\frac{1}{C\omega}\right)^2}} = \frac{1000}{(30^2 + (78.125)^2} = 11.95 \text{ amperes.}$$

Tan θ (angle between *E.M.F.* and current)

$$= \frac{\text{reactance}}{\text{resistance}} = \frac{-78.125}{30} = 2.6042 = 69^\circ,$$

and current leads.

Cos $\theta = .35837$, and power transmitted is

$$EI \cos \theta = 1000 \times 11.95 \times .35837 = 4282 \text{ watts.}$$

The drop over the resistance is $11.95 \times 30 = 358.5$ volts.

" " " " capacity is $11.95 \times 78.125 = 933.4$.

6. *Resistance, Inductance, and Capacity.*—The current will be

$$L = \frac{E}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} = \frac{1000}{\sqrt{30^2 + (75.4 - 78.125)^2}}.$$

$$= 33.11 \text{ amperes.}$$

$$\text{Tan } \theta = \frac{\text{reactance}}{\text{resistance}} = -\frac{2.725}{30} = -.0908 = 5^\circ\ 12',$$

and current leads.

Cos $\theta = .99588$, and power transmitted is

$$EI \cos = 1000 \times 33.11 \times .99588 = 32973 \text{ watts.}$$

The drop over the resistance is 33.11 × 30 = 993.3 volts.
" " " " inductance is 33.11 × 75.4 = 2496.4 volts.
" " " " capacity is 33.11 × 78.125 = 2586.7 volts.

In this case the potential difference over the condenser is over two and a half times the applied *E.M.F.*

447. Graphical Methods. — While the preceding paragraphs have given an outline of the algebraic treatment of alternating current circuits essential to an elementary conception of the subject, the same problems may be handled geometrically by graphical methods. In many cases, these methods are far simpler than analytical solutions, and always present the advantage of appealing directly to the eye in such a manner as to insure immediate detection of error. *Electrographics* has already received considerable attention from many eminent electricians,[1] to which the reader is referred for more detailed descriptions. As alternating current problems are most conveniently handled by the use of vectorial algebra, it is advisable to define the elementary uses of vectors before proceeding to the consideration of the problems.

448. Vector Quantities. — When the symbols of ordinary algebra are assigned a definite meaning, they usually become *scalar* quantities, that is, quantities which simply have a numerical value, or are mere numbers. When dealing with geometrical magnitudes, it is not only necessary to consider the numerical value of various lines, but also to consider the *direction* of each line. A vector quantity, therefore, is one in which the direction of the quantity is considered as well as its scalar magnitude. Direction is considered as positive when it is reckoned upwards, and negative when it is downwards from a horizontal base line. Right-handed rotation is negative, and left-handed positive. Suppose $\overline{AB}$, No. 1, Fig. 276, "Diagram of Operations on Vectors," to be a straight line of two units in length, inclined at an angle of 35° to the base line $\overline{AC}$. Let $\overline{DE}$, No. 2, be any other straight line inclined at an angle of 75° to the same base and of four units in length. These lines are plain vectors, of which the scalar magnitudes are, respectively, two and four units.

[1] See Fleming's *Alternating Current Transformer*; Blakesley's *Alternating Currents*; Kapp's *Dynamos, Alternators, and Transformers*; Gerard's *Leçons sur L'Électricité*; Kennelly *Trans. A. I. E. E.*, April, 1893, and *Electrical World*, vol. 22, p. 306; vol. 23, p. 17; Rimmington, *Electrical Review* for 1893, p. 664; Emmett's *Alternating Current Wiring*.

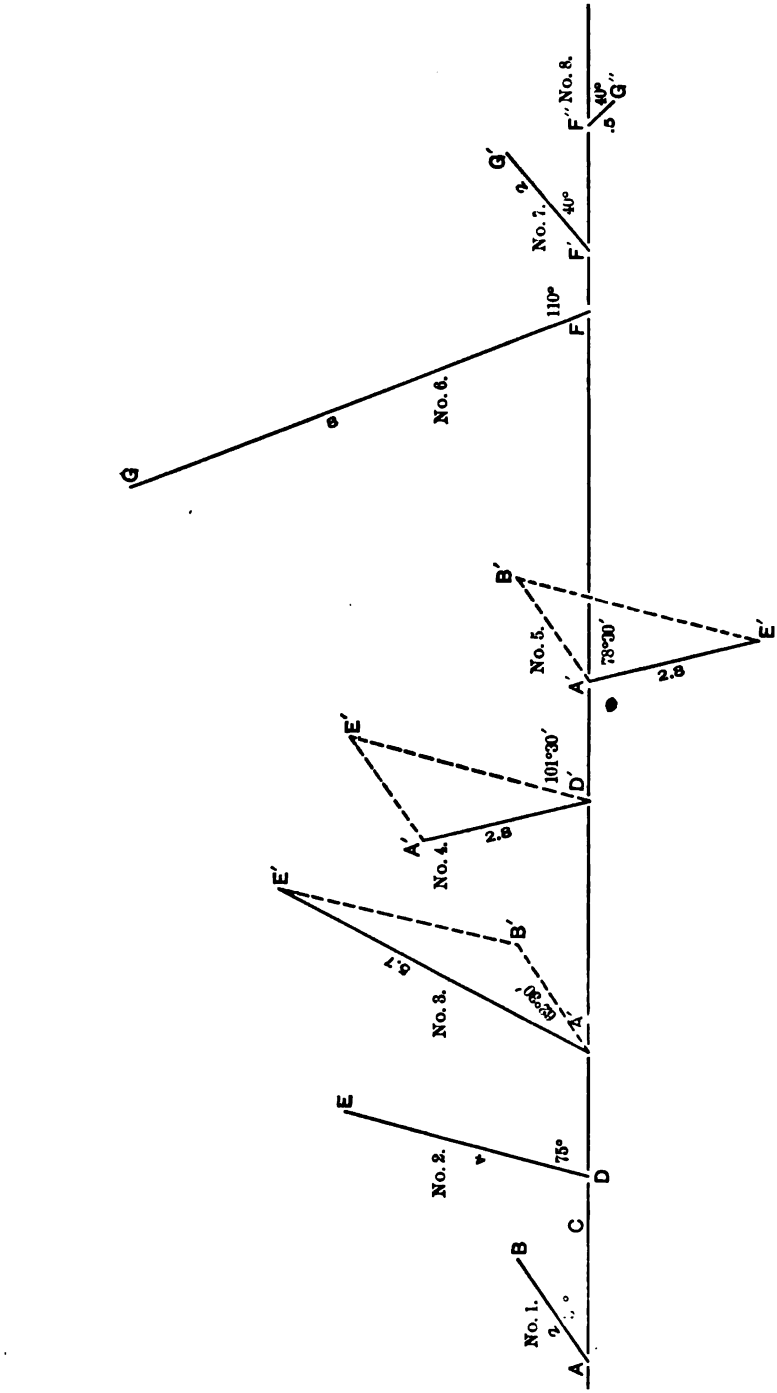


Fig. 270 Diagram of Operations on Vectors

449. — The addition of vectors is accomplished by joining them end to end and then connecting their extremities; the line connecting the extremities, being the vector sum of the two vectors to be added. Thus, to add $\overline{AB}$ and $\overline{DE}$, draw $\overline{A'B'}$, as in No. 3, parallel and equal to $\overline{AB}$, and from B′ draw $\overline{B'E'}$ parallel and equal to $\overline{DE}$ and join $\overline{A'E'}$. Then $\overline{A'E'}$ in direction and magnitude is the sum of $\overline{AB}$ and $\overline{DE}$. In this case $\overline{AB}$ plus $\overline{DE}$ equals 5.7, and is inclined to the base line at an angle of 62° 30′.

450. — Similarly, subtraction is performed. Thus, to perform the operation $\overline{DE} - \overline{AB}$, draw, as in No. 4, $\overline{D'E'}$ equal and parallel to $\overline{DE}$. From E′ lay off $\overline{E'A'}$ equal and parallel to $\overline{AB}$. Join $\overline{D'A'}$. Then $\overline{D'A'}$ measured positively is the desired result; in this case $\overline{D'A''} = 2.8$, and is inclined to the base at 101° 30′. To perform the operation $\overline{AB} - \overline{DE}$, draw, as in No. 5, $\overline{A'B'}$ equal and parallel to $\overline{AB}$. From B′ draw $\overline{B'E'}$, *negatively*, equal and parallel to $\overline{DE}$. Join $\overline{A'E'}$. In this case $\overline{A'E'}$ has the same numerical value as in No. 4, but it is a negative quantity, and not a positive one, as in No. 4. Also in No. 4 the angle of inclination is positive and 101° 30′, while in No. 5 it is negative and is 78° 30′.

451. — Multiplication of vectors is performed by multiplying the scalar magnitudes and taking the sum of their angular directions. Thus, the product of $\overline{AB}$ and $\overline{DE}$ is the plain vector $\overline{FG}$, No. 6, equal to $2 \times 4 = 8$ units in length, and inclined $35 + 75 = 110°$ to the base line.

452. — Conversely, division is performed by dividing the scalar magnitudes, and taking the difference of the angles. Thus, in No. 7, $DE / AB = 4 / 2 = 2$, inclined at an angle of $75° - 35° = 40°$ to the base line. Also, in No. 8, $AB / DE = 2 / 4 = .5$, inclined at an angle of $35° - 75° = -40°$.

453. — The reciprocal of the vector is a plain vector having a scalar magnitude equal to the reciprocal of the scalar of the original vector, but inclined to the base line at the same angle as the original vector.

454. — The solution of the following problems will now be given:

1. Composition and resolution of electro-motive forces.

2. Electrical properties of simple circuits with one resistance and one inductance in series.

   a. The resistance variable.

   b. The inductance variable.

3. Electrical properties of simple circuits with several resistances and inductances in series.

4. Electrical properties of simple circuits with one resistance and one capacity in series.

a. Resistance variable.

b. Capacity variable.

5. Electrical properties of simple circuits with several resistances and capacities in series.

6. Electrical properties of simple circuits with resistance, inductance, and capacity in series.

7. Electrical properties of simple circuits with several resistances, inductances, and capacities in series.

8. Electrical properties of circuits with resistance, inductance, and capacity in multiple arc.

9. Electrical properties of mutual inductive circuits.

455. 1. Composition and Resolution of Electro-Motive Forces. — For continuous current circuits it has been shown that the effective *E.M.F.* was equal to the algebraic sum of all the *E.M.F.s* acting on the circuit. For alternating currents it must now be proved that a similar proposition holds true, provided the vector, or geometrical sum, is taken.

In "Diagram of Composition of *E.M.F.s*," Fig. 277, suppose the line $\overline{AB}$ represents one *E.M.F.* acting on a circuit, and $\overline{AB'}$ represents another, the two *E.M.F.s* to have the same period, and separated by angle $\overline{BAB'}$. Draw $\overline{BB''}$ and $\overline{B'B''}$ respectively equal and parallel to $\overline{AB'}$ and $\overline{AB}$, forming the parallelogram AB′B″B. Draw the diagonal $\overline{AB''}$. Since $\overline{BB''}$ is equal and parallel to $\overline{AB'}$, the projection of $\overline{BB''}$ on $\overline{AC}$ is equal to the projection of $\overline{AB'}$; that is, $y' y'' = Ay$. The projection of $\overline{AB}$ is Ay', hence the sum of the projections of $\overline{AB}$ and $\overline{AB'}$ is Ay'', and, from the geometry of the figure, this is equal to the projection of $\overline{AB''}$, or the projection of the diagonal of the parallelogram. Suppose the parallelogram AB′B″B revolves about A as a center, all of the lines retaining the same angular relation. The sum of the projections of $\overline{AB}$ and $\overline{AB'}$ will, in every position, be the projection of $\overline{AB''}$; and as these lines revolve harmonically, they will trace three sine curves, indicated by the heavy line, the light line, and the dotted line, numbered I, II, III. As $\overline{BB''}$ is equal and parallel to $\overline{AB'}$, it will be at once per-

ceived, by the previously outlined rules for vector quantities, that $\overline{AB''}$ is the vector sum of $\overline{AB}$ and $\overline{AB'}$; hence the sine curve III, is the vector sum of I and II, and at all points represents the resultant of the harmonic *E.M.F.s* acting on the circuit. Should more than two *E.M.F.s* act on a circuit, the same train of reasoning may be extended by selecting any two *E.M.F.s* and combining them into a single, resultant curve. This resultant and any other *E.M.F.* may then be united into a third resultant, and the process repeated until the final curve is obtained. In a like manner it can be shown that any number of *E.M.F.s* of varying *periods* may give rise to a single resultant *E.M.F.*, while the converse of this proposition is equally obvious; namely, that a single *E.M.F.* may be resolved into any two components. The similarity between this construction and

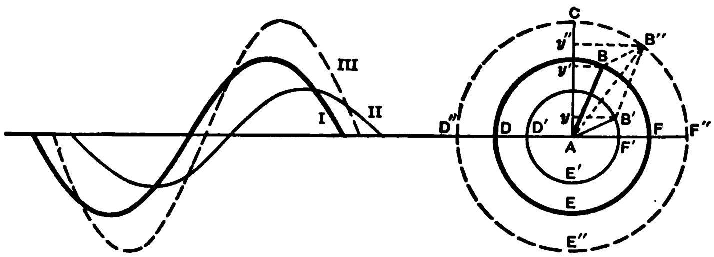


Fig. 277. Diagram of Composition of E.M.F.s.

that employed by the science of mechanics, in the parallelogram of forces, is obvious. If, therefore, in a complex circuit, the *E.M.F.s* in the various branches are given in magnitude and direction, the resultant *E.M.F.*, or that which it is necessary to impress on the circuit, is readily deducible. Given the electrical properties of the various branches of a compound circuit and the several currents required, the impressed *E.M.F.* may be decomposed into components having magnitudes and directions suitable to produce the desired currents in each branch; or, finally, given the resultant *E.M.F.*, and all but one of the components, the missing component may be found, and the electrical relation of the circuits adjusted to suit. In the solution for the Energy Equation, as applied to circuits containing resistance, inductance, and capacity, it was shown that the energy delivered to the circuit split into three parts, the RI component in phase with the current; $L\omega I$, 90° behind the current,

and $I / C\omega$ 90° in advance. As these components may assume a great variety of values, and as in multiple arc circuits other phase relations may obtain, it is easy to see that the maximum values of the components may be greater, equal, or less (numerically), than that of the resultant. The geometrical construction will always, in a clear and simple manner, elucidate any case of this description.

456. 2. **Electrical Properties of Simple Circuits with One Resistance and One Inductance in Series.** — Referring to Fig. 275, "Diagram of Harmonic Motion," assume that the full line KRMSO represents the current curve due to an harmonically varying *E.M.F.* in a simple circuit having resistance and self-induction. The counter *E.M.F.*, due to self-induction of the current, originates in the varying magnetic field set up by the changing current, and is directly proportional to the rate of change of this field. The rate of variation of the current, at any time, is measured by a tangent to the curve at the point of time under consideration. A simple inspection of the curve indicates that the maximum value of this tangent occurs at the points O and π, while at $\pi / 2$ and $3\pi / 2$; the tangent is horizontal, and its value 0; therefore the points K and M correspond to the maximum ordinates of the curve of *E.M.F.* due to inductance, indicating that this curve is similar to the current curve in period and shape, but lags behind it at an angle of 90°. Such a curve is represented by the dotted line cutting the axis of X 90° behind the current curve. It is, therefore, possible to represent geometrically the relations of an alternating current circuit containing resistance and inductance, by a right-angle triangle. In Fig. 278, "Diagram of *E.M.F.* in a Simple Circuit Containing Resistance and Inductance," draw a line horizontally from A to B in a positive direction. At A lay off AB to any convenient scale, equal numerically to R. At B draw BC perpendicularly and positively to AB, and lay off BC to the same scale of a value equal to $L\omega$. Draw AC, then AC to the

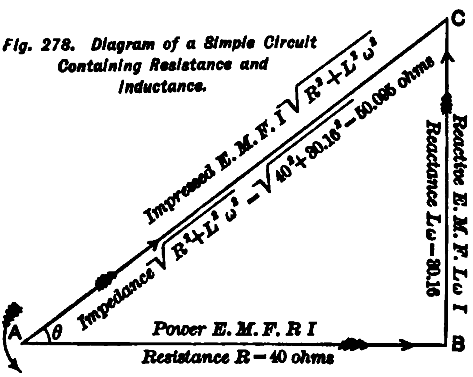


Fig. 278. Diagram of a Simple Circuit Containing Resistance and Inductance.

same scale represents the $\sqrt{R^2 + L^2\omega^2}$. Considering the equation $I\sqrt{R^2 + L^2\omega^2} = E$, it is seen that the impressed *E.M.F.* may be divided into two components: —

457. *First.* The component acting in the direction of the current and expended in overcoming the ohmic resistance. This component is often termed "The Power Electro-motive Force," and is numerically equal to RI.

458. *Second.* "The Reactive Electro-motive Force" in quadrature with the current, and employed in balancing the counter *E.M.F.* of inductance, and numerically equal to $L\omega I$.

459. Reactance. — The quantity $L\omega$, which is the measure of this effect, has formally been denominated "Reactance" by the American Society of Electrical Engineers, and is defined as "numerically *equal to the component of the impressed E.M.F. at right angles to the current, divided by the current.*" The reactive *E.M.F.* in any circuit may arise from inductance, mutual inductance, capacity, or from the introduction of a counter electro-motive force due to any exterior cause; and the impressed *E.M.F.* may always be regarded as the vector sum of two components, one of which transmits power, and the other which balances "reactance." In circuits containing mutual inductance, the reaction due to the current in the secondary coil may be resolved into two components, one in the same direction as the primary, and the other at right angles to it, thus obeying the foregoing definition. Some objection to this broad use of the term "reactance" has been made by Continental electricians, who hold that the employment of words ending in "*ance*" should be restricted to apply to the constants of a circuit; thus resist*ance*, conduct*ance*, permitt*ance*, are invariables for any given circuit; while under the above definition, "reactance" would vary when applied to circuits containing motors or mutual inductance. For such circuits all confusion may readily be avoided by using the term "equivalent reactance," or equivalent resistance in cases where such quantities can be variable.

460. Reactance is measured in ohms. In many respects it closely resembles resistance, but no power is expended in overcoming reactance, as it is at right angles to the current; and, therefore, the product of this *E.M.F.* component and the current is zero watts. As will presently be shown, reactance may arise from other influences than simple inductance. From an inspection of the diagram,

it will be seen that reactance tends to produce a phase difference between the impressed electro-motive force and the current. If θ represents this angle,

$$\tan\theta = -\frac{BC}{AB} = -\frac{\text{Reactance}}{\text{Resistance}} = -\frac{L\omega}{R},$$

the current being in advance of the impressed electro-motive force when θ is positive, and lagging behind it when it is negative.

461. Impedance. — The quantity $\sqrt{R^2 + L^2\omega^2}$, represented by the hypothenuse of the triangle, as the vector sum of the resistance and the reactance, has been termed "The Impedance of the Circuit," denoting the total opposition to transfer experienced by the current. In a simple circuit containing only resistance and inductance, the power *E.M.F.* is equal to the ohmic *E.M.F.*, or RI, and the reactive *E.M.F.* is equal to the inductive *E.M.F.*, or $L\omega I$. This may be indicated in the diagram by simply changing the scale sufficiently to introduce the numerical factor I. As an example, assume in the diagram, Fig. 278 —

$$\begin{aligned} \overline{AB} &= R = 40 \text{ ohms}, \\ \overline{BC} &= L = .08 \text{ henry}, \\ n &= 60, \\ \omega &= 377\,; \end{aligned}$$

then $L\omega = 2 \times 3.1415 \times 60 \times .08 = 30.16 = \overline{BC}$, and the impedance AC equals $\sqrt{40^2 + 30.16^2} = 50.095$ ohms, say 50 ohms. With a maximum impressed *E.M.F.* of 1000 volts the maximum current will be $\frac{1000}{50} = 20$ amperes. The power *E.M.F.* $= 40 \times 20 = 800$ volts. The reactive *E.M.F.* $= 20 \times 30.16 = 600$ volts, while the current will lag behind the impressed *E.M.F.* by an angle whose tangent is $\frac{30.16}{40} = .755$, or $37°$. The arrows indicate the direction of the forces.

Sec. *a*. — The Resistance Variable.

462. Suppose that in any given circuit E and L remain constant, while R becomes variable, what is the effect on I? With a continuous current, I varies directly as R; but in an alternating current circuit, I varies as the vector sum R and $L\omega$, or as $\sqrt{R^2 + L^2\omega^2}$. From inspection, it is evident that when $R = 0$, $I = E / L\omega$. Therefore, when R vanishes, the current can never attain a greater value than $E / L\omega$. When $R = \infty$, I becomes 0; thus these values indicate the

limit of I in both directions. To determine the successive values of I between these boundaries, construct a triangle of energy, as shown in Fig. 279, "Diagram of Current Values in Circuits containing Resistance and Inductance with Variable Resistance," by drawing $\overline{AB}$ positively and equal to RI, $\overline{BC}$ perpendicularly to $\overline{AB}$ positively and

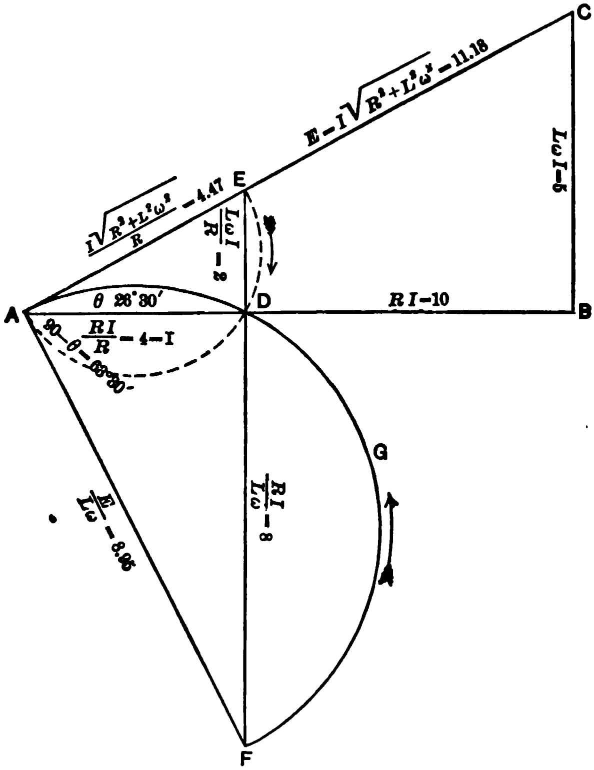


Fig. 279. Diagram of Current Values in Circuits containing Resistance and Inductance with Variable Resistance.

equal to $L\omega I$, then $\overline{AC} = I \times \sqrt{R^2 + L^2\omega^2}$. Divide RI by R to find the value of I. Suppose this to be $\overline{AD}$, then by similar triangles—

$$\overline{DE} = \frac{L\omega I}{R}, \qquad \text{and} \qquad \overline{AE} = I \times \frac{\sqrt{R^2 + L^2\omega^2}}{R}.$$

463. The maximum value of I is $E / L\omega$; and when the current has this value, the power component of the impressed *E.M.F.* is 0, and the current is 90° behind the impressed *E.M.F.* When R is infinitely large, the current is infinitely small, the angle of lag

becomes infinitely small, and the vanishing current coincides w the impressed *E.M.F.* From A draw $\overline{AF}$ perpendicularly $\overline{AC} = E / L\omega$. By its magnitude and direction, this line represents the maximum value of the current. Also the point A represents in direction and magnitude the vanishing value of the current, and coinciding with $\overline{AC}$ represents the minimum value of the current. All other values must lie between these two. On $\overline{AF}$ as a diameter, draw a semicircle AGF. By geometry, all triangles drawn as ADF on $\overline{AF}$, and limited by the semicircle AGF, are right-angled at D; but the maximum and minimum limits of these triangles are the maximum and minimum limits of the current values, hence all current values may be represented by the varying values of the vector $\overline{AD}$.[1] As a concrete example, assume —

$$\begin{aligned} R &= 2.5 \text{ ohms}, & \omega &= 314.15, \\ I &= 4 \text{ amperes}, & L\omega &= 1.25, \\ n &= 50, & RI &= 10, \\ L &= .004 \text{ henry}, & L\omega I &= 5; \end{aligned}$$

then,

$$E = I\sqrt{R^2 + L^2\omega^2} = 4\sqrt{2.5^2 + 1.25^2} = 11.18.$$

$$\frac{E}{R} = \frac{11.18}{2.5} = 4.47; \quad \frac{E}{L\omega} = \frac{11.18}{1.25} = 8.95; \quad \frac{RI}{L\omega} = \frac{10}{1.25} = 8.$$

$$\tan.\,\theta = \frac{L\omega I}{RI} = \frac{L\omega}{R} = \frac{5}{10} = .5 = 26^\circ\,34'$$

$$90^\circ - \theta = 63^\circ\,26'.$$

Sec. *b*. — Inductance Variable.

464. When L varies, the current limits are 0 when L is infinity, and E / R when L is 0, the current equation then reducing to Ohm's formula. By a similar train of reasoning to that employed in Sec. *a*, it is easily shown that when L is variable, the current variation is given by a vector drawn from point E in the previously mentioned diagram, and limited by a semicircle drawn on AE. This is indicated in the illustration by dotted lines. The arrows indicate the direction of the current variation in both Secs. *a* and *b*.

465. 3. **Electrical Properties of Simple Circuits with Several Resistances and Inductances in Series.** — In the case of a circuit containing a number of distributed resistances and inductances, the

[1] This proposition was first demonstrated by Messrs. Bedell and Crehore, see *Alternating Currents*, p. 223.

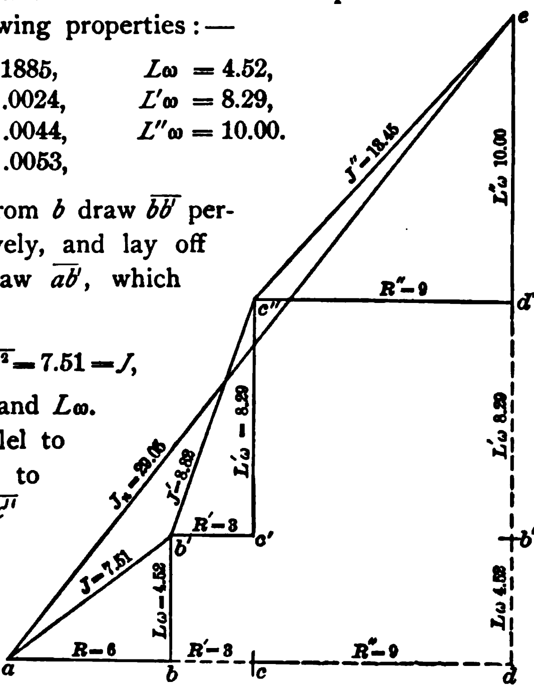
impedance is calculated by obtaining the vector sum of all resistances and the sum of all inductances. This is most conveniently done diagrammatically, as indicated in Fig. 280, "Diagram of *E.M.F.* in a Circuit containing Several Resistances and Inductances in Series." There are two methods, each attaining the same result, though the first to be described has the advantage of more clearly featuring all points of the circuit, and indicating more explicitly the distribution of currents and potentials. Assume as an example a circuit having the following properties:—

$R = 6$ ohms,	$\omega = 1885$,	$L\omega = 4.52$,
$R' = 3$ ohms,	$L = .0024$,	$L'\omega = 8.29$,
$R'' = 9$ ohms,	$L' = .0044$,	$L''\omega = 10.00$.
$n = 30$.	$L'' = .0053$,	

Lay off $\overline{ab} = 6$. From b draw $\overline{bb'}$ perpendicularly and positively, and lay off $\overline{bb'} = L\omega = 4.52$. Draw $\overline{ab'}$, which will be equal to

$$\sqrt{R^2 + L^2\omega^2} = \sqrt{6^2 + 4.52^2} = 7.51 = J,$$

the impedance of R and $L\omega$. From b' draw $\overline{b'c'}$ parallel to $\overline{ab}$, and make it equal to $R' = 3$. From c' draw $\overline{c'c''}$ parallel to $\overline{bb'}$, and lay off $\overline{c'c''} = L'\omega = 8.29$. Draw $\overline{b'c''}$, then $\overline{b'c''}$ equals—



$$\sqrt{R'^2 + L'^2\omega^2} = \sqrt{3^2 + 8.29^2} = 8.82 = J'.$$

Fig. 280.
Diagram of E.M.F.s in a Circuit Containing Several Resistances and Inductances in Series.

the impedance of R' and $L'\omega$.

Proceed in a like manner with R'' and $L''\omega$, obtaining point e, then $c''e$ will be the impedance of R'' and $L''\omega$, or $J'' = 13.45$. Join a and e. Then $\overline{ae}$ will represent $J_n = 29.05$, the impedance equivalent to the vector sum of all resistances and all inductances. The same result may be gained as shown by the dotted lines, by which the sum of $R + R' + R''$ is laid off horizontally, positively, as $ab + bc + cd$, then the sum of $L\omega + L'\omega + L''\omega$ is laid off vertically, positively, as

$db' + b''d' + d'e$, thus reaching the same point e as in the previous construction, obtaining $J_n = 29.05$, as before. The angles of lag of the current, and the components of the *E.M.F.*, are calculated in the manner already indicated. By the latter method, the total impedance of the circuit obtained is the same as that given by the former; and while more speedy of execution, the former gives a much clearer and more vivid idea of the component parts of the circuit.

466. 4. **Electrical Properties of Simple Circuits with one Resistance and one Capacity in Series.** — Turning to Fig. 275, and remembering that the tangent to the current curve KRMSO has a maximum value at the points O and π, it is evident that at these points there will be the greatest difference of potential exerted on the capacity of the circuit, and a maximum current will flow. It is also evident that the condenser current will oppose the line current; for as the current in the line decreases, the charge in the condenser will flow out, tending to continue the line current by the amount of charge due to the capacity, while, when the current is increasing, the condenser will absorb electricity, thus tending to reduce the line flow.

467. As an aid to the conception of the *rôle* played by a condenser, and its effect to introduce an *E.M.F.* 90° in advance of the impressed *E.M.F.*, consider the mechanical analogy of the common hydraulic elevator supplied with an air compression tank. The elevator is operated by a piston traveling to and fro in a cylinder. As the elevator falls, water is forced into the tank, and the air compressed; while, as it rises, the pressure of the compressed air tends to balance the weight of the car. To stretch the analogy a little, suppose the elevator to make regular trips, thus moving harmonically, and suppose that when it is at mid-stroke, the air in the tank is at atmospheric pressure. The motion of the elevator will be swiftest at mid-stroke and zero at either end, and may typify the current curve; the top, middle, and end of the stroke corresponding to the points $\pi / 2$, π, and $3\pi / 2$, of Fig. 275. The counter *E.M.F.* set up by the condenser has its analogy in the air pressure in the tank, while the charge is represented by the *amount* of water forced in. When the car is at mid-stroke, it is moving most rapidly, the air pressure is zero, and the water occupies one-half the space devoted to it in the tank. This state corresponds to the points O and π in Fig. 275.

As the car falls, the water is forced into the tank, the air pressure increases, the tank is filled, and the motion of the car decreases to zero; corresponding to a quarter period on the curve from O to $\frac{\pi}{2}$. At the points O and π the charge and counter *E.M.F.* are zero, and the current is a maximum. Between O and $\pi / 2$ the current decreases; the charge and counter *E.M.F.* increase. From mid-stroke to the end the increasing air pressure opposes the fall of the car, as the increasing counter *E.M.F.* opposes the current, while the increasing volume of water typifies the augmenting condenser charge. As the air pressure balances the car, it is evidently equal, and opposite to it, and must be 90° in advance. The effect, therefore, of the condenser is to introduce an *E.M.F.* 90° in advance of the impressed *E. M. F.* This condition is indicated in the diagram by the broken line III or VWYZ. The same relation is shown algebraically in the solution of the General Energy Equations. Thus, Capacity introduces a Reactance measured by $-\frac{1}{C\omega}$.

Remembering the negative nature of this Reactance, the triangle of *E. M. F.* may be constructed vectorially, as already indicated in the sections applying to Resistance and Impedance.

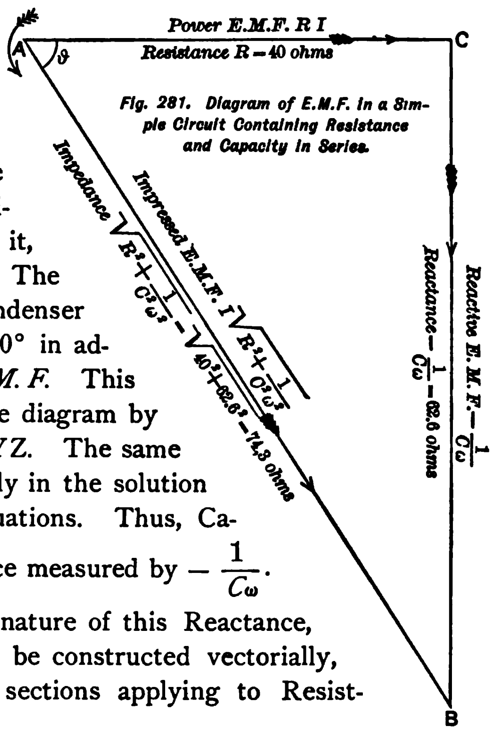


Fig. 281. Diagram of E.M.F. in a Simple Circuit Containing Resistance and Capacity in Series.

468. In Fig. 281, "Diagram of Electro-Motive Force in a Simple Circuit containing Resistance and Capacity in Series," draw $\overline{AC}$ horizontally and positively equal to R. From C draw $\overline{BC}$ negatively, and to the same scale equal to the quantity $-\frac{1}{C\omega}$. Draw $\overline{AB}$, then $\overline{AB}$ represents $\sqrt{R^2+\frac{1}{C^2\omega^2}}$. Adopting a similar notation to that employed in the diagram of electro-motive force in a simple circuit containing resistance and inductance in series, the horizontal

line $\overline{AC}$ measures the resistance of the circuit, and by a simple change in scale, to introduce the factor I, will measure that component of the impressed electro-motive force which is in phase with the current, usually denominated "Power Component." The line $\overline{BC}$ measures the reactance $-\frac{1}{C\omega}$, or the reactive component of the impressed electro-motive force $-I/C\omega$; while $\overline{AB}$ measures the $\sqrt{R^2+\frac{1}{C^2\omega^2}}$, and is the impedance of the circuit. When the factor I is introduced in the two sides of the triangle, the hypothenuse measures the total energy of the circuit EI. To illustrate by a concrete example. Suppose in the diagram the same value for the resistance R, 40 ohms, and $n = 60$ as was assumed in the diagram of electro-motive force in a simple circuit containing resistance and inductance, then $\omega = 377$. Let C equal .00000425 farad, then $-\frac{1}{C\omega} = -62.6$ and $\sqrt{R^2+\frac{1}{C^2\omega^2}} = \sqrt{40^2+62.6^2} = 74.3$ ohms. The properties of impedance and reactance, as given in this diagram, are similar to those described in the section treating of resistance and inductance in series; namely, the impedance of the circuit is the effective resistance or opposition encountered by the current, and which, when substituted for R in Ohm's law, renders his formula equally applicable to the alternating current circuits. The reactance of the circuit also possesses the same properties as indicated in the previous example; remembering, however, that the effect of inductance is to introduce a positive reactance, while the effect of capacity is to introduce one which is negative. Thus, it is apparent that capacity tends to oppose and neutralize inductance, and by proper proportioning of these quantities in any circuit, one may be so designed as to counteract and neutralize the other.

SEC. *a*. — RESISTANCE VARIABLE.

SEC. *b*. — CAPACITY VARIABLE.

469. In the paragraph treating of electrical properties of simple circuits with one resistance and one inductance in series, two sub-headings were given, indicating a method of geometrical construction whereby the different values of the current could be ascertained when either the resistance or the inductance in the circuit was supposed to vary from zero to infinity. As capacity has

been shown to introduce a ***negative*** reactance in the circuit, it is evident that the same construction may be used to determine the varying current in a circuit containing resistance and capacity, by constructing a diagram precisely similar to the one already alluded to, in which the reactance of the circuit is laid off ***negatively*** instead of positively; thus, under these circumstances, in a circuit containing resistance and capacity, when these quantities vary from zero to infinity, varying values of the current will be found as vector quantities bounded by semicircles drawn upon diameters having the values of $E / 1 / C\omega$ when R is variable, and upon a diameter equal to $\dfrac{I\sqrt{R^2 + 1 / C^2\omega^2}}{R}$ when C is variable. In a construction so obvious it is not necessary to repeat the diagram.

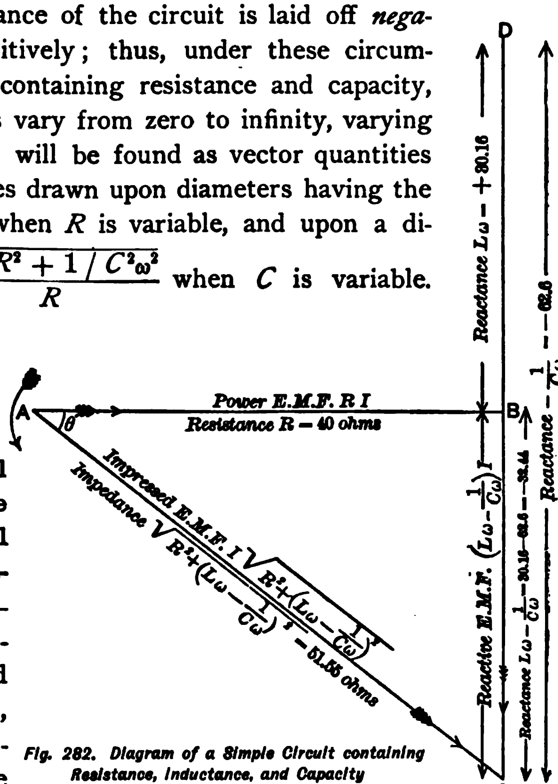


Fig. 282. Diagram of a Simple Circuit containing Resistance, Inductance, and Capacity in Series.

470. 5. Electrical Properties of Simple Circuits with Several Resistances and Capacities in Series. — In the case of a number of resistances and capacities in series, the diagram of electromotive force may be constructed as indicated in No. 3, bearing in mind the negative value of the reactances, and drawing the vectors representing them negatively downwards. In every other particular the construction is precisely the same as that outlined in No. 3, and the result may be obtained by the directions there given.

471. 6. Electrical Properties of Simple Circuits with Resistance, Inductance, and Capacity in Series. — To treat this case, it is requisite to recollect that the reactance of the circuit must be the vector sum of the positive and negative values of the two reactances developed by the inductance and the capacity. The case is illus-

trated in Fig. 282, "Diagram of Electro-Motive Force in a Simple Circuit containing Resistance, Inductance, and Capacity in Series," by assuming the values given in the previous examples, Nos. 2 and 4, namely:—

$$R = 40, \qquad L\omega = 30.16,$$
$$N = 60, \qquad 1 / C\omega = 62.6,$$
$$C = .0000425, \qquad \omega = 377.$$

From any point B, draw $\overline{BD}$ perpendicularly and positively equal to $L\omega = 30.16$. From D draw $\overline{DC}$ negatively downwards equal to $-1 / C\omega$. For a certain distance $\overline{DC}$ will coincide with $\overline{DB}$, but as $1 / C\omega$ is greater than $L\omega$, $\overline{CD}$ will be longer than $\overline{DB}$. The difference, BC, will be equal to the vector sum of $L\omega - \frac{1}{C\omega} = 30.16 - 62.6 = -32.44$.

From B draw $\overline{BA}$ horizontally and equal to R, in this case equal to 40. Draw $\overline{AC}$, then the vector $\overline{AC}$ is equal to $\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$.

In this diagram, as it is constructed, the vector sum of the three quantities $R + L\omega - \frac{1}{C\omega}$ has been obtained, which is evidently the impedance of the circuit in question. As in the previous diagrams, the horizontal line $\overline{AB}$ represents the resistance, or by a change of scale sufficient to introduce the factor I, represents the power component of the impressed electro-motive force. The vertical line $\overline{BC}$ represents the net reactance of the circuit, or the vector sum of the positive reactance due to impedance and negative reactance due to capacity; while $\overline{AC}$ represents the impedance of the circuit, or the impressed *E.M.F.* when the factor I is introduced.

472. 7. Electrical Properties of a Simple Circuit with Several Resistances, Inductances, and Capacities in Series. — By the principles already laid down, it is evident that where a number of resistances, inductances, and capacities are joined in series in a single circuit, the solution of the problem may be directly obtained by constructing an appropriate triangle of electro-motive forces, giving the vector sums of all the quantities producing impedance. Either of the methods given in No. 3 may be used, care being taken

to attribute to each vector its appropriate direction, positively and negatively.

478. 8. Electrical Properties of Circuits with Resistances, Inductances, and Capacities in Multiple Arc. — In the previous chapter it has been shown that in the case of a continuous current, the total resistance of a number of branch circuits joined in multiple arc is given by the reciprocal of the sum of the reciprocals of all the resistances. It has been shown that Ohm's formula applies to alternating current circuits when the apparent resistance or impedance of the circuit is substituted for R in the ordinary formula; so, in the case of alternating currents when traversing circuits in multiple arc, if for R the impedance of the various circuits be substituted, a correct solution is immediately arrived at. Therefore if, in accordance with the principles already laid down, the impedances of a number of multiple arc circuits be determined, and the sum of the reciprocals of these impedances be obtained, the total impedance of the circuit will be the reciprocal of this sum. To illustrate the case by a concrete diagram, suppose in Fig. 283, "Diagram of Electro-Motive Forces in a Complex Circuit containing Several Resistances, Inductances, and Capacities in Parallel," that G represents the diagram of the circuit. Here X is the generator to which five circuits, A, B, C, D, and E are joined in multiple arc. The circuit A has a simple resistance of 60 ohms; the circuit B has a resistance of 30 ohms in series with an inductance of .06 henry; the circuit C has a simple capacity of 13 mf.; the circuit D has a resistance of 50 ohms in series with a capacity of 5 mf., and an inductance of .18 henrys. The circuit E has a simple inductance of .09 henrys. To determine the total inductance of the circuit, assume any base line as ax. On this base line lay off $\overline{ab}$ equal to R, equal to 60. As there is no inductance, the impedance in this circuit $J_A = 60$.

At any other point on the base line, construct a triangle of the electro-motive forces for the second circuit B, by laying off $\overline{cd} = R = 30$; $\overline{de}$ vertically and positively $= L\omega = 45.24$. Join $\overline{ce}$ to obtain the impedence $J_B = 54.27$. Proceed in a like manner with the remaining circuits, C, D, and E, obtaining the inductances —

$$J_C = 102.02, \qquad J_D = 138.7, \qquad J_E = 67.86.$$

[1] The frequency n in this example is 120 per second.

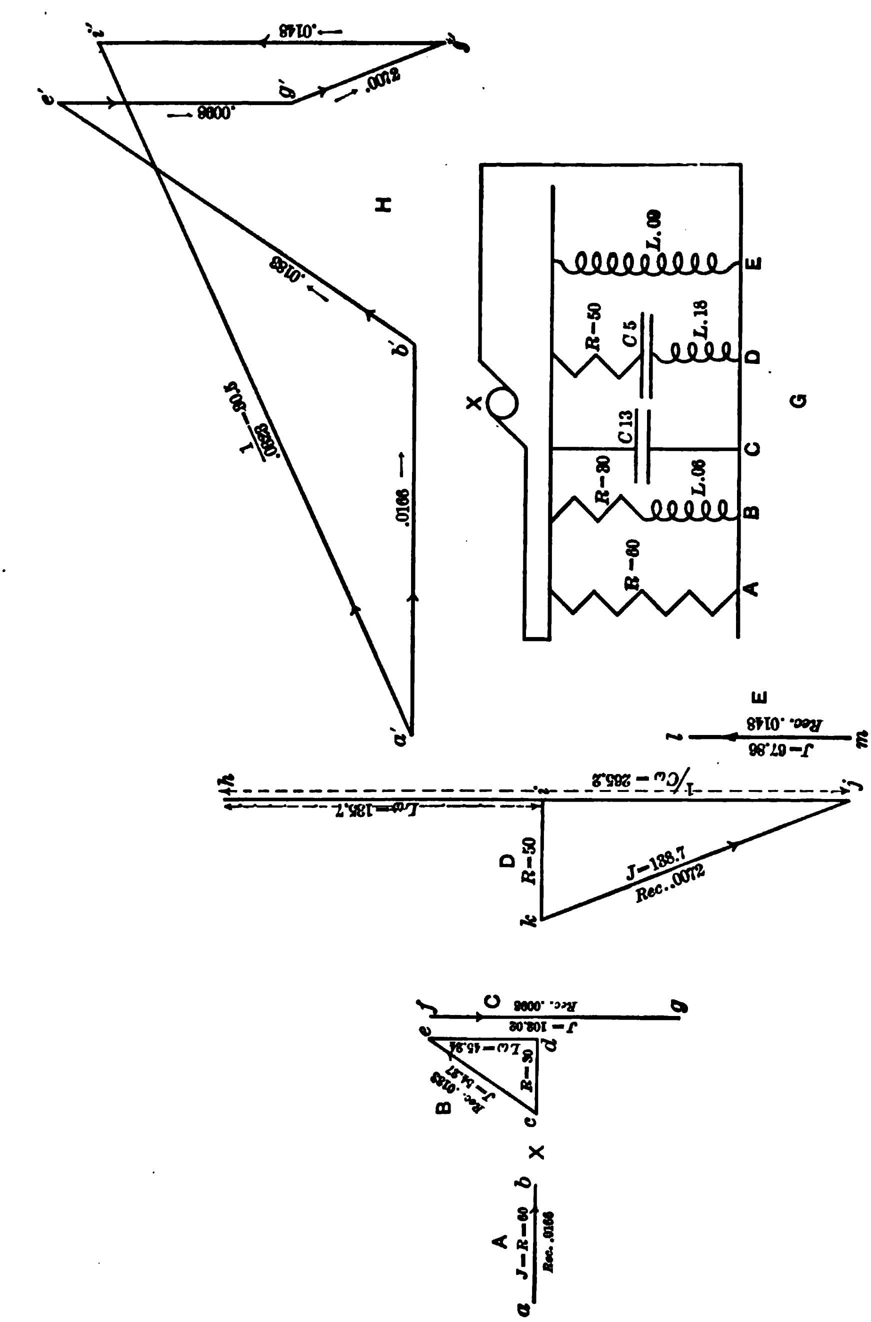


Fig. 283. Diagram of E.M.F.s in a Complex Circuit with Several Resistances, Inductances, and Capacities in Parallel.

Each of these are plain vectors. Obtain now the reciprocal of each one, remembering that the reciprocal of a vector is a plain vector having a scalar magnitude equal to the reciprocal of the original vector and lying in the same direction. Thus, —

the reciprocal of $J_A = .0166$,	the reciprocal of $J_D = .0072$.
$J_B = .0183$,	$J_E = .0148$.
$J_C = .0098$,	

Assume any point, a', and draw $\overline{a'b'}$ parallel to $\overline{ab}$, and make it to any convenient scale equal to the reciprocal J_A.

From b' draw $\overline{b'e'}$ parallel to $\overline{ce}$, making $\overline{b'e'}$ equal to the reciprocal J_B. From e' draw $\overline{e'g'}$ parallel to $\overline{fg}$, making $\overline{e'g'}$ equal to the reciprocal of $\overline{fg}$. From g' draw $\overline{g'j'}$, parallel to $\overline{kj}$, making it equal to the reciprocal of J_D. From j' draw $\overline{j'i'}$ parallel to $\overline{ml}$, making it equal to the reciprocal of J_E. Join the points, a' and i', then the line $\overline{a'i'}$ will, in direction, represent the resultant electro-motive force acting in the circuit, and in magnitude will be the sum of the reciprocals of all the impedances in circuit. Obtaining the reciprocal of this sum, in this particular case equal to 30.5, the total impedance of the circuit is given as 30.5 ohms.

The phase of the impressed electro-motive force, with reference to the currents in the various branches or parts of the branches, may be found by the previously given rules. The direction of the arrows in the diagram indicates the direction of the current in the various parts of the circuit.

474. Method of Equivalent Resistance and Inductance. — When a number of circuits in multiple arc are acted upon by an electro-motive force, it is possible, theoretically, to replace the several resistances, inductances, and capacities, by an equivalent resistance and inductance, remembering that a capacity is equivalent to a negative inductance. The equivalent resistance and inductance would be such a resistance and inductance as would cause the same current (both in magnitude and phase) to flow in the main leads, as would pass when the several parallel circuits were connected. The substitution of such an equivalent inductance and resistance evidently produces no change in the main circuit, and could displace the branch circuits without producing any variation, either in magnitude or in phase, in the original current. The employment of such a

hypothetical substitution as this simplifies, in some cases, the solution of problems in alternating currents, when applied to a number of branch circuits. The application of this method is a direct cor-

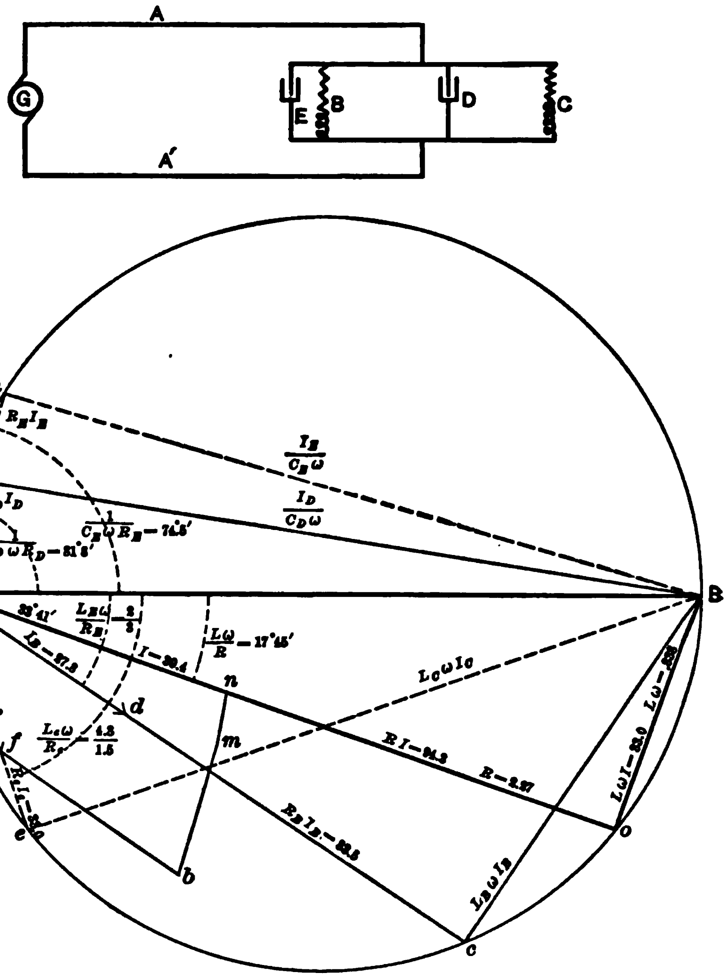


Fig. 284. Diagram of the Method of Equivalent Resistance and Inductance.

ollary to the proposition of Art. 462, and can be, perhaps, most clearly described by the use of an example. Suppose in Fig. 284 the generator G supplies two leads A and A′, extending from the generator to a center of distribution. From this point suppose four

circuits B, C, D, and E to be placed in parallel across the leads, and let each circuit be respectively denominated by its appropriate letter. Let the frequency be 159.15, so that $\omega = 1000$. For the circuit B:—

$R_B = 3$ ohms, $L_B = .002$ henry, $L_B\omega = 2.$

For the circuit C:—

$R_C = 1.5$ ohms, $L_C = .0043$ henry, $L_C\omega = 4.3.$

For the circuit D:—

$R_D = 2.5$ ohms, $C_D = .00006$ farad, $C_D\omega = .06$, $1 / C_D\omega = 16.67.$

For the circuit E:—

$R_E = 1.5$ ohms, $C_E = .00019$ farad, $C_E\omega = .19$, $1 / C_E\omega = 5.26.$

From the above data the impedance of each branch circuit may be directly calculated, as,—

$J_B = 3.60$, $J_C = 4.56$, $J_D = 16.72$, $J_E = 5.47.$

475. Now, assume any convenient electro-motive force to act at the center of distribution, uniformly affecting all of the branch circuits. For this purpose it is very convenient to assume 100 volts, as then all the quantities to be derived from the solution will be in percentage, and may be conveniently and easily handled. With the assumption of 100 volts as the electro-motive force, calculate from the impedance as above obtained, the currents in each of the branches, obtaining,—

$I_B = 27.8$, $I_C = 21.9$, $I_D = 5.98$, $I_E = 18.2.$

Now, referring to the diagram, draw any line AB to any convenient scale, making $\overline{AB}$ equal to 100 volts. At A lay off $\overline{Ac}$, making the angle BAc equal to $\tan^{-1}\frac{L_B\omega}{R_B}$. If the line AB represents the electro-motive force, then the line Ac is equal to I_BR_B, and the line $\overline{cB}$ is equal to $L_B\omega I_B$. In a similar manner construct other triangles AeB, AgB, and AhB, remembering that in circuits containing inductance the angle θ must be laid off from the line of electro-motive force negatively, while in circuits containing capacity it must be laid off positively. From what has previously been demonstrated, it is obvious that the points e, c, B, h, g, and A will lie on the circumference of a circle drawn upon $\overline{AB}$ as a diameter. On each of the lines $\overline{Ac}$, $\overline{Ae}$, $\overline{Ag}$, and $\overline{Ah}$, lay off $\overline{Ad}$, $\overline{Af}$, $\overline{Ak}$, and $\overline{Ai}$, respectively,

equal to the currents in each of the branches, or in other words, divide these lines by the resistance in each branch. The lines $\overline{Ad}$, $\overline{Af}$, $\overline{Ak}$, and $\overline{Ai}$, will then represent, in direction and magnitude, the currents in each of the branch circuits. From what has previously been shown, it is evident that the vector sum of all the currents would be equivalent to the resultant current, or the current in the main lines A, A′. To obtain this resultant current select in the diagram any current vector as $\overline{Af}$. From the point f draw a line parallel to the next current vector $\overline{Ad}$, and lay off $\overline{fb}$ equal to $\overline{Ad}$. From the point b draw $\overline{bm}$ parallel and equal to $\overline{Ai}$. From the point m draw $\overline{mn}$ equal to $\overline{Ak}$. The broken line $Afbmn$ represents the vector addition of all the lines representing the currents, or in other words, forms a polygon of currents. This construction is parallel to the polygon of forces in mechanics. To find the resultant current, namely, the vector sum of all the component currents, draw $\overline{nA}$, thus completing the current polygon. Then the line $\overline{nA}$ will represent, both in direction and magnitude, the current in the main leads. Prolong $\overline{nA}$ until it intersects the circumference drawn upon the line $\overline{AB}$ at O. Then, evidently, the lines $\overline{AO}$ and $\overline{OB}$ represent respectively the product of the current in the main leads by such resistance and such inductance as is equivalent to the vector sum of all the inductances acting in the branch circuits, or $\overline{AO} = RI_A$ and OB $= L\omega I_A$. By dividing these lines by the current, the respective desired equivalent resistance and inductance is immediately obtained. In this particular example the current in the main leads, —

$$I_A = 39.4, \qquad RI_A = 94.3 \qquad R = 2.27 = \text{equivalent resistance.}$$
$$L\omega I_A = 33, \qquad L\omega = .838, \qquad L\omega / \omega = .000838 = \text{equivalent inductance.}$$

The tangent of the angle of lag is obtained in the usual manner.

476. By this method the currents in each of the branch circuits, and the equivalent resistance and inductance necessary to produce in the main leads the same current as would flow with all of the parallel circuits working under the given conditions, are obtained. The assumption, however, has been made of an electro-motive force of one hundred volts. If, now, any other electro-motive force is operative, it is simply necessary to change the scale of the entire diagram by the proportion which 100 bears to the real electro-motive force. To complete the solution of the problem, it must be recollected that so

far no account has been taken of the circuit AA′, extending from the generator to the "Center of Distribution." The entire solution of the problem is evidently obtained by taking the vector sum of the resistance and inductance of the main leads, together with the equivalent inductance and resistance of the branch circuits, as given

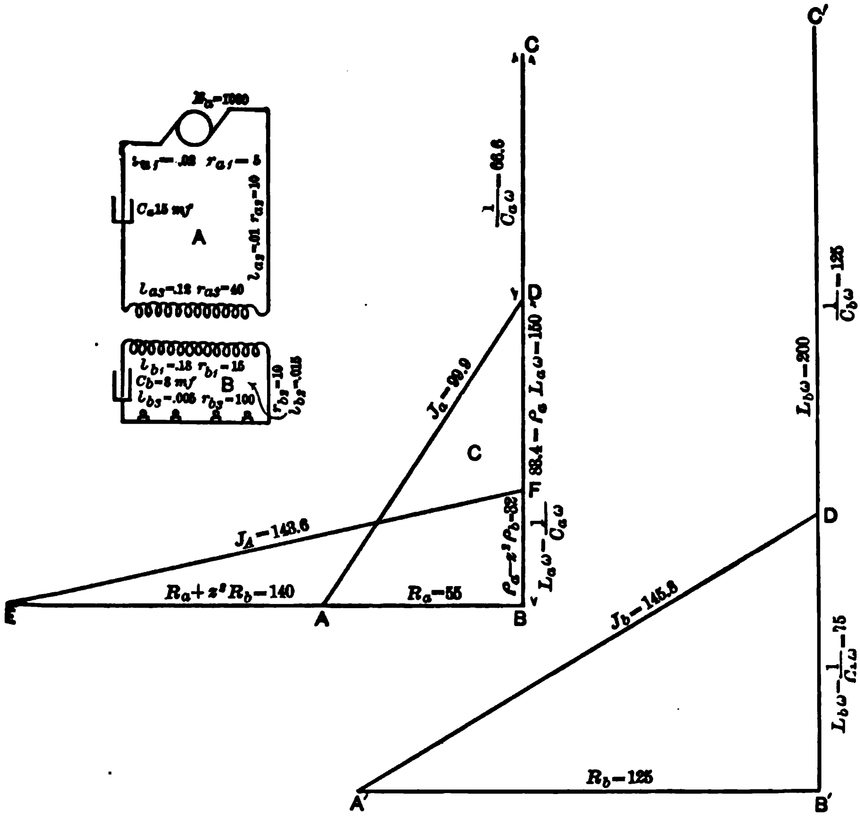


Fig. 285. Diagram of E.M.F. in Mutually Inductive Circuits.

by this problem. The method of obtaining the vector sum of two inductances and resistances in series has already been given.

477. 9. **The Properties of Circuits containing Mutual Inductance.** — The most frequent and important cases of mutual inductance are to be found in the construction of dynamo machinery; the common transformer forming a convenient example. Here two circuits are in close proximity to each other, in one of which an impressed

E.M.F. acts, producing by mutual inductance a useful *E.M.F.* in the neighboring circuit. Mr. Kennelly[1] is the deviser of the neat method of graphically solving this problem, of which the following example is an illustration.

Assume two circuits A and B, indicated in Fig. 285, "Diagram of *E.M.F.* in Mutually Inductive Circuits." For simplicity, a non-magnetic medium is predicated; though by the simple introduction of the permeability factor μ in the formulæ, the same treatment will apply to any media. Suppose the circuit A to consist of a generator supplying an *E.M.F.* denoted by $E_a = 1000$ volts. Let the resistance and inductance of the generator be respectively $r_{a1} = 5$ ohms, and $l_{a1} = .02$ hen. Let the line have a resistance of $r_{a2} = 10$ ohms, and an inductance of $l_{a2} = .01$ hen., with a capacity of $C_a = 15$ mf. Let the resistance and inductance of the primary coil be $r_{a3} = 40$ ohms, and $l_{a3} = .12$ hen. Then the total resistance $R_a = r_{a1} + r_{a2} + r_{a3} = 5 + 10 + 40 = 55$ ohms, and the total inductance $L_a = l_{a1} + l_{a2} + l_{a3} = .02 + .01 + .12 = .15$ hen. For the secondary circuit, suppose the resistance and inductance of the coil to be $r_{b1} = 15$ ohms, and $l_{b1} = .18$ hen., and for the leads $r_{b2} = 10$ ohms, and $l_{b2} = .015$ hen., with a capacity of $C_b = 8$ mf.; with finally a resistance and inductance in the receivers of $r_{b3} = 100$ ohms, and $l_{b3} = .005$ hen. Then the total resistance and inductance of the B circuit is $R_b = r_{b1} + r_{b2} + r_{b3} = 15 + 10 + 100 = 125$ ohms, and $L_b = l_{b1} + l_{b2} + l_{b3} = .18 + .015 + .005 = .20$ hen. Let the mutual inductance be $M = .12$ hen., and the frequency 159.15 (in round numbers 160), so that $\omega = 2\pi n = 2 \times 3.14 \times 159.15 = 1000$.

478. As a preliminary to the final solution, suppose the circuit B to be absent, then in circuit A the impedance—

$$J_a = \sqrt{R_a^2 + \left(L_a\omega - \frac{1}{C_a\omega}\right)^2} = \sqrt{55^2 + 83.4^2} = 99.9 \text{ ohms.}$$

Denote the reactance of circuit A, $\left(L_a\omega - \dfrac{1}{C_a\omega}\right) = 83.4$, by ρ_a. The current in circuit A is $i_a = E_a / J_a = 1000 / 99.9 = 10$ amperes (about). The triangle of *E.M.F.* is drawn as at C by laying off from A $\overline{AB} = R_a = 55$ ohms. From B draw $\overline{BC}$ perpendicularly and positively equal to $L_a\omega = 150$. From C lay off $\overline{CD}$ perpendicularly

[1] *Electrical World*, vol. xxii., p. 306.

and negatively equal to $\frac{1}{C_a\omega} = 66.6$, thus leaving $\overline{DB} = \rho_a = 83.4$. Draw $\overline{AD}$, obtaining $J_a = 99.9$.

479. Now suppose circuit B to be brought into such relations with circuit A that the coefficient of mutual induction M shall have the previously assigned value of .12 hen. The first effect of the current i_a in circuit A is to initiate an induced *E.M.F.* in circuit B measured by $\omega Mi_a = E_b = 1000 \times .12 \times 10 = 1200$ volts; tending to produce in B a current $i_b = E_b / J_b$. But —

$$\left(L_b\omega - \frac{1}{C_b\omega}\right) = \rho_b = 200 - 125 = 75,$$

Also

$$J_b = \sqrt{R_b^2 + \left(L_b\omega - \frac{1}{C_b\omega}\right)^2} = \sqrt{125^2 - 75^2} = 145.8.$$

and hence

$$i_b = \frac{1200}{145.8} = 8.2 \text{ amperes.}$$

Construct now the triangle of *E.M.F.* in the secondary circuit B, as shown at D by the methods already given. The current in the circuit B will react in turn on A, tending in that circuit to set up an *E.M.F.* that would give rise to a current superimposed on the current i_a that is already passing. The modified primary current will again react on the secondary, causing a new adjustment of current value, this process continuing till equilibrium is attained. Denoting by i_A the final value of the current in the A circuit, this value could be derived from the expression $i_A = E_a / J_A$, in which J_A is different from J_a. The value J_A of the impedance, which will give the true amount of the final current in the A circuit, may be termed the "Effective Impedance;" and is shown to be derived by increasing the resistance of the A circuit by a quantity z^2R_b, and diminishing the reactance by $z^2\rho_b$; in which $z = \omega M / J_b$. The final primary current then becomes —

$$i_A = \frac{E_a}{\sqrt{(R_a + z^2R_b)^2 + (\rho_a - z^2\rho_b)^2}}. \tag{175}$$

In this example, $z = 1000 \times .12 / 145.8 = .828$; $z^2R_b = 85.5$; and $z^2\rho_b = 51.5$; therefore, —

$$i_A = \frac{1000}{\sqrt{(55 + 85.5)^2 + (83.4 - 51.5)^2}} = 7 \text{ amperes.}$$

It also follows that the true secondary *E.M.F.* will be equal to $\omega M i_A = 1000 \times .12 \times 7 = 840$ volts, and the current in the circuit $B = i_B = E_b / J_b = s i_A = 840 / 145.8 = 5.8$ amperes. This result is graphically shown at C by increasing $\overline{AB}$ to $\overline{AE}$, making $\overline{AE} = s^2 R_b$; and decreasing $\overline{BD}$ by $\overline{FD} = s^2 \rho_b$. Then the effective impedance J_A is $\overline{FE} = 143.6$. The angles of phase may be determined in the usual manner.

In this example, only one of the circuits has been given an impressed *E.M.F.*; but the same method can readily be extended to embrace an impressed *E.M.F.* in both circuits, by taking in circuit B the vector sum of the impressed and induced *E.M.F.* No allowance is made for hysteresis, which will doubtless limit to a certain extent this method.

480. Impedance Tables. — From the preceding considerations, it is perceived that inductance and capacity, when of sensible amount, play an exceedingly important part in modifying the current, both in magnitude and direction. For convenience in treatment, the subject may be divided into two parts: —

CASE I. — CIRCUIT CONTAINING RESISTANCE AND INDUCTANCE.

SEC. *a*. Two parallel aerial wires as a complete metallic circuit.
SEC. *b*. One aerial wire with ground return.
SEC. *c*. Concentric cables.

CASE II. — CIRCUITS CONTAINING RESISTANCE, INDUCTANCE, AND CAPACITY.

SECS. *a*, *b*, and *c*, as above.
SEC. *d*. Effect of adjacent bodies.

CASE I. — CIRCUITS CONTAINING RESISTANCE AND INDUCTANCE.

481. SEC. *a*. — Two parallel overhead wires, as a complete metallic circuit.

From the energy equation the general value of the impedance J in any circuit containing resistance and inductance is $\sqrt{R^2 + L^2\omega^2}$. If, in this expression, R be the value of the ohmic resistance for a unit of length of the conductor, the value of J may be arranged as a simple numerical factor, to be used as a multiplier; and if l be the length of any circuit, R its resistance per unit

of length, and J the impedance factor, the total impedance of the circuit becomes JRl. The values of J may be determined graphically, with sufficient accuracy for common practice, by the aid of the accompanying tables, with the avoidance of much tedious calculation. As the tabular values are given for commercial copper, it is only necessary, when the impedance factor is ascertained, to multiply it by the resistance of one unit length of the proposed conductor, and by the length of the circuit, to determine the total impedance.

482. The coefficient of inductance L for two indefinitely long parallel wires is given on p. 408, as $.5 + 2 \log_e \frac{d}{r}$, where L is the value per centimeter of length, when d is the distance between the centers of the conductors, and r the radius of the wire, in the same units. For demonstrations of this formula, the reader is referred to Mr. Kennelly's paper on Impedance.[1] The resistance of the conductor per centimeter, when ρ is the specific resistance, is $R = \rho / \pi r^2$. Substituting these values in the general expression for impedance, —

$$J = \sqrt{1 + \frac{L^2\omega^2}{R^2}} = \sqrt{1 + \frac{4\,\pi^4 n^2 r^4}{\rho^2}\left(.5 + 2 \log_e \frac{d}{r}\right)^2}. \tag{176}$$

By inspection, this expression is resolvable into four parts; viz.: —

$$\frac{4\,\pi^4 n^2}{\rho^2}, \qquad r^4, \qquad \text{and} \qquad \left(.5 + 2 \log_e \frac{d}{r}\right)^2.$$

Each of these parts, or components, may be plotted as a curve, and the value of the entire quantity obtained rapidly by summing the *separate parts*. It is the object of TABLE No. 72 (Pocket) to facilitate this process.

483. The base line of the portion of Sec. 1, on the right of the double line, is divided into 100 equal parts allotted to the diameter of the conductor. The top of the sheet is similarly allotted to the distance between the axes of the conductors. As the scales are decimal, either, or both, may be multiplied or divided by any power of 10, in order to extend the range of the Table. The vertical axis in the center of Sheet 1 gives the values of d/r. Thus, the portion of Sheet 1 marked **a**, bounded by the top and bottom lines of the sheet, the vertical axis on the left, and including the diagonals to the equal part scale on the base line, will give the value of d/r for a circuit of

[1] Trans. A. I. E. E.; vol. x., No. 4., p. 203.

any size of wire from .001″ to 1″ in diameter, and having the axes of the conductors separated from .1″ to 100″ apart.

484. On the left hand of the double line, the curve **b** on Sec. 2 gives the values of $\left(.5 + 2 \log_e \frac{d}{r}\right)^2$, the vertical axis and scale for this curve being the same as that for ***d/r***. On Sheet 2, curve **d**, Sec. 3, on the left hand of the sheet is an extension to higher ranges of the curve **b** of Sheet 1, giving extended values of $\left(.5 + 2 \log_e \frac{d}{r}\right)^2$. The curve **c**, Sheet 1, Sec. 1, gives on the extreme right-hand scale the values of r^4 in the same units as the base line of the sheet; so if the base line be multiplied or divided by any power of 10, this axis must be similarly multiplied or divided by the same *power* of 10 *raised* to the *fourth power*.

On the right hand of Sheet 2, Sec. 4, is the frequency curve **e**, giving the value of $4\pi^4 n^2/\rho^2$.

485. An example will illustrate the use of the tables. Let it be required to find the impedance factor of two parallel wires, No. 18 Am. W. G., one-half inch apart, working under a frequency of 150. The dotted lines on the tables indicate the course to be followed in obtaining the components of the factor. The diameter of No. 18 wire is 40 mils. On Sheet 1 find the diagonal ending at 40. Follow this diagonal until it intersects a vertical line passing through .5 of an inch, on the top base line for the distance separating the wires. In this case, the vertical through .5 in. does *not* intersect the diagonal through 40; therefore, take the vertical through .05, or, in other words, divide the distance between the wires by 10. From the point where the vertical through .05 intersects the diagonal through 40, follow a horizontal line to the left, finding in the column marked "Values of d/r" the quantity 2.5. As the upper base line used as the distance between the wires was *divided* by 10, the value found for d/r must be *multiplied* by 10, making 25 for the value of d/r, thus determining one of the desired components of the impedance factor. As the curve **b** for values of $\left(.5 + 2 \log_e \frac{d}{r}\right)^2$ on Sheet 1 does not run as high as 24.5, turn to the extension of the same curve **d** on Sheet 2. Find 25 on the scale marked d/r; follow a horizontal from this value to the left, to the intersection of the curve. From

this point follow the vertical line downward to the base line, finding 48 as the value of $\left(.5 + 2 \log_\epsilon \frac{d}{r}\right)^2$, corresponding to $d/r = 25$, giving the second component. To find the value of r^4, return to the diagonal ending in 40 on Sheet 1. From the foot of the diagonal, follow a vertical upward to the intersection with curve **c**, then follow a horizontal to the right to the vertical axis marked "Values of r^4," finding .000006 as the value of r^4, the third component. To ascertain the value of $4\pi^4 n^2/\rho^2$, turn to the frequency curve **e** in Sheet 2, find the frequency (150 in this example) on the vertical axis on the right; follow a horizontal to the intersection with the curve, and then a vertical down to the base line, obtaining the value of $4\pi^4 n^2/\rho^2$ as 3.1 for the fourth component.

486. To recapitulate, the four components now stand: —

1st. $\dfrac{4\pi^4 n^2}{\rho^2} = 3.1$, 2d. $r^4 = .000006$,

3d. $\dfrac{d}{r} = 25$, 4th. $\left(.5 + 2 \log_\epsilon \dfrac{d}{r}\right)^2 = 48$;

then,

$$J = \sqrt{1 + [3.1 \times .000006 \times 48]},$$
$$J = \sqrt{1.0008928},$$
$$J = 1.000446.$$

487. When the *decimal part* of the quantity under the radical sign is less than .1, the square root may be found with sufficient accuracy by dividing the *decimal part* of the *quantity* by 2, and prefixing 1 to the quotient. For greater values than this, consult any good table of square roots. The value of J thus found is the value for one unit of length. To obtain the total impedance of any circuit, it is now necessary to multiply this factor by the resistance of the conductor per unit of length (to be obtained from any wire table), and by the length of circuit expressed in the same units. In all circuits falling under this case, the value of J will be greater than unity, indicating that the effect of inductance is to increase the resistance. As J varies as d, it is evident that this factor may be materially reduced by bringing the two conductors as close together as possible. With uninsulated aerial lines, the wires must be separated at least six inches, or more, to prevent crosses. In conduit lines, with careful construction, this distance may be greatly decreased, while in

concentric cables, d may be reduced to a fraction of an inch. The table may also be applied to determining the impedance of circuits carrying polyphase sine currents of equally effective intensity, provided the component parts of the circuit are equally distant from each other. The assumption is also made that the current density is uniform throughout the entire conductor, and that the current waves penetrate equally throughout its entire mass. For currents of ordinary frequency, this supposition is essentially true, attention having been already directed to "Skin Effect." The determination of the impedance factor, by this method, is accurate only when the form of the current wave is a sine curve. Any departure from this form serves to increase the value of the impedance factor, and must be calculated from the particular shape of the wave employed. As the departure from the sine curve is most apparent in poor dynamo machinery working on a light load, and as transmission calculations are always made for full load with good machinery, the agreement of the current wave to the theoretical form is very close, the method may be regarded as practically accurate (see page 611).

488. SEC. *b*. — An aerial line with ground return.

When a circuit is composed of one aerial wire placed at a height "h" above the ground, and the earth used as a return, Mr. Heaviside[1] has shown that by the method of "Images" the ground may be replaced by an imaginary wire situated at an interaxial distance from the real wire equal to $2h$. Such a circuit immediately reduces to Case 1, by making "d" in the formula equal to twice the height of the line above the ground.

489. SEC. *c*. — Concentric cables.

Suppose one conductor to be rolled out into a thin sheet and formed into a tube surrounding the other conductor, this forming a concentric cable, in which the same amount of metal is employed, and the distance from the central conductor to the surrounding ring is maintained, the same as in the case of two parallel wires. Evidently, the resistance of the circuit is unchanged, and, also, each element in the ring is at the same interaxial distance as in the original circuit. The geometrical relations of the currents of the two conductors are unaltered, and the impedance may be calculated by the preceding methods, by substituting for d, in the preceding nota-

[1] See *Jour. Tel. Eng.*, vol., vii. p. 303.

tion, the value r' of the radius of the external conductor in the concentric cable.

CASE II. — CIRCUITS CONTAINING RESISTANCE, INDUCTANCE, AND CAPACITY.

490. SEC. *a.* — Two parallel aerial wires as a complete metallic circuit.

The determination of the impedance factor for circuits containing resistance and inductance has been shown to be a simple matter. While both inductance and capacity are always present in all forms of electrical apparatus, the capacity effect is usually much less apparent, and may be more safely neglected, than that presented by inductance. Moreover, in a single circuit, inductance always manifests itself in series with the rest of the circuit, either sensibly, concentrated at a single point, as in the case of a very short line supplying transformers, or else distributed from point to point along the line, as exemplified in a pair of transmission mains. Contrariwise, capacity usually exhibits itself as a high resistance *shunt*, acting as a *branch circuit* between the conductors, and must therefore be treated by the law of divided circuits. Occasions arise, as in the construction of some forms of dynamo machinery and in certain telephone circuits, where a large amount of capacity in the shape of *condensers* is placed in series at one point, in the circuits. Such cases, however, do not fall within the scope of transmission problems as usually understood, and when encountered may be solved by direct application of the formula $\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$. Consider the case of an aerial line. Here are two indefinitely long parallel conductors, each having a surface equal to the length of *one-half* the circuit multiplied by the circumference of the conductors, and separated by a stratum of air equal to the distance between the wires. Evidently this combination possesses all the characteristics of a condenser placed across the conductors. If, now, the circuit be supposed to be subdivided into a large number of equal parts, each of one linear unit in length, and each part on one conductor be conceived of as joined to the corresponding part on the other conductor by a condenser having a capacity equal to the capacity of the line per unit of length, the line may be represented as the sum of a great number of branch circuits, each

containing a capacity equal to the capacity per unit of length. The total impedance of such a circuit is the reciprocal of the vector sum of all the reciprocals of the branches. Moreover, as even dry air is not a perfect dielectric, and as aerial circuits are rarely, if ever, immersed in even moderately dry air, there will also be a certain amount of leakage across the conductors per unit of length; and thus the branch circuits along the conductor may be regarded as circuits having a capacity equal to the capacity per unit of length of the line, in series with a resistance equal to the insulation of the line per unit of length. The line may now be regarded as a number of branch circuits, containing resistance, inductance, and capacity, and treated accordingly. The full and exact solution of this problem leads to the use of hyperbolic functions and complex algebraic quantities. There are three methods of approximation which avoid mathematical difficulties, and which may be quickly and rapidly applied.

491. 1. When the line is not over three to five miles in length, the total capacity and leakage may be considered as concentrated in an equivalent condenser placed across the center of the line. There are, then, two parts of the circuit to consider. First: The portion extending from the generator to the center of the line, having a resistance and inductance equal to one-half of the total resistance and inductance of the line. Second: At the center of the line is a branch circuit consisting of two parts; one having a resistance and inductance equal to one-half of the total line resistance and inductance, and the other a resistance and capacity equal to the total line capacity and line insulation. The joint impedance of these branches is to be obtained by either the method given on p. 439 or that on p. 442. Having obtained this joint impedance, it is necessary to add it to the impedance of the first portion of the circuit, remembering that the vector sum is the desired quantity.

492. 2. A closer approximation may be obtained by dividing the line into any desired number of parts, attaching to each its proper resistance, inductance, and capacity, and obtaining the joint impedance of all these branches, as above indicated. In this way accuracy may be carried to any desired limit that the patience of the operator will permit.

493. 3. As capacity is equivalent to a negative inductance, it can be shown[1] that for an aerial line uncomplicated by the resistance,

[1] See *Traité de Télégraphie*, par T. Tomas, p. 313.

inductance, and capacity of the receivers at the end of the line, the impedance may be expressed by —

$$J = \sqrt{R^2 + (L - \tfrac{1}{8} CR^2)^2 \omega^2}. \qquad (177)$$

494. Tabular Values. — For all these methods, the capacity per unit of length of the line is required. Unfortunately, the capacity of a circuit is a function, not only of the geometrical relations of the conductors and the potential acting, but is also affected by the geometrical relations of the circuit to all other neighboring bodies. Thus, for the simple case of an ordinary aerial line, to accurately ascertain the capacity, consideration must be given not only to the two conductors, but to the presence of the poles, insulators, and cross-arms, or other supports, and also to the earth itself. If other conductors are in the immediate vicinity, the results are still further involved, while if the neighboring wires are under electrical action, the mutual reactions present a problem of the greatest complexity. If the mutual effect of two parallel wires of a radius r and separated by a distance d, is considered, while the reaction of neighboring bodies, the earth included, be neglected (which in a majority of cases is sensibly true), Mr. Heaviside[1] shows that the capacity is determined by the expression —

$$C = \frac{1}{4 \log_\epsilon \dfrac{d}{r}}.$$

Here the value of C is in electrostatic C. G. S. units.

On Sheet 3, TABLE No. **72, f** and **f′** are plotted for this value of C. To use these curves, the ratio of d/r is found from the diagonal scale **a**, Sheet 1, and the value of C is ascertained by following a horizontal line from the value of d/r (found as previously described on Sheet 1), on the left-hand scale, to its intersection with the curve **f** or **f′**, Sec. 5, and then a vertical line to the top or bottom of the sheet. Here, on the scale marked —

$$\frac{1}{4 \log_\epsilon \dfrac{d}{r}},$$

will be found the value of C. To illustrate: Assume two bare wires, No. 00, are placed on insulators in a conduit 6″ apart. As the conductor is 365 mils diameter, $r = 183$ mils (.463 cm.) and $d = 6''$

[1] *Electrical Papers*, vol. i., p. 43.

(15 cm.). On Sheet 1, as already explained, find the value of d/r as 32.4, then turning to Sheet 3, find 32.4 on the left-hand scale marked "d/r"; follow a horizontal line to the curve f, value of

$$\frac{1}{4 \log_e \frac{d}{r}},$$

then follow a vertical to the top scale of the Table marked Values of

$$\frac{1}{4 \log_e \frac{d}{r}},$$

finding .074 as the desired capacity of the wires in electrostatic C. G. S. units, per centimeter of length.

495. Usually capacity in microfarads per unit of length is a more convenient quantity for the purpose of calculation. The following, TABLE No. 73, gives the necessary multipliers sufficing to transform E. S. C. G. S. units into M.F. for each of the customary units of length.

TABLE No. 73.

Multipliers to Transform E. S. C. G. S. Units into M.F. per Unit of Length.

E. S. C. G. S. UNITS MULTIPLIED BY	EQUALS M.F. PER	E. S. C. G. S. UNITS MULTIPLIED BY	EQUALS M.F. PER
.19	Mile of 5280 feet.	.000108	Yard.
.036	1000 feet.	.1205	Kilometer.

496. Thus, in the example, $.074 \times .19 = .01405$ M.F. per mile, it will be seen that the vertical scales d/r, $2\,h/r$, and $2\,h/d$ have two sets of numbers, one in heavy type, and one in light. There are two curves for each expression, one drawn with a heavy line, and one with a light line. Also, the horizontal scales for the heavy line curves will be found at the bottom of the sheet, and for the light line curves at the top. The heavy type on the vertical scales correspond to the heavy curves, the values of which must be read off on the scales at the *bottom* of the sheet, while the light-face type on the vertical scales corresponds to the light curves, values of which must be read on the scales on the *top* of the sheet. The heavy curves are drawn for the small values of d/r from 0 to 20, and the small values of $2\,h/r$ and $2\,h/d$ from 1 to 2,000; while the light curves are for larger values of d/r from 20 to 500, and the large

values of $2\,h\,/\,r$ and $2\,h\,/\,d$ from 2,000 to 20,000. Having, by means of the Tables, ascertained the value of the line capacity per unit of length, the impedance may be determined by either of the above methods at the discretion of the operator.

497. Sec. *b.* — One aerial wire with ground return.

By means of the method of "Images," as indicated on page 452, it can be shown that the capacity of an aerial wire with a ground return is given by the expression —

$$C = \frac{1}{4 \log_e \frac{2\,d}{r}},$$

and the value of this formula may be at once derived as just described, by substituting $2d$ for d.

498. Sec. *c.* — Concentric Cables.

When one of the conductors is rolled into a cylinder surrounding the other, forming a concentric cable, the geometrical relations of the circuit are not altered, and the capacity may be expressed by the same formulæ, by substituting r', the radius of the outer conductor, for d. In the formula —

$$C = \frac{1}{4 \log_e \frac{d}{r}},$$

the value of C is for one *unit* of conductor length, and the total capacity is obtained by multiplying by the entire length of the circuit. In speaking of concentric cables, it is usual to consider the *length* of the *cable*, which is only *one-half* the length of the circuit contained *by* the *cable;* and if the formula —

$$C = \frac{1}{4 \log_e \frac{r'}{r}}.$$

used to give the capacity of a cable, be multiplied by the length of the cable, the result will be only one-half the desired amount. It is necessary that the value of C be multiplied by the cable circuit or twice the length of the cable for the true capacity, and the formula reduces to the common expression —

$$C = \frac{1}{2 \log_e \frac{r'}{r}}.$$

499. SEC. *d.* — Effect of adjacent bodies.

In the consideration of capacity effect, attention has only so far been given to the mutual reactions of the two conductors forming the circuit. If circuits could be thought of as electrically separated from all other bodies, there would be no further modification; but this is never the case and it becomes necessary to recognize the presence of other bodies. Take the simple case of a single wire of radius r, set at a height h above the ground, with a ground return as modified by the presence of an additional wire of the same size, set at a distance d from the first wire. Mr. Heaviside shows (in the article above referred to) that, under these circumstances,—

$$C = \frac{.4343\left(2\log\frac{2h}{r}\right)}{\left(2\log\frac{2h}{r}\right)^2 - \left(2\log\sqrt{1+\left[\frac{2h}{d}\right]^2}\right)^2}. \qquad (178)$$

In TABLE No. 72, Sec. 6, there will be found, on the right-hand side of the center vertical scale, a set of diagonals **a′** for ascertaining the values of $2h/d$ and $2h/r$. The height above the ground h is on the top horizontal scale, while the diameter of the conductors and the distance between them will be found on the bottom scale; the portion of the table is used in a manner similar to that given for the **a** on Sheet 1. As an example, assume a No. 00 wire 10 ft. from the ground, with a second No.00 wire 6″ from it. Here, the diameter of the wire is 365 mils, hence $r = 183$, $d = 6'' = h = 10$ ft. $= 120''$. Find $2h/d = 40$, by following a diagonal from 6 on the lower bottom scale to its intersection with a vertical through 120 on the top scale, and then a horizontal to the scale marked "$2h/d$," finding 40 as the desired value. Find $2h/r = 1{,}310$, in a similar manner. Then determine the values of —

$$.4343\left(2\log\frac{2h}{r}\right) \quad \text{and} \quad \left(2\log\frac{2h}{r}\right)^2,$$

by following a horizontal from 1,310 to the intersection with the respective curves **g′** and **h′**, and then a vertical to the lower scales, getting respectively 2.7 and 38.8 as the desired values. The value for $\left(2\log\sqrt{1+\left[\frac{2h}{d}\right]^2}\right)^2$ is gained in a similar manner, as 10.26,

by following a horizontal through 40 to the intersection with the curve i, and then a vertical to the top scale, obtaining 10.26; then,

$$C = \frac{2.7}{38.8 - 10.26} = .094 \text{ electrostatic C. G. S. units.}$$

In a similar manner the effect of more than one adjacent wire may be obtained. The problem, however, soon becomes so complex as to be very difficult of solution. The addition of a second wire increases the capacity about 11 per cent, and with three more wires the increment is about 24 per cent. Probably the most important case of the effect of adjacent bodies is the consideration of the reaction of the earth on a complete metallic circuit. For this combination Mr. Heaviside shows that the capacity is given by the expression —

$$C = \frac{.4343\left(2 \log \sqrt{1 + \left[\frac{2h}{d}\right]^2}\right)}{\left(2 \log \frac{2h}{r}\right)^2 - \left(2 \log \sqrt{1 + \left[\frac{2h}{d}\right]^2}\right)^2}. \qquad (179)$$

The solution of this formula is made in the same manner as given for the preceding case. Assuming the same data as in the last example, —

$$C = \frac{1.39}{38.8 - 10.26} = .048 \text{ electrostatic C.G.S. units.}$$

500. Character of Dielectric. — In the preceding formulæ for capacity, the value of the specific inductive capacity of the dielectric has been assumed as 1, the value for air. Should any other substance be used, the formulæ must be multiplied by the proper coefficient k, for difference in specific inductive capacity; the value for k will be found in TABLE No. 74.

TABLE No. 74.

Specific Inductive Capacity.

NAME OF SUBSTANCE.	SPECIFIC INDUCTIVE CAPACITY.	NAME OF SUBSTANCE.	SPECIFIC INDUCTIVE CAPACITY.
Air	1	Carbon Di-oxide	1.00066
Glass	1.90 to 3.013	Hydrogen	.99967
Shellac	1.95 to 2.740	Vacuum	.99941
Sulphur	1.93	Yellow Wax	1.86
Gutta-percha	2.580 to 4.20	Resins	1.80
Ebonite	2.284	Hooper's Composition	3.10
India-rubber	2.220 to 3.70	Mica	5.00
Turpentine	2.160	Flint Glass, extra dense	6.55 to 10.10
Petroleum	1.6 to 2.070	Distilled Water	76.00
Paraffine	1.98 to 2.00	Ozokerite	2.13
Carbon Bi-sulphide	1.810	Pitile	1.80

CHAPTER XIV.

SERIES DISTRIBUTION.

Art. 501. Origin. — In the earliest attempts to distribute energy by means of electricity, one source of supply, or generator, was connected directly with a single device for utilizing the energy produced. The generator and receiver were separated by but a short distance; and thus a simple circuit of wire, sufficient to carry the small amount of current produced by the early dynamos, was amply sufficient for the purpose required.

502. The next step in the development of distribution was the introduction of two or more receiving mechanisms placed successively upon the same circuit. From this as a starting-point systems for supplying electrical energy have gradually grown until they have attained their present complexity, involving miles of mains receiving from central stations of immense size amounts of energy to be measured by thousands of horse-power, and distributing the same over many square miles of territory. So long as a single generator supplied but one receiver, the load was a constant one, the receiver, when running, absorbing all the energy delivered by the generator, and the generator operating under no load when the receiver was cut out of service. In modern systems the load not only varies in quantity from time to time, thus varying the demands placed both on the distributing system and upon the generators; but often the load is a movable one, its position with reference to the generating-station constantly varying. Thus it is apparent that with the developments of new and additional methods for utilizing electrical energy many complicating factors have been introduced into the problem of distribution.

503. Classification. — The quantity of energy carried by any circuit is measured by the product of two factors, one, the electromotive force or pressure, being that unknown quality of this form of energy by means of which it is enabled to overcome resistance, and the other the quantity or amount of electricity, which, by the aid of

the electro-motive force, is set in motion, and is therefore capable of doing work. Three methods may therefore be employed for varying the amount of energy delivered by any circuit. If the quantity of electricity remains constant, the quantity of energy will vary directly as the electro-motive force. If the voltage is kept constant, and the quantity of current varied, the amount of energy transmitted will be in direct proportion to this variation. To state the relation in mathematical language, if Q be the quantity of energy, V the voltage of the circuit, and I the amount of current in amperes, Q varies directly as the product of VI. Thus it is apparent that by varying either of the factors, the amount of energy transmitted to any point may be consequently changed in any desired degree. It is also plain that a similar change could be effected by varying both the current and electro-motive force. For most purposes, however, the variation of both factors introduces undesirable complications to such an extent that this latter method is rarely, if ever, adopted. To recapitulate, therefore, circuits may be treated:—

First. As constant current circuits.

Second. As constant potential circuits.

The problem of distribution under either of the preceding divisions may still further be varied by the relative position of the generator and receivers. Under the supposition either of a *constant current* or a *constant potential* circuit, the receivers may either be placed at a constant distance from the generating-station, or may, from time to time, occupy a varying position with reference to the same. So four conditions arise under which distribution may be considered.

1. Constant Current Circuits having the generators and receivers at fixed distances respecting each other.

2. Constant Current Circuits having the generators and receivers at varying distances respecting each other.

3. Constant Potential Circuits having the generators and receivers at constant distance respecting each other.

4. Constant Potential Circuits having the generators and receivers at varying distances respecting each other.

504. 1. *Constant Current Circuits with Generators and Receivers at Fixed Distances.* — From the first attempts involving a single generator delivering all of its energy to one receiver, the next

step was to embrace along one circuit two or more receivers, placed one after the other. As the receivers were arranged succeeding each other, having the same current pass through all of them, this kind of circuit came to be known under the name of "Series Distribution." The current through the entire circuit being a constant one, this method is particularly adapted to installations covering a large territory, in which the load throughout the entire area is essentially uniform. Municipal illumination, whether by arc or incandescent lamps, is properly arranged by the series system. The operation of motors upon series circuits is perfectly feasible, especially if the motor load be so reasonably constant that the machines may run steadily and uniformly. As all of the receivers are traversed by the same current, the conductor that successively unites them is most simply arranged along the sides of an irregular polygon of which the various receivers form the apices. The location of the line should therefore be designed by a careful examination of the proposed site of the circuit, in order to select among all of the possible locations that which will give a polygon having the shortest total perimeter. Frequently the arrangement of city streets, or regulations of city authorities, militate against the selection of the shortest and most direct route for the circuit. The dictates of economy, however, indicate that special attention should be given to arranging the circuit with a view to attaining the minimum length of conductor that can possibly be selected.

505. Location of Station. — After the location of the circuit is determined, it is entirely immaterial at which point upon the route the central station is placed. Should it be impracticable to locate the plant exactly upon the line of the route, it should be situated as near to it as circumstances will permit; and all locations giving the same distance measured along the line of the conductor from pole to pole of the generators are equally favorable. This latitude in the location of the central station is one of the most valuable properties of the series system; for it allows the selection of the site of the central station to be entirely controlled by such conditions as economy in cost of real estate, availability of fuel, water supply, etc.

506. Current Density in Main Circuit. — As soon as the location of the circuit is selected, it becomes possible to design the line. Here the engineer must make such a selection between the dimen-

sions for the conductor as indicated by strict rules of economy, and those proscribed by the commercial limitations of manufactured goods, as will lead to the best and most economical design. The nature of the service to which the plant is to be applied is usually the chief governing condition; so a reasonably accurate knowledge of the number of receivers, the current and electro-motive force of each, and the resistance of the line, must be known, together with a parallel knowledge of the properties of the generators obtainable, in order that the supply and demand of station and line may be mutually adjusted. As the plant is to be a constant current one, in which all parts of the circuits are traversed by the same current, it is apparent that all the receivers must be capable of operating under this imposed current, and that only such receivers as can do so must be placed in the circuit. Only such generators as can supply this predetermined current can be used in the station. By varying the electro-motive force at the terminals of the receivers, different amounts of power can be supplied to different customers.

Let I = the current selected for the line in amperes.
E = the electro-motive force of the station.
e, e', e'', etc. = electro-motive forces of the different receivers.
n, n', n'', etc. = the number of each kind of receiver.
R = the resistance of the line.
L = the length of the line from pole to pole of the station.
S = the cross-section of the conductor.
ρ = the specific resistance of the conductor.

The energy demanded by the receivers is evidently—

$$\Sigma I\,(ne + n'e' + n''e'' + \text{etc.}). \tag{180}$$

The resistance of the line is—

$$R = \frac{\rho L}{S}.$$

In order to deliver a current of I amperes to the customers, an amount of energy equal to $\rho LI^2 / S$ must be expended in the line; the station, therefore, must supply energy to the amount of—

$$EI = \Sigma I\,(ne + n'e' + n''e'' + \text{etc.}) + \frac{\rho LI^2}{S}. \tag{181}$$

The number of receivers, the current, and electro-motive force required by them, with the length of the conducting circuit, are fixed by the general condition of service that the proposed plant is intended to

perform; so in equation (181) there remains E, ρ, and S as possible variables whose value is to be determined by the designer according to the best economic condition. Experience has eliminated all materials except copper from circuits designed to supply power; ρ, therefore, may always be assumed as the specific resistance of this metal. In the selection of E, the engineer is limited to the existing commercial forms of dynamos, or some combination of them. It is advisable to keep E as low as possible; for with high potentials the danger and difficulties to be encountered, the probabilities of interruption to service, and the expense of maintenance are largely increased, thus, in reality, S becomes the important variable; solving, then, for S, —

$$S = \frac{\rho L I}{E - \Sigma(ne + n'e' + n''e'' + \text{etc.})}. \qquad (182)$$

For an exact solution of this equation the value of E must be known. This, however, as has been seen, may vary within what may be called commercial limits. Now, as the cost of the line varies quite closely with S, it becomes important to inquire into the conditions governing S and E that shall, in the most commercial manner, reduce original outlay and maintenance.

The quantity I / S is the current density per unit of area of the conductor, and is frequently used, lines being simply proportioned so that the current density shall not exceed a certain predetermined amount.

507. Economical Conditions. — So long as electrical distributions were comparatively of small magnitude, involving but a single generator supplying one receiver and requiring but a limited circuit, the question of economy in the conductors occupied but a small and subordinate field of consideration. A wire amply large enough to transmit all of the energy was introduced with but little thought as to the cost of the circuit. As soon, however, as systems of distribution commenced to ramify over areas of magnitude, the cost of the copper conductors immediately arose to a position of great importance, in many cases equalling, if not exceeding, the cost of the remainder of the plant; therefore rendering it imperative that their design should be treated with the utmost care along the lines of the most rigid economy.

In designing a system of conductors, eight points must be carefully considered in order to secure the best results.

1. The conductors *must* be so proportioned that the energy transmitted through them will not cause an undue rise of temperature.

2. The conductors *must* have such mechanical properties as to enable them to be successfully erected, and so durable as to require a minimum of annual maintenance.

3. The conductors *may* be so designed as to entail a minimum first cost in line construction.

4. The conductors *may* be designed to attain a minimum first cost for station construction.

5. The conductors *may* be so designed to reduce first cost of plant, and cost of operation and maintenance to a minimum.

6. The conductors *may* be designed to secure minimum total first cost of installation.

7. The conductors *may* be so designed as to secure maximum conditions of good service.

8. The conductors *may* be so designed as to attain a maximum of income with a minimum of station first cost.

508. Careful consideration of the foregoing conditions indicate such a degree of incompatibility between them that it is impossible to fully realize all in any one plant. The skill of the designer is, therefore, to be exhibited in such a selection of governing conditions as will, in each particular case, develop a maximum service with a maximum economy. Compliance with the first two conditions is *necessary* in all distributing installations; for if either the safe heating limit, or working strength of the conductors be exceeded, the lines become positive sources of danger to life and property.

509. 1. **Design for Heating Limit.** — In every conductor a certain amount of energy is transformed into heat and wasted by being radiated from the conductor itself.

The most economical size of conductor to be used for a particular installation will then depend largely upon the cost of producing energy; for, if the station operating expenses are low, so that the cost of production is small, and cost of the conductors comparatively high, it is obvious that the least metal section consistent with safety should be selected, in order that the interest on the cost and the maintenance expenses of the conductor may be a minimum, and balance the cost of the amount of energy lost by transformation into heat.

510. On the contrary, where station operating expenses are high, and cost of conductor installation is comparatively low, the converse will hold true; it being under these circumstances advisable to put a larger investment of capital into the conductors in order to reduce the losses in the line to a minimum.

511. It is conceivable that, under the first conditions, the cost of producing the energy lost in the conductors may be so great, that to attain the most economical arrangement the conductors should be so small that the energy transformed into heat would be sufficient to raise the conductors to a dangerous temperature.

Notwithstanding the masterly investigations of Mr. Kennelly into the subject of the heating of conductors, to which reference has been made in Chapter XII., there is yet hardly as much experimental knowledge on the subject as could be desired, so that electrical circuits are often located in situations to which Mr. Kennelly's rules do not fully apply, or in which the service is of unexpected severity.

The case of concentric conductors is one of peculiar interest. Here one conductor, being entirely within and inclosed by a second, can have little or no chance for radiating the heat developed in it unless the limit be kept to so low a point that the heat in the interior conductor may pass through the insulating envelope, through the second conductor, and thence through the exterior envelope, into the air, without developing in the central conductor a destructive temperature. Under such circumstances, a very large factor of safety must be allowed in the heating limit assigned. This point is particularly emphasized in Mr. Kennelly's deductions. Circuits of this kind are particularly liable to injury from overheating, as they are used to transmit very large quantities of electrical energy, and the pressure brought to bear on the designer to effect a saving of copper is usually severe. Such circuits are also usually inclosed in some form of conduit structure where the chances for radiation are exceedingly poor. It is true that the conduits being buried in the earth are constantly surrounded by a low mean temperature, which greatly adds to the safety of the inclosed circuits; yet, on the other hand, the lack of air circulation and poor conductivity, of either the earth or conduit structure, must not be lost sight of in planning for the dissipation of the heat inevitably evolved. For interior wiring, special pains should be exercised in the design of the conductors to

keep their maximum temperature well under control. While the rules of the various boards of Fire Underwriters (See Appendix to Chapter III.), if followed, are designed to afford ample protection to buildings carrying electrical circuits, there always exists a temptation on the part of the designer, as well as of the contractor and builder, to effect economy by using a minimum amount of copper, protected by a low grade of insulation; by employing the cheapest and least efficient forms of interior conduits; and to reduce the number of safety appliances to a minimum. The consumer, on the other hand, usually plans for less electrical service than his future requirements are certain to demand. Thus there is the constant tendency on the one side toward insufficient conductors and dangerous installations, and on the other toward the use of a current exceeding that even for which the circuits, conduits, and other appliances were designed. It is therefore essential to use particular care and to check the designs for size of conductors with the most unfavorable circumstances that can be applied to the location in which they are placed, as indicating the probable temperature that may be attained by the circuits.

512. 2. **Mechanical Strength.** — It frequently occurs that the safe heating limit indicates a wire of so small a size as, mechanically, to be impracticable. All circuits, whether overhead, underground, or in interior conduits, require a certain amount of mechanical strength in order that the conductors may be introduced in their appropriate places with a minimum amount of installation expense, and without endangering the integrity of the line. The lines must likewise have sufficient strength to withstand for a reasonable period of time the natural wear and tear to which plants of this kind are subjected. It would at first sight appear that the conductors, after being installed either in underground conduits, or as house-wiring, should be exempt from disturbing influences, and would constantly retain their integrity. On the contrary, numerous causes are operative, constantly exposing the circuits to disturbing influences, such as settlement and shrinkage of the structures in which the circuits are inclosed, the mischief done by rats and mice, necessary changes and rearrangement of the lines, and many other causes of similar description — all tending to affect the mechanical integrity of the conductors.

In aerial circuits unusual snow or sleet loads, high winds, the

abrasion of tree branches, etc., are constantly tending to destroy the conductors. Thus, in many cases, the safe heating limit may indicate a conductor too weak from a mechanical standpoint to be successful; and the introduction of such a circuit, while perhaps more economical in first cost, will, in a very short time, prove to be enormously expensive from the standpoint of maintenance.

513. 3. Minimum First Cost of Line. — The first cost of any circuit is separable into three distinct elements:

First. The actual cost of the copper necessary to transmit the required amount of energy.

Second. The cost of insulating or protecting the same electrically.

Third. The cost of installing or erecting the circuits.

For installations simply for temporary purposes, such as the illumination of, or operation of motors for, the construction of public works, etc., in which the area covered is of small extent, and the plant only expected to run for a limited portion of time, the most rigid economy should be exercised in the design of the circuit, and in the provisions made for its introduction; for it is obvious that the circuit being in use but for a short period of time, and in a location where the wire is likely to undergo considerable injury, will be subject to but very little salvage when the work for which it is installed is completed. Thus, under these circumstances, the cheapest kind of pole-line, with meager insulation, sufficient only for purposes of safety to the workmen, and embracing the smallest possible amount of copper, is the one to be selected.

From such circuits it is rare to obtain more than one-third or one-fourth of the value of the line as salvage.

The generating-plant, on the contrary, so far as dynamos and engine consist, is but little injured by service of this kind, and may, at the end of the work, be credited back at almost its full value. In such lines it usually pays to waste a large amount of energy in the conductors in order to reduce the first cost of the line to a minimum.

514. With permanent distribution plants of large magnitude, such as are usually tributary to central stations, the first cost of the circuit, while it should receive careful consideration, should never be allowed to militate against the introduction of the very best possible

style of conductors adequately designed for the work thrown upon them, and protected by all the best known means, either in underground conduits or on the strongest and most substantially constructed pole-lines.

515. 4. **Minimum First Cost of Station.** — The minimum first cost of station is obviously incompatible with the minimum first cost of conductors, for, if the amount of metal in the conducting circuit be reduced to the lowest point of safety, a very much larger amount of energy will be lost in the circuit, and still an additional amount usually escapes through leakage due to defective insulation.

To obtain minimum first cost on station plant, it is essential to expend a much larger capital in the line, in order that the station plant may be enabled to deliver the requisite amount of energy to the various receivers, without being loaded with line losses.

516. In city locations, where underground conduits are a necessity, the cost of the circuits is the largest item in the installation of the plant. In order to avoid constant reopening of the streets to accommodate enlargements or extensions, it is advisable to work out the design of the conductors on a sufficiently large scale to meet all of the business that is likely to accrue for several years. The conductors under these circumstances will be much larger, and will cover a very much greater territory, than the immediate demands of the business will indicate, and will necessitate a corresponding investment. Yet a structure of this kind, carefully arranged to reduce the annual maintenance to a minimum is, under such circumstances, a paying investment. The station, on the other hand, may be planned for a minimum of first cost, and the buildings so arranged that additional generating units may be added from time to time as the business grows. The utility, under such circumstances, of a super-abundance of copper in the conductors is also apparent, as it evidently affords to the station the ability to carry the load thrown upon it with the least expenditure of energy lost in the conductors themselves, and with the least initial investment of capital.

517. 5. **Minimum First Cost of Plant and Minimum Cost of Maintenance and Operation.** — To reduce the initial cost of the conducting system to a minimum, it is necessary to employ the smallest mains consistent with safety. This plan causes consid-

erable waste of energy in the leads by transformation into heat, thus increasing the cost of the operating expense by the amount required to produce this lost energy, and also necessitating such an additional expense in the construction of the station as is required to provide the additional amount of plant necessary to produce the energy wasted in the conductors, over and above that which is essential to supply the demands of the customers. Thus false economy introduced by reducing the size of the conducting system may increase both the total cost of the plant and the cost of operation. On the contrary, by using large mains of low resistance, the lost energy and cost of additional station capacity may be reduced to any desired amount, but only by a corresponding increase in the expense of the conducting system.

518. There evidently exists in every plant a certain relation between the cost of station equipment, conducting system, and lost energy that will reduce the sum of these three quantities to a minimum, indicating the conductor that in the long run will be the cheapest, both as regards the gross expense of installation and the cost of operation. The determination of this, the most economical cross-section of the conductor, is somewhat complicated, and must be made for each plant under its peculiar conditions of operation, with special reference to the following considerations: —

First. Cost of station per watt of output.
Second. Cost of producing energy per watt.
Third. Cost of conductor per unit of cross-section and length.
Fourth. Cost of conductor insulation per unit of cross-section and length.
Fifth. Cost of erecting or installing the line (such as pole-line or conduit expense).
Sixth. Rate of interest on total invested capital.
Seventh. Rate of depreciation upon capital invested in the station.
Eighth. Rate of depreciation upon the cost of the metallic portion of the conducting system.
Ninth. Rate of depreciation upon the cost of the insulating portion of the conducting system.
Tenth. Rate of depreciation upon the cost of the conduit, or pole-line.

519. This problem was first proposed to electrical engineers by Sir William Thomson in 1881. The solution then suggested predicated that the total cost of the conducting system varied directly as the weight of the material employed for the conductors, and that it was simply essential to make the annual interest and depreciation upon the cost of the conducting system equal to the cost of the energy wasted therein. Closer investigation, however, indicates the advisability of considering as variables all of the afore-mentioned quantities.

520. Scrutinizing the cost of conductors, their expense may evidently be divided into two parts — one the cost of the metal employed, and the other the cost of the insulating material. The expense of bare wire and copper strips evidently varies as their weight or cross-section; and the expense of the material for uninsulated lines may be expressed by the equation —

$$y = bS, \tag{183}$$

in which y is the cost per unit of length, S being the cross-section of the conductor expressed in any desired units, and b a constant depending upon the varying price of line material per unit of weight.

Stranded cable is slightly more expensive than solid conductors, but this simply increases the value of b.

521. While the amount of insulating material necessary to protect wires and cables does not vary exactly with the area, the rate of variation for all of the more common forms commercially employed is so nearly proportional to the cross-section, that this rate may be assumed without serious error. So, for any given class or kind of insulation, the expense of the conducting system may, with sensible accuracy, be expressed by the equation, —

$$y = a + bS, \tag{184}$$

in which a and b are constants, depending upon the mode of manufacture, and the kind and quality of the insulation, and the current market price of the material used. To determine the constants of this equation for any particular make of conductor, or class and quality of insulation, the prices for three or four cross-sectional areas should be obtained, and their values plotted on a sheet of cross-section paper by assuming the axis X to be the axis of the areas, and that of Y the axis of cost. By obtaining three or four points in this

way, and drawing through them a line, a curve of prices is obtained, the tangent to which, at any point, is expressed by the equation, $y = a + bS$, from which the cost of any desired size of conductors may be readily obtained. Some examples of such curves will be found in Chap. XVIII.

522. By means of a similar train of reasoning and graphical construction, the cost of pole-lines, conduits, subways, or other structures necessary for the installation of the conducting system, may be expressed and obtained by a similar equation —

$$y' = a' + b'S, \qquad (185)$$

in which y' is the cost per unit of length of the structure, and a' and b' are constants, depending upon the kind of line to be built, while S is the area of the conductor as before.

523. The cost of the line installation, however, cannot be nearly so exactly determined for variations in the size of the conductors, as it is evident that the style of installation which is adopted is a very large factor in the rate of variation of the cost. In the ordinary pole-line, the cost will be almost precisely the same for a very large variation in the cross-section of the conducting system; for a single line of poles may be made to carry either one very small conductor, or a great many of large cross-section, the only additional expense entailed upon the additional number of wires being that necessary for the insulators, and the labor of putting the lines into place. Thus, for pole-line construction, the constant a' is a large proportion of the value of y, $b'S$ being relatively small.

524. In a similar manner, that fraction of the cost of underground conduits, which is embraced in the items of paving, excavation, construction of manholes, etc., is very nearly constant over very wide ranges of conduit capacity and line area, the cost of the material used for the ducts, and labor of placing the same, being the chief items that vary to any great extent with the size of the conductor. For a concrete conduit, for example, with bare wire mains, the value of b' is zero; for this description of conduit can contain any desired cross-section of conductor, with no variation in the expense of construction. The cost of placing the conductors in position should be included in the term b', and will also be found to be sensibly constant for all cross-sections, excepting for conductors of very

large size, but will vary considerably if the required conductor section is split into several parts.

525. Equations for minimum first cost of plant, and minimum cost of operation and maintenance.

Let i = the rate of interest charged against the plant in per cent.
d_l = the rate of depreciation charged against the line in per cent.
d_c = the rate of depreciation charged against the conduit in per cent.
L = the length of the conducting system in any desired units.
U'' = the annual charge against the line for interest and depreciation.

The cost of the line will be —

$$Ly = L\,[(a + bS) + (a' + b'S)]\,; \tag{186}$$

then,
$$U'' = L\,[(a + bS) \times (i + d_l) + (a' + b'S)\,(i + d_c)]. \tag{187}$$

For simplification, let

$$\alpha = L\,(a\,(i + d_l) + a'\,(i + d_c)),$$

and
$$\beta = L\,(b\,(i + d_l) + b'\,(i + d_c));$$

then,
$$U'' = \alpha + \beta S. \tag{188}$$

Let F = the number of hours per annum that the plant operates.
K = the cost of producing energy per watt-hour, K.W.-hour, or H.P.-hour,

then assuming the notation on page 463, $\rho I^2 L / S$ gives the energy lost in the line, and as ΣI $(ne$ + etc.) is the energy supplied to the customers, the station must supply —

$$\Sigma\, I\,(ne + \text{etc.}) + \frac{\rho I^2 L}{S} \text{ watts.}$$

The cost per annum of the energy lost in the line will be $F\rho I^2 LK / S$. If K' be the cost per watt of output for equipping the station, and i and d_s the rates of interest and depreciation on the station, then

$$\frac{\rho I^2 LK'}{S}\,(i + d_s)$$

will be the annual charge for interest and depreciation on this expense. If U' be the total cost per annum of the lost energy, then

$$U' = \frac{\rho L I^2}{S}\,[FK + K'\,(i + d_s)]. \tag{189}$$

For simplification, let

$$\lambda = \rho L I^2\,[FK + K'\,(i + d_s)]\,;$$

then,
$$U' = \frac{\lambda}{S}.$$

Let $U = U' + U''$, then

$$U = \alpha + \beta S + \frac{\lambda}{S}; \tag{190}$$

differentiating with respect to S,

$$dU = \beta dS - \frac{\lambda dS}{S^2};$$

$$\frac{dU}{dS} = \beta - \frac{\lambda}{S^2} = 0;$$

$$S^2 = \frac{\lambda}{\beta}, \qquad S = \sqrt{\frac{\lambda}{\beta}}. \tag{191}$$

526. A consideration of this equation will reveal several important deductions.

First. It will be noticed that the value of S obtained makes that fraction of U' which varies with S equal to U'', indicating that the most economical area of the conductor to be employed is that in which the annual cost of energy expended in it is equal to the sum of the interest and depreciation on that fraction of the total capital outlay which is proportional to the weight of the conductor employed.

527. It is also to be seen that inasmuch as E and L do not enter into this equation, the most economical section of the conductor depends simply upon the amount of current in the circuit, and is entirely independent, either of the voltage at the generators, or at the distance to which the energy is transmitted.

528. In selecting the values for the various constants in the preceding discussion, considerable judgment should be exercised.

The value of i, the rate of interest upon the total capital invested, will vary according to the location, and will naturally be made to conform to the prevailing rates of interest for money at the location of the plant.

529. The rate of depreciation on the station, d_s, will naturally subdivide itself into four constituents, the rate on the buildings being the least of these, which for fire-proof construction may be taken as low as 2 to 3 per cent, while for buildings of wood or of less permanent character this constant will vary from 5 to 8 per cent. The depreciation on dynamos, provided standard types of machines are selected, and are not allowed to be dangerously overloaded, is also exceedingly small, varying from 2 to 4 per cent.

530. For the prime movers, whether steam or water motors are

selected, the rate should be considerably higher, varying from 5 to 10 per cent, while on the boilers, in the case of the steam plant, the rates of depreciation are greatest, and should be assumed at from 8 to 16 per cent.

531. For the constant d_c, the depreciation upon the conduit or pole-line part of the conducting system also varies between widely different limits.

532. Permanent structures, such as cement-lined or iron-pipe concrete conduits, or earthen-pipe conduits, undergo little or no depreciation, and for these structures d_c may be assumed not to exceed 2 per cent per annum.

533. For wooden conduits or pole-lines, on the contrary, the value of d_c should be from 10 to 20 per cent, depending on the location. In a similar manner d_l, the depreciation on the value of the circuits, may extend over a wide range. For lead-covered cables with the highest kind of insulation, placed in underground circuits, this factor may be almost neglected. For rubber-covered wire in underground conduits, or in exposed pole-lines in thickly settled cities, this constant should have a value of 20 per cent, or more, as the insulation is very rapidly deteriorated by the effects of gas and water. For the best insulation on heavy aerial lines d_l should vary from 5 to 10 per cent; but for the poorer kinds, such as underwriters' wire, it should be 20 to 30 per cent. In cases where there are many trees, d_l may be as high as 40 to 60 per cent.

534. It is thus evident that, in determining the factors entering into the interest charge upon the cost of the plant, much careful consideration must be given, as usually the tendency is to place these factors so low that, after a short time of operation, the maintenance charges are found to be very much larger than was first estimated, and consequently sad inroads are made into the net profits of the plant.

535. The determination of the factor K' is one which will vary considerably with the character of the plant under consideration. Apparently this value would be most properly computed by determining the cost per watt of output, then assigning K' such a fractional part of this sum as is represented by the ratio of the lost energy to the total output. In many instances this value is correct. However, in the case of a large station, with a very short line, this would proba-

bly give K' too great a value; while, on the contrary, in the case of a small station with a very long line, it would give K' too small a value. It is, therefore, essential to canvass each particular instance for itself, and assign to K' such a proportionate assessment of the total station value as seems to fit the particular circumstances.

536. In a similar manner, in assigning a value to K, consideration must be given to the mutual relations of the line and station. Apparently K would be given by dividing the total operating expenses by the total output in watts; and while this value in many cases is partially correct, there are frequent situations in which it departs widely from the true amount. The values of both K' and K, and in fact all quantities of this nature, are most accurately ascertained by plotting a curve as indicated in Fig. 286, in which the axis of X indicates the varying output of the station, and that of Y either (in the case of K') the cost of installation, or (in the case of K) the cost of producing energy, and selecting for the desired value of K or K', that obtained from the equation of a tangent to the curve at that particular point, representing the circumstances in question. By this process, assuming W to represent the capacity of the station, dK' / dW or dK / dW is obtained, instead of K / W or K' / W; for in all such calculations the rate of variation of the factors entering into the problem is the true value desired.

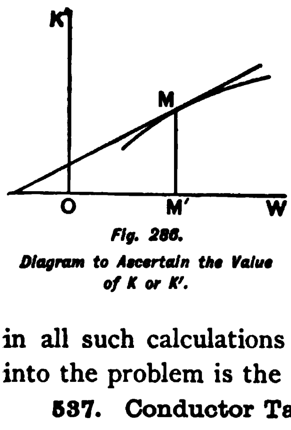


Fig. 286.
Diagram to Ascertain the Value of K or K′.

537. Conductor Tables. — To facilitate calculations for the most economical conductor cross-section, Professor Forbes in England, and Professor Cartwright in this country, have calculated a series of tables, involving the cost of erecting or laying one ton of copper and the interest and depreciation charges allowed upon the plant, from which the most economical current density per unit, of actual cross-section of conductor, can readily be ascertained. Extracts from these Tables are given in Table No. 75, A and B (pp. 478 and 479).

In section A, the left-hand vertical column contains the rate of interest and depreciation, while the top horizontal line gives figures for the cost of laying, or erecting, one ton of copper. This cost is supposed to cover the entire cost per ton of the wire or cable,

with its insulation, pole-line, conduit, or other supporting structure.

In section B, the left-hand vertical column indicates the cost of energy, in terms of one electrical horse-power, at the terminals of the generating station, while the top horizontal lines give the area of the conductor in square inches, or in circular mils. This Table is calculated for a current of 100 amperes.

The use of the Table may be best indicated by an example. Suppose, for instance, that the cost of laying one ton of copper is $600, and that 12 per cent is allowed for the sum of the interest and depreciation upon the conducting system. Following the horizontal line opposite 12 per cent in the left-hand vertical column of A, until this line intersects the column headed $600, the number .144 is obtained.

Assume also that the cost of producing one electrical horse-power is $60 per year. Taking the horizontal line opposite $60 in the first left-hand vertical column of TABLE No. 76, follow the horizontal line along until the nearest corresponding number to 144 is obtained (in this particular example the number is exactly 144, being found opposite 60). Running up this column to the top of the table, $\frac{22}{100}$ of a square inch, or 280,104 circular mils, is obtained for the requisite cross-section of the conducting system to carry 100 amperes.

If the desired current in the conducting system is any other quantity than 100 amperes, the cross-section of the conductor is obtained by solving a direct proportion thus: —

100 (amperes) : proposed current :: (the tabular area for 100 amperes) : the desired area.

538. 6. **The Conductors may be so designed** as to secure a total minimum first cost of installation, irrespective of operation and maintenance.

There arises frequent occasion to use an electric plant on work of more or less temporary nature, in which the total cost of the machinery and operation must be charged against the work in question, as the circumstances are such as to preclude the credit of any salvage. Usually, under such conditions, the cost of operation cuts too small a figure to be regarded. The cost of line and generating-plant must for this case be made a minimum. Assuming the previous notation, the station must have a capacity to supply —

$$EI \text{ watts} = \Sigma I(ne + n'e' + n''e'' + \text{etc.}) + \frac{\rho LI^2}{S};$$

TABLE NO. 75. — SECTION A.

Cost of Laying One Additional Ton of Copper.

Annual Allowance for Interest and Depreciation in per cent.	$300	$325	$350	$375	$400	$425	$450	$475
5	.030	.033	.035	.038	.040	.043	.045	.048
6	.036	.039	.042	.045	.048	.051	.054	.057
7	.042	.046	.049	.053	.056	.060	.063	.067
8	.048	.052	.056	.060	.064	.068	.072	.076
9	.054	.059	.063	.068	.072	.077	.081	.086
10	.060	.065	.070	.075	.080	.085	.090	.095
12	.072	.078	.084	.090	.096	.102	.108	.114
14	.084	.091	.098	.105	.112	.119	.126	.133
16	.096	.104	.112	.120	.128	.136	.144	.152
18	.108	.117	.126	.135	.144	.158	.162	.171
20	.120	.130	.140	.150	.160	.170	.180	.190
25	.150	.163	.175	.188	.200	.213	.225	.238

Annual Allowance for Interest and Depreciation in per cent.	$500	$550	$600	$650	$700	$750	$800	$900
5	.050	.055	.060	.065	.070	.075	.080	.090
6	.060	.066	.072	.078	.084	.090	.096	.108
7	.070	.077	.084	.091	.098	.105	.112	.126
8	.080	.088	.096	.104	.112	.120	.128	.144
9	.090	.099	.108	.117	.126	.135	.144	.162
10	.100	.110	.120	.130	.140	.150	.160	.180
12	.120	.132	.144	.156	.168	.180	.192	.216
14	.140	.154	.168	.182	.196	.210	.224	.252
16	.160	.176	.192	.208	.224	.240	.256	.288
18	.180	.198	.216	.234	.252	.270	.288	.324
20	.200	.220	.240	.260	.280	.300	.320	.360
25	.250	.275	.300	.325	.350	.375	.400	.450

Annual Allowance for Interest and Depreciation in per cent.	$1000	$1100	$1200	$1400	$1600	$1800	$2000
5	.100	.110	.120	.140	.160	.180	.200
6	.120	.132	.144	.168	.192	.216	.240
7	.140	.154	.168	.196	.224	.252	.280
8	.160	.176	.192	.224	.256	.288	.320
9	.180	.198	.216	.252	.288	.324	.360
10	.200	.220	.240	.280	.320	.360	.400
12	.240	.264	.288	.336	.384	.432	.480
14	.280	.308	.336	.392	.448	.504	.560
16	.320	.352	.384	.448	.512	.576	.640
18	.360	.396	.432	.504	.576	.648	.720
20	.400	.440	.480	.560	.640	.720	.800
25	.500	.550	.600	.700	.800	.900	1.000

Table No. 76. — Section B.

Sectional Area for 100 Amperes in Square Inches and Circular Mils.

Circular Mils.		127,320.	140,052.	152,784.	165,516.	178,248.	190,980.	203,712.	216,444.	229,176.	241,908.	254,640.	267,372.	280,104.
Sq. Ins.		.10	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20	.21	.22
Annual cost of one electrical horse-power at generator terminals. (Inclusive of interest and depreciation on buildings, motive power, and generator.)	25	291	240	202	172	148	129	114	101	090	081	073	066	060
	30	349	289	242	207	178	155	136	121	108	097	087	079	072
	35	407	337	283	241	208	181	159	141	126	113	102	092	084
	40	465	385	323	275	238	207	182	161	144	129	116	105	096
	45	524	433	364	310	267	233	204	181	162	145	131	118	108
	50	582	481	404	344	297	259	227	201	180	161	146	132	120
	55	640	529	445	379	327	285	250	221	198	177	160	145	132
	60	698	577	485	413	356	310	273	241	216	193	175	158	144
	65	757	625	526	448	386	336	295	261	234	209	190	171	156
	70	815	673	566	482	416	362	318	281	252	225	204	185	168
	75	873	721	606	517	445	388	341	302	270	241	219	198	180
	80	931	769	647	551	475	414	364	322	287	257	233	211	192
	85	989	817	687	585	505	440	386	342	305	274	248	224	204
	90	1047	865	727	620	534	466	409	362	323	290	262	237	216
	95	. .	914	768	654	564	491	432	383	341	306	277	251	228
	100	. .	. .	808	689	594	517	455	403	359	322	291	264	240
	105	. .	. .	. .	723	624	543	477	423	377	339	306	277	252
	110	. .	. .	. .	. .	653	569	500	443	395	355	320	290	264
	115	. .	. .	. .	. .	. .	595	523	463	413	371	335	304	276
	. .	. .	. .	. .	. .	. .	. .	546	483	431	387	349	317	288
	. .	. .	. .	. .	. .	. .	. .	. .	505	449	403	364	330	301
	. .	. .	. .	. .	. .	. .	. .	. .	. .	467	419	378	343	313
Circular Mils.		292,836.	305,568.	318,300.	331,032.	343,764.	356,496.	369,228.	381,960.	394,692.	407,424.	420,156.	432,888.	445,620.
Sq. Ins.		.23	.24	.25	.26	.27	.28	.29	.30	.31	.32	.33	.34	.35
Annual cost of one electrical horse-power at generator terminals. (Inclusive of interest and depreciation on buildings, motive power, and generator.)	25	055	051	047	043	040	037	035	032	. .	. .	. .	. .	. .
	30	066	061	056	052	048	045	042	039	036	. .	. .	. .	. .
	35	077	071	065	060	056	052	048	045	042	040	. .	. .	. .
	40	088	081	074	069	064	059	055	052	048	045	043	. .	. .
	45	099	091	084	077	072	067	062	058	054	051	048	045	. .
	50	110	101	093	086	080	074	069	065	061	057	053	050	048
	55	121	111	103	095	088	082	076	071	067	063	059	055	052
	60	132	121	112	103	096	089	083	076	073	068	064	060	057
	65	143	131	121	112	104	097	090	084	079	074	069	065	062
	70	154	141	131	120	112	104	097	091	085	080	075	070	067
	75	165	152	140	129	120	111	104	097	091	085	080	076	071
	80	176	162	149	138	128	119	111	103	097	091	086	081	076
	85	187	172	158	146	136	126	118	110	103	097	091	086	081
	90	198	182	167	155	144	134	125	116	109	102	096	091	086
	95	209	192	177	164	152	141	131	123	115	108	102	096	090
	100	220	202	186	172	160	148	138	129	121	114	107	101	095
	105	231	212	195	181	168	156	145	136	127	119	112	106	100
	110	242	222	204	189	176	163	152	142	133	125	118	111	105
	115	253	232	214	198	184	171	159	149	139	131	123	116	109
	. .	264	242	223	207	192	178	166	155	145	136	128	121	114
	. .	275	253	233	215	200	186	173	162	151	142	134	126	119
	. .	286	263	242	224	208	193	180	168	157	148	139	131	124

if K' is the installation cost per watt of output, —

$$K'\left[\Sigma I\,(ne + n'e' + n''e'' + \text{etc.}) + \frac{\rho L I^2}{S}\right]$$

is the cost of the station.

The cost of the line is (see equation (186)) —

$$L\,[(a + bS) + (a' + b'S)];$$

so that the total cost of the plant is —

$$U = K'\left[\Sigma I\,(ne + n'e' + n''e'' + \text{etc.}) + \frac{\rho L I^2}{S}\right] + L\,[(a + bS) + (a' + b'S)], \qquad (192)$$

for which a minimum must be obtained.

Differentiating,
$$dU = \frac{d\,(\rho L I^2)\,K'}{S} + d\,[L\,(bS + b'S)]; \qquad (193)$$

$$dU - \frac{K'\rho I^2 dS}{S^2} + L\,(b + b')\,dS = 0;$$

$$\frac{dU}{dS} = \frac{K'\rho I^2}{S^2} - L\,(b + b');$$

$$\frac{K'\rho I^2}{S^2} = L\,(b + b');$$

$$\frac{K'\rho I^2}{L\,(b + b')} = S^2;$$

$$S = \sqrt{\frac{K'\rho I^2}{L\,(b + b')}}. \qquad (194)$$

539. The value of S thus obtained must be used with due regard to the precautions indicated on page 464. It is also necessary to consider carefully whether the length of time during which the plant will be in use, and whether the circumstances of operation, are such as to cause interest, depreciation and cost of lost energy to become an appreciable factor.

540. 7. **Design for the Accomplishment of Best Service.** — The preceding paragraphs have treated at length the method for determining the minimum cost of a plant to accomplish a given service. In many instances, however, this factor is *not* the most important one in the solution of the problem, for the reason that the conditions of minimum expense will militate against the accomplishment of a satisfactory service to the consumers. In series circuits, where the line is intended for a constant current, the calcu-

lation of the conductor can usually be accomplished along the lines indicated under the previous headings.

541. In other forms of distribution, however, as, for example, upon the parallel system, the conductors must be so arranged as to deliver to the consumer a certain definite pressure. Inasmuch as the variation in the potential at the different points along the mains is a function of the amount of current transmitted, and as the amount of current will depend upon the demands of the conducting system, it becomes essential to so design conductors, irrespective of economy, that the pressure required at the various points of the conducting system shall not vary too greatly.

542. Under such conditions, service requirements, rather than the dictates of maximum economy, must govern the design of the conductors. This case, however, will be more extensively treated in the sections upon multiple arc distribution. Other circumstances, however, frequently arise in which service conditions should govern the size selected for the conducting system. The endeavor of the capitalist is always to reduce initial investment to a minimum, but there is no better guaranty of a paying investment than uniformly successful service.

543. 8. **Minimum Cost of Plant to Attain a Maximum Income.** — The income to be derived from a distributing-plant must not be lost sight of in the design of the conducting system; and in some cases, though rarely, this becomes so important a factor as to govern the design.

In locations where power is cheap, and transportation facilities are such as to largely increase the cost of materials, it would, from the standpoint of economy solely, be advisable to design the conducting system according to Sec. 3. In many cases, however, this might lead to the expenditure of so large an amount of the station output in the conducting system that the load on the station might be so close to the total station capacity that the losses entailed in the conducting system would prevent service to the maximum number of consumers that could otherwise be placed upon the line. To increase the station capacity sufficiently to serve a very small additional proportion of consumers, might add so largely to the station cost as to be prohibitive, on account of the commercial size of the units of machinery obtainable. On the contrary, by increasing the size of the

conductors, so as to reduce the losses in the line, the station may be able to supply sufficient additional power to accommodate the desired customers. Under these circumstances it may be exceedingly advisable to increase the size of the mains, and correspondingly, their cost, to such an extent as to allow the station to supply additional customers without incurring the expense of a large building and additional prime movers and their generators.

544. Calculation of Loads. — In order to properly arrive at the most advantageous proportion for the relation of the line to the station, it is essential to accurately determine the conditions of load under which the plant will operate.

For series circuits the solution of the problem is facilitated by the fact that the amount of current is a constant quantity during the entire time that the circuit is under operation. Therefore, to obtain the requisite data for calculating the load, it is simply essential to ascertain for each day in the year the number of hours that the circuit is likely to be in operation, and take the sum.

As series circuits are chiefly employed in lighting installations, TABLES Nos. 77, 78, and 79 are given as indicating the average number of daily hours of service for each month in the year. Circuit loads can by this means be easily estimated.

545. Regulation. — Systems to work under series distribution can only be regulated by varying the voltage or the pressure at the central station to correspond with the changes in load thrown upon the distributing system. From this cause, automatic regulation can only be perfectly mechanically attained in the simple example of the transmission of power between two similar dynamos, one serving as a generator while the other acts as a motor.

546. For ordinary distributing-plants two methods are adopted to secure regulation under the varying load. If it is desired to throw out of service one or more of the receivers, it is necessary to short-circuit those whose service is to be discontinued, in order not to interrupt the rest of the line. If this is done, it is evident that the resistance of the entire line has been decreased by the amount due to the receivers that have thus been short-circuited. Thus, the equilibrium of the line has been disturbed, and the current increased just in proportion to the diminution of the resistance. It is practicable to maintain equilibrium by substituting for the short-circuited

TABLE No. 77.

Hours of Lighting. — Giving Approximate Daily Number of Hours from Sunset to Sunrise, and from Sunset to Midnight for each Month in the Year. Standard Time, Latitude 42° N.

NAME OF MONTH.	No. OF HOURS FROM Sunset to Sunrise.	No. OF HOURS FROM Sunset to Midnight.	NAME OF MONTH.	No. OF HOURS FROM Sunset to Sunrise.	No. OF HOURS FROM Sunset to Midnight.
	H. M.	H. M.		H. M.	H. M.
January	14.20	7.0	July	9.11	4.30
February	13.20	6.28	August	10.05	5.10
March	12.12	5.50	September	11.33	5.52
April	10.40	5.20	October	12.48	6.40
May	9.37	4.50	November	14.00	7.17
June	8.48	4.28	December	14.24	7.26

TABLE No. 78.

Showing Hours of Lighting Exclusive of Sundays and Four Holidays.

Taken from actual records of the average time of lighting during three years, including fogs and dark days.

HOURS OF LIGHTING.	PERIOD OF THE YEAR During which Light is required at these hours.	TOTAL NUMBER OF HOURS of lighting per annum.
6 A.M. till daylight.	October 1 to March 15.	200
Dusk till 5.30 P.M.	October 1 to March 1.	150
Dusk till 6.30 P.M.	September 7 to April 1.	300
Dusk till 7.15 P.M.	August 15 to May 1.	400
Dusk till 7.45 P.M.	August 7 to May 11.	600
Dusk till 8 P.M.	July 28 to June 5.	800
Dusk till 9 P.M.	All the year.	1,050
Dusk till 10 P.M.	All the year.	1,440
Dusk till 11 P.M.	All the year.	1,800
Dusk till midnight.	All the year.	2,150
Dusk till 2.15 A.M.	All the year.	3,000
All night.	All the year.	4,300

TABLE No. 79.

Showing Hours of Lighting Throughout a Year of 8,760 Hours.

DAILY LIGHTING.	JANUARY.	FEBRUARY.	MARCH.	APRIL.	MAY.	JUNE.	JULY.	AUGUST.	SEPTEMBER.	OCTOBER.	NOVEMBER.	DECEMBER.	TOTAL PER ANNUM.
From sundown to 8 P.M.	125	89	67	36	6	. .	. .	21	54	87	117	140	742
From sundown to 9 P.M.	156	117	98	66	37	20	25	52	84	118	147	171	1091
From sundown to 10 P.M.	187	145	129	96	68	50	56	83	114	149	177	202	1456
From sundown to 11 P.M.	218	173	160	126	99	80	87	114	144	180	207	233	1821
From sundown to midnight . . .	249	201	191	156	130	110	118	145	174	211	237	264	2186
From sundown to 2 A.M.	311	257	253	216	192	170	180	207	234	273	297	326	2916
From sundown to 4 A.M.	373	313	315	276	254	230	242	269	294	335	357	388	3646
From 4 A.M. to sunrise	125	92	69	32	3	. .	. .	24	51	75	103	154	728
From 5 A.M. to sunrise	94	64	38	2	. .	. .	. .	. .	21	44	73	123	459
From 6 A.M. to sunrise	63	36	7	. .	. .	. .	. .	. .	. .	13	43	63	254

receiver an equivalent resistance. But by this device no economy is introduced, for the entire circuit is then expending precisely the same amount of energy as it was previously called upon to deliver. Even in cases where the power costs little or nothing, it is necessary to complicate the installation by separate pieces of apparatus to accomplish this short-circuiting that shall be capable of absorbing and destroying, by conversion into heat, the amount of energy usually taken by the receiver.

547. In order to avoid loss of power, it is sometimes practicable to introduce devices for short-circuiting the receivers, which shall substitute for the receiver an apparatus which will introduce into the circuit an opposing electro-motive force equivalent to, or producing the same effect, as the receiver itself. This method is widely applied to alternating current circuits under the various forms of choking or reactive coils. The arrangement consists of an electro-magnet, having a divided core, preferably with a closed magnetic circuit. The self-induction of the coil is so calculated as to produce at its terminals an electro-motive force of opposite sign equivalent to that of the apparatus which has been short-circuited. Under these circumstances, neither the current nor electro-motive force of the generators has been changed; but their difference of phase has been slightly altered, thereby effecting a saving of energy equal to that previously expended in the short-circuited receiver. Alternating circuit devices of this kind practically save all of the energy that would be otherwise expended in the receivers, excepting the small amount absorbed by the reaction coil, which is usually inappreciable.

548. A second method of regulation, which is less simple and less economical in its effect upon the energy dispensed in the circuit, may be applied to constant current machines, in which the reactive coil is inoperative. This scheme of regulation consists in applying to the dynamo machine a regulating apparatus which shall affect the potential delivered at the terminals of the machine itself. All devices of this kind involve an electro-magnet, which is excited by the current delivered by the machine. When, by the short-circuiting of any of the receivers, the resistance of the line is decreased, there is a proportionate increase in the quantity or current sent out by the generator. This addition to the line current, flowing through the

regulator, excites the electro-magnet, forming a part of the apparatus, to a greater degree, thereby setting in motion a train of mechanism which may be arranged to accomplish either of the three following results : —

549. 1. The regulator may be so arranged as to shunt or diminish the current flowing through the field magnets of the generator. Under these circumstances, a decrease in the current flowing through the fields decreases the number of magnetic lines in the magnetic circuit of the generator ; and this weakening of the magnetism is followed by a proportionate decrease in the voltage of the machine, thereby restoring the balance of the circuit.

550. 2. The regulator may be so arranged as either to increase the air-gap, or to short-circuit a part of the magnetic circuit of the generator, thereby accomplishing the same result in decreasing the voltage of the machine.

551. 3. The regulating mechanism may be arranged so that on the increase of current flowing through the circuit, the regulator shall automatically shift the brushes on the dynamo away from their position of maximum voltage to some other place on the commutator, thus giving a decrease in the pressure developed by the machine.

552. While many of these devices have mechanically been brought to great perfection and are eminently successful, yet this method of government is attended by difficulties involving a loss of economy or danger to the commutator, or other parts of the generator, to such an extent that series circuits are rarely selected for distribution under any circumstances excepting those involving loads which are expected to be reasonably constant during the greater part of the time in which service is expected.

553. Series distribution, therefore, possesses the advantage that the amount of current can never exceed a certain predetermined limit. This presents security against the chances of danger from short-circuiting, for a sensible loss of current is immediately indicated by the irregular action of the receivers that may lie between the points of leakage. This quality is not possessed by other methods of distribution. On the contrary, the series system has the disadvantage of a lower efficiency for the percentage of energy expended in the circuits, and is only constant so long as the resistance and the current remain mutually unchanged. Therefore the efficiency falls

in proportion to the number of receivers that are put out of commission.

554. Automatic Cut-outs for Series Circuits. — A great number of devices have been arranged for the purpose of automatic cutting out of the various receivers on series circuits. These devices may be divided into two classes.

Lamp Cut-outs. — These devices, operating on and especially adapted to arc lamps, are so arranged as to cut the lamp out of circuit as soon as the carbons are entirely consumed. All such contrivances are based upon a differential magnet, so planned that when the resistance of the circuit, due to the increase in the length of the arc, becomes sufficiently great, a portion of the circuit will be shunted into a fine wire coil on the differential magnet, and by closing its armature, will cut the lamp out of series.

555. *Time Cut-outs.* — Other automatic cut-outs are arranged upon the principle of allowing the translating device to operate for a certain number of hours, and then cut it out of the circuit. These devices are usually based upon the application of a clock to a shunt operated by an electro-magnet, so arranged that after the receiver has operated a certain number of hours, the clock mechanically closes the shunt, throwing the current around the receiver. Such devices are applied to cut out constant current motors, and also to cut out arc lamps that are contracted to burn a certain, definite number of hours each day.

While contrivances of this kind have evinced remarkable ingenuity on the part of their inventors, and while on some circuits they form valuable adjuncts, they add so great a degree of complexity, and require so much additional maintenance expense, that their utility is, in many cases, quite questionable.

556. Designs for Series Circuits. — In the use to which series circuits are most frequently applied, namely, for municipal illumination, it is obvious that the greater part of the plant load will be thrown on at about sundown, and will remain essentially constant throughout the entire hours of the night, all of the lamps being simultaneously extinguished at the succeeding sunrise. Thus, under these circumstances, the plant load is essentially a constant quantity during its entire time of service; and while during different periods of the year the varying lengths of night and day, or the demand

caused by cloudy and stormy weather, is of such a nature that, while it increases or decreases the length of time that the plant is at work, it does not vary to any appreciable extent the load which the plant is called upon to carry.

557. Arc-light installations are frequently designed to supply commercial lights in addition to those used for city lighting. The commercial lights in interior locations may be called upon to run at very different periods of time than those demanded for municipal illumination; but the conditions giving rise to the demand for such lights will naturally be tolerably constant throughout the territory that would ordinarily be embraced by the lighting plant. So while the length of time that commercial lamps would be required to burn might be very different from that called for by the city lighting, yet both the commercial and municipal loads would be reasonably constant quantities. Many attempts have been made to so plan arc circuits that the commercial load will be separated or rendered distinct from the municipal load. This can always be done by running independent circuits for each kind of service; yet this plan naturally entails a certain amount of waste conductor material, for while the commercial lights may be required at an earlier hour than the municipal lights, and also may be extinguished at an earlier hour, yet for the great proportion of the time both kinds of service are simultaneous.

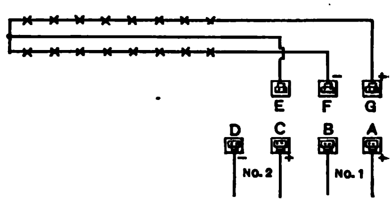


Fig. 287. Diagram of Series Lighting.

Any design of circuits, therefore, which can be made so that at least for a part of each day one circuit may be used for both sets of lamps, will result in a corresponding saving in copper expense for the original circuit.

558. One method for introducing a saving in the copper required for circuits containing both commercial and municipal arcs has been proposed by Mr. Sharpstein, in the *Electrical Engineer.* This method is shown in Fig. 287.

Two machines were installed in the station indicated at No. 1 and No. 2, and the circuits so arranged that all the commercial lamps were on wire G, while all the municipal lamps were on wire F.

When the commercial lamps were placed in operation, machine No. 1 was started, and with the switchboard, cables, and plugs, A and G were connected, and B and E. When service was needed for the municipal lamps, machine No. 2 was started, the brushes being placed at the point of minimum working capacity. Then, by means of the remaining hole in the terminal B, machine No. 1 was connected with C, and by means of the remaining hole in E the third wire leg was connected with D. As a result, machine No. 2 was in circuit with no load; and if the brushes had not been placed at the point of minimum capacity, a burn-out would have occurred. Now, by connecting E and B, both machines are placed upon the commercial circuit, and in order to cut E out and get F into circuit, one plug of the cable, just removed from the switchboard, should be placed in the remaining hole at D. The other plug is put into the right hand, and held near F until the plug in E is withdrawn far enough to draw a short

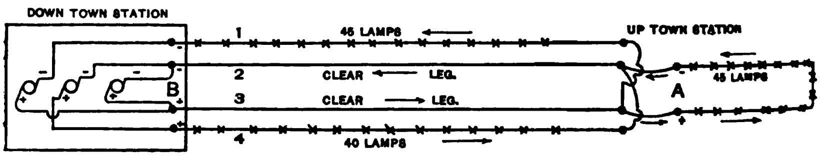


Fig. 288. Arc-Lamp Circuit.

arc, when the plug in the right hand is put into one of the holes in F, when the arc at E is extinguished, and both lamp-legs are on both machines.

559. There are many obvious objections to running dynamos in series, as is required by the preceding method. The geographical location of the respective commercial and municipal arcs is not always such as to enable the saving of an appreciable amount of copper. Mr. C. G. Young has indicated two methods, as shown in the accompanying illustrations, which avoid the difficulties of placing the dynamos in series, and yet accomplish a notable saving of copper. In Fig. 288 three dynamos are shown as operating three circuits, which may be arranged either to run conjointly or independently, with four wires instead of six. A little study of the diagram will render the operation of the currents entirely clear. If all the lamps are in operation at once, it is evident that wires Nos. 2 and 3 will carry double the current of 1 and 4. If the dynamos are worked

fully up to their capacity, an extra allowance of copper must evidently be provided in this part of the circuit. It will often happen, however, that there is sufficient spare voltage, or the extra pressure may be obtained by a slight increase in speed, so that no extra copper is needed. In the case in point, No. 6 wire was used throughout the entire circuits, and proved entirely successful.

560. Where the station load can be subdivided into three parts, operating at different times in the 24 hours, and geographically so located as to be separable one from the other, the arrangement shown in Fig. 289 effects a reduction in line material. Under these circumstances continuous service is given on line A, day service on

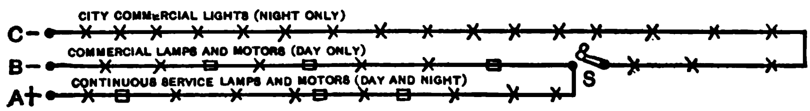


Fig. 289. Arc-Lamp Circuit.

lines A and B, and night service on lines A and C, thereby saving one-half the copper that would be called for by three independent circuits. A switch introduced at S serves to isolate line C, in order to protect the trimmers.

561. Division 2, *Constant Current Circuits, Embracing Generators and Receivers at Varying Distances* from each other, has as yet received little or no practical development. Several attempts have been made to introduce the series system upon electric railways; but so far the practical difficulties have been found commercially insurmountable, and the attempts have been abandoned.

Divisions 3 and 4 of the classification of circuits on page 373, treating of constant potential circuits, covering at present the most important electrical plants, will be considered in a succeeding chapter.

CHAPTER XV.

PARALLEL DISTRIBUTION.

Art. 562. The Evolution of the Parallel System. — In the discussion upon series distribution, in Chapter IX., it has been shown that the development and extension of this method are limited in several directions. As the current in a series system is constant throughout the entire circuit, a variation in the number of customers, or in the amounts of energy supplied to respective customers, can only be obtained by a corresponding variation in the *potential* of the system. Every additional receiver increases the tension proportionally to the amount of energy required to supply the additional demand; and the practical limit of possible difference of potential is reached, when a comparatively small number of translating devices have been placed upon the circuit. Experience has, thus far, demonstrated the inadvisability of increasing the potential of direct current circuits beyond 8,000 or 10,000 volts. Occasionally installations have been operated as high as 10,000 volts; and though the tendency is toward higher pressure, such tensions require more careful and constant supervision and maintenance, and the gravity of injury arising from an accidental short-circuit is very largely increased. Again, the series circuit finds itself at a disadvantage when widely different amounts of energy are desired by various consumers along the line. As the quantity of energy to be delivered to each customer can only be varied by changing the potential between the terminals of the translating devices supplying the different subscribers, a customer using a large amount of energy must, necessarily, receive mains having great difference of potential. This has always been found to be a source of difficulty and danger; experience having shown the hazard to the community at large of introducing high potential circuits directly into residences, or the places of business, of the subscribers, where they are likely to be under the management of those little skilled in electrical manipulation. In order to attain any reasonable degree of economy, it has been shown that the load upon a series

circuit must be nearly constant and uniform throughout the whole time that the circuit operates, and that a series plant becomes decidedly uneconomical when applied to the service of customers demanding widely varying supplies of energy, extending over different periods of time. In the development of electrical industries, central stations soon reached a sufficient magnitude to bring the limitations of the series circuit into sharp conflict with desired business extensions. To enable the central station to supply a large number of customers, without introducing potentials that are impracticable, the first step in electrical evolution was to equip the station with a number of generators, each one of which was arranged to operate upon a separate and independent circuit. By this means dangerous potentials were avoided; but still all of the individual circuits were open to the remaining objections of the series method, and the large number of independent machines proved decidedly expensive in operation. The multiplicity of circuits soon became confusing, and much duplication of wire was necessary in order to cover a reasonable amount of territory. To improve the economy of the station, large dynamos were planned, capable of supplying a number of different circuits, upon each one of which the various receivers were placed in series. Such an arrangement is indicated in Fig. 290.

563. It should be noted, in the examination of all the illustrations giving diagrammatically the outlines of various circuits, that the sketches serve merely to illustrate the principles of the circuit, without having special reference to the kind of receivers, or translating devices, which may be employed upon installations of differing design. Though the multiplication of circuits from one machine formed a step in advance, enabling the station to operate somewhat more flexibly and economically than the single series circuit, as indicated in Fig. 290, in so far as losses in the dynamos themselves were concerned, it in no wise obviated the other limitations to which the series circuit is subjected. As each of the series circuits from the generator is supplied with the requisite number of receivers to exhaust the potential of the dynamo, the tension of the system may, evidently, be reduced to any desired safe and practical limits, by multiplying the number of circuits, and proportionally reducing the number of receivers which are placed upon each one. Another advantage accrues from the ability to arrange the differing circuits in such a manner that

they may be thrown in and out of commission, in a way to allow a much greater variation of the load upon the station. In the case of an electric lighting plant intended to supply both municipal and commercial arcs, it is feasible to arrange a multiple circuit generator of sufficient capacity to supply the current required for both circuits, placing all the municipal arcs upon one, and all the commercial lights upon the other. The two circuits would thus be entirely separate and independent of each other, and could be operated during differ-

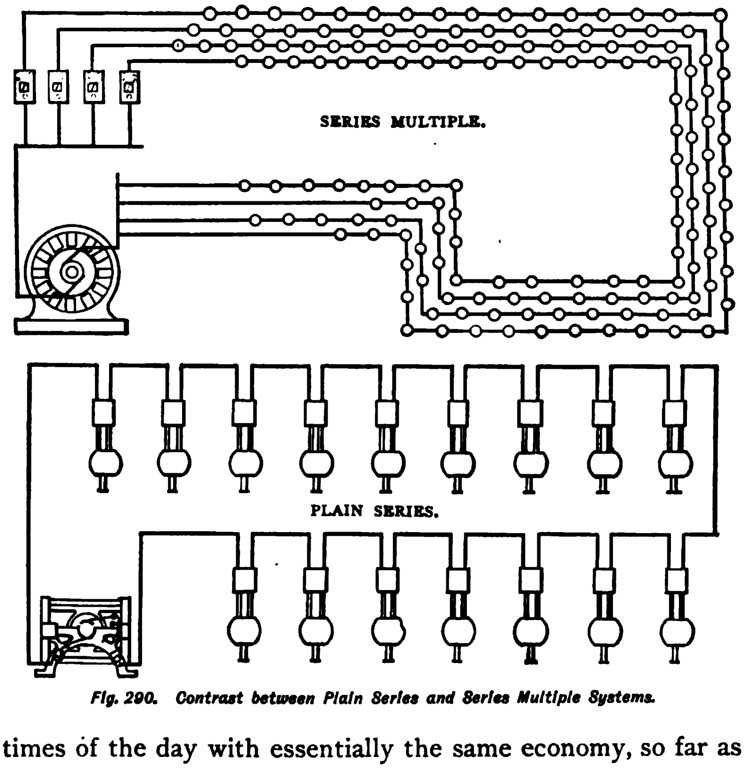


Fig. 290. Contrast between Plain Series and Series Multiple Systems.

ent times of the day with essentially the same economy, so far as the losses in the circuit were concerned, as would accrue provided all the lamps were placed upon a single line. While this method introduces economy in the series system so far as the circuit losses are concerned, unless the generator be worked for a greater proportion of the time at its full load, the dynamo losses tend, to a considerable extent, to counterbalance the economy gained. While this method presents a partial solution of the problem, it in no wise provides any

ability to deliver to the different customers varying amounts of energy, or to render the various customers independent of each other, in order that they may throw in and out of service, at pleasure, their receivers. This is really the most important disability of the series circuit.

564. If the multiplication of separate circuits should be carried to its limit, each receiver would be supplied with a separate and independent wire from the generating-station, as shown in Fig. 291, and then all the chief objections to the series circuit disappear. In this case the potential of the generating-station is reduced to the highest pressure required by any receiver that may be placed in service. As each receiver is supplied with a separate and independent circuit extended from the receiver to the generating-station, every translat-

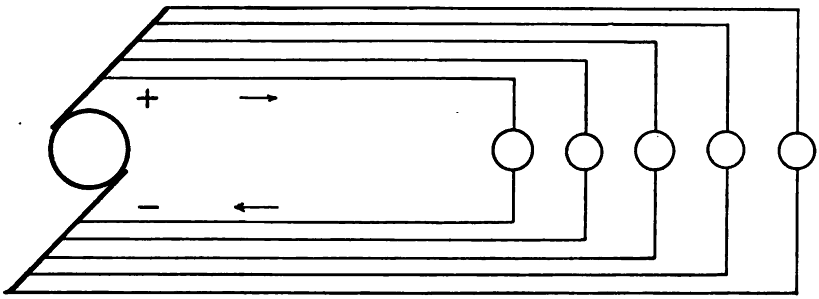


Fig. 291. Diagram of Independent Circuit for Each Receiver.

ing device is entirely independent from every other one, and may be thrown in or out of the circuit, without interfering in the slightest with the service of any other consumer.

565. From the independence of the individual circuits, the amounts of energy supplied to the different receivers may be varied, by varying the quantity of current, without changing the pressure of any of the circuits. Thus, any subscriber may be supplied with any desired number of translating devices of different powers, and the amount of energy supplied varied at pleasure, by varying the quantity of current entering each translating device. As the various receivers may be adjusted to work upon any convenient electrical pressure, the circuits can be easily designed to never exceed safe limits; and by increasing the quantity of current supplied by the station, it becomes possible to distribute energy over a very large territory and to a great

number of customers. The independence of the receivers also allows the customers to throw their loads on and off at pleasure, or to vary them to any extent. It is now evident that all receivers, instead of operating under a constant current and a varying pressure, operate under a constant pressure and a varying amount of current. The evolution, therefore, of electrical distribution has, evidently, taken place by a differentiation of the series method, the early single circuit being finally split up into such a number of parts as will practically give an independent line to each of the respective customers. To serve a large territory, however, by actually giving to each customer a circuit

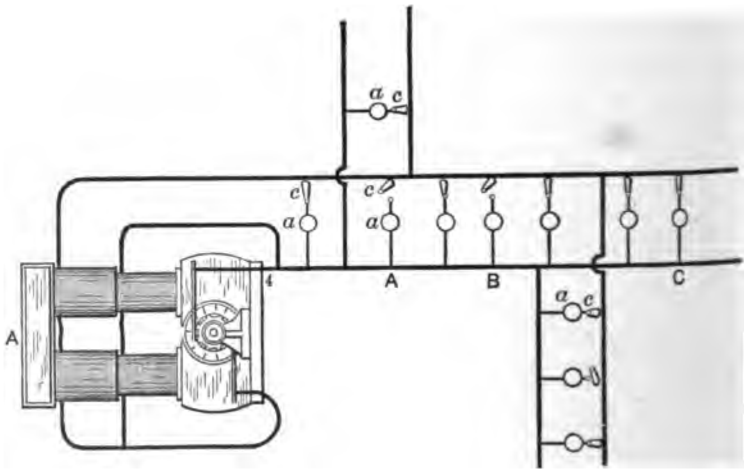


Fig. 292. The Parallel System.

completely his own, extending from his receiver back to the generating-station, would introduce such complexities of wiring as to prohibit the introduction of this plan in installations of any magnitude. To avoid this objection, the next step consisted in uniting the adjacent receivers into bunches, the various groups being placed in parallel to each other across the line, the system finally developing into the plan indicated in Fig. 292, in which the essential independence of the individual receiver is manifest. From the characteristic parallelism of the individual receiver circuits, the system has derived its name of the "Parallel" or "Multiple Arc System."

566. Methods of Distribution. — The arrangement whereby each translating device is supplied with an entirely independent cir-

cuit, extending from the generator to the receiver, gives the individual customer the best possible service and the greatest independence. As each receiver is absolutely separate from every other one in the entire installation, it may be thrown on or off the circuit, or the amount of energy absorbed varied, without affecting in the slightest degree any other customers. With the individual circuit arrangement, provided the speed of the generator at the station be maintained constant, and the dynamo is not overloaded, the service delivered to all of the customers will attain the greatest uniformity. The inconvenience of this method, giving rise, in stations supplying a large number of customers, to utterly impracticable multiplication and complexity of circuits, has been noted, and the method of obviating this, by uniting the various receivers into groups, and placing them in parallel across a common set of conductors, indicated. A difficulty here arises from the fact that the fall of potential along the conductors is not only a function of the resistance of the mains, and so an inseparable concomitant of the distance of the various customers from the station, but is also a function of the amount of current which, at the time being, is passing through the mains. Thus, referring to Fig. 292, and assuming the plant represented to be a lighting circuit, the current at A will be much greater when all of the lamps are in operation than when the group at C only is in service. As the fall of potential depends, not only on the resistance of $\overline{AC}$, but also upon the amount of current flowing through the mains, the decrease in the pressure at A will be much greater when all of the lamps at A, B, and C are lighted, than when a single group is alone in service. On the supposition that the generator always produces a constant potential, if the mains are so calculated as to give B precisely the required tension, when all the lamps are in service, the pressure at B will be too high when A and C are extinguished; or, if the mains are calculated so as to give the required tension at B when the other lamps are extinguished, if A and C are in service, the tension at B will be too low, and the lamps will not burn with their required brilliancy. To obviate this difficulty, many systems of wiring have been devised, all of which may be finally reduced to four elementary forms. Before giving the fundamental systems the necessary careful consideration, it is advisable to review hastily the various plans of wiring. The ordinary features of the parallel system

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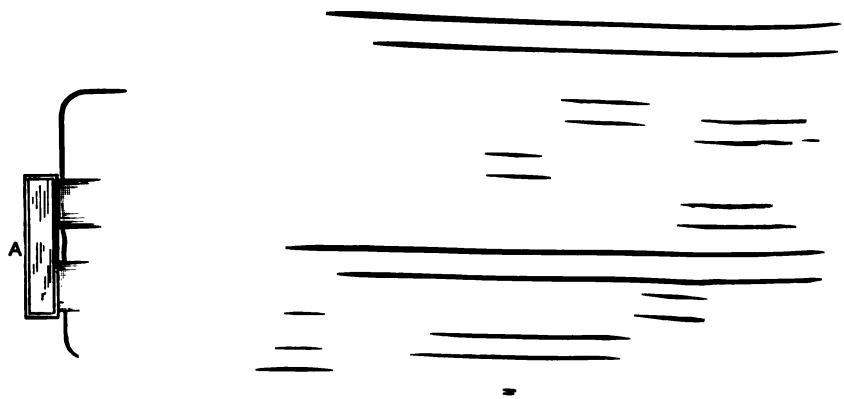

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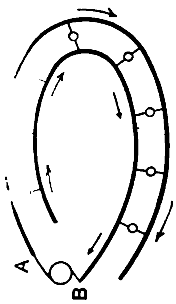


Fig. 295. The Spiral Loop.

Fig. 294. The Loop System.

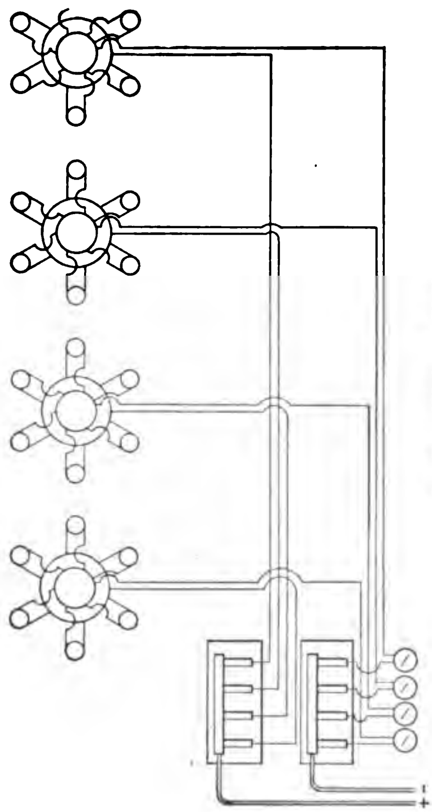


Fig. 297. The Closet System.

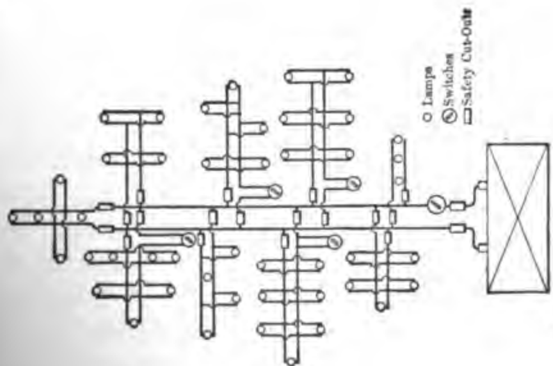


Fig. 296. The Tree System.

are exemplified in Figs. 292 and 293. From the generator two or more sets of mains are extended through the district to be served, the various receivers being placed in bunches across the mains, as indicated in the illustrations. A very slight consideration of the diagrams will show that the electrical distance from the generator to the various receivers varies with the successive translating devices, and that the simple fact of the variation in distance from the receivers to the generator would preclude the possibility of supplying a uniform pressure throughout the entire system. Evidently the electrical distance from the generator to the group A, Fig. 292, is much less than it is from the generator to the groups B and C. As there is no known substance which may be employed for the conducting

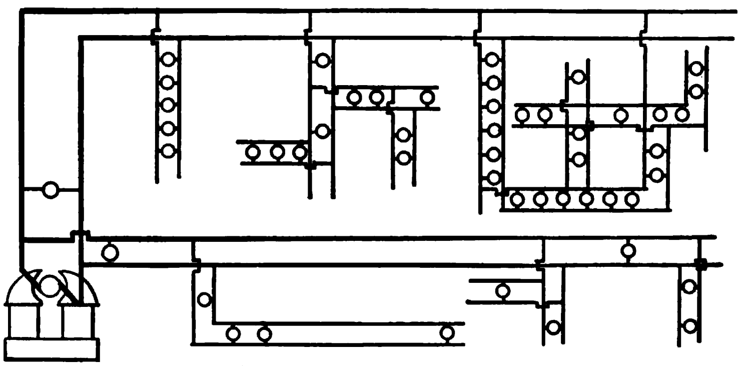


Fig. 293. Complete Multiple Arc System.

system having no resistance, it is impossible on this account to render the pressure at A the same as it is at B and C.

567. The Loop System. — The loop system is an endeavor to so design the conducting circuit as to render the electrical distance from the generator to each of the receivers the same throughout the entire circuit.

Thus, in Fig. 294, one of the conducting leads *aa'*, from the generator A, extends directly away from the dynamo to the end of the system, having the receivers placed in succession along its length. The main BC*b*, on the contrary, extends from the generator to the most remote point of the circuit *b'*, without being attached to any of the receivers. At the point *b'* it returns upon itself, toward the generator, having upon this branch the connections to all of the receiv-

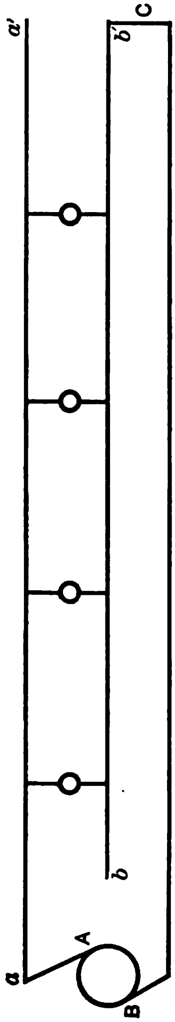


Fig. 294. The Loop System.

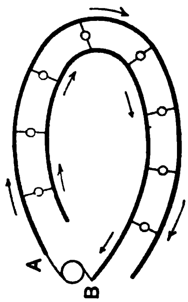


Fig. 295. The Spiral Loop.

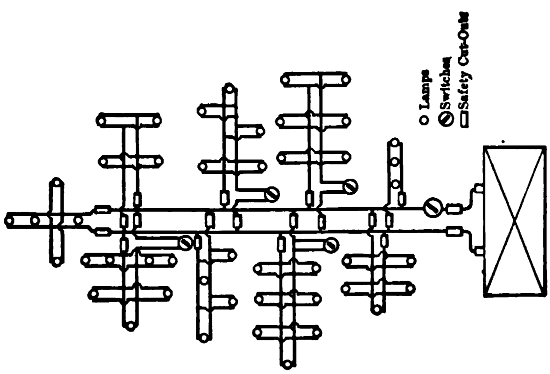


Fig. 296. The Tree System.

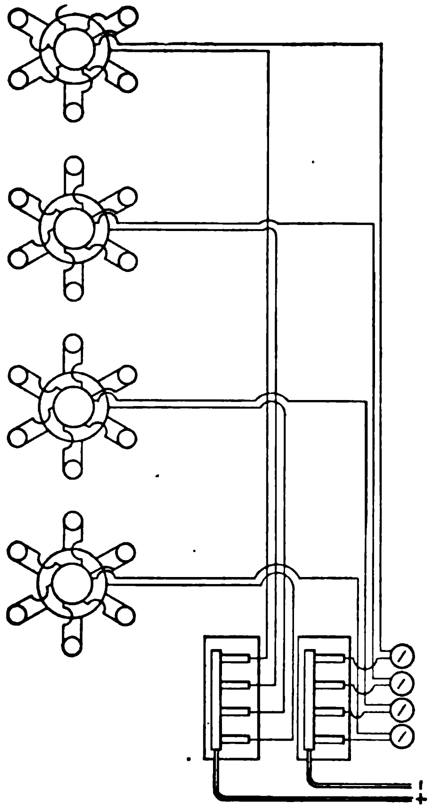


Fig. 297. The Closet System.

ers. An inspection of the diagram will show that, under these circumstances, the distance from the pole A of the generator to the pole B, throughout any of the translating devices, is precisely the same; so the pressure in all of the receivers is not affected by their proximity or remoteness from the generating-station; and were it not that the fall of pressure is a function of the amount of current flowing, the receivers would always obtain a constant potential.

568. The Spiral Loop. — Another loop arrangement is indicated in Fig. 295, in which the parallel conductors, A and B, are extended in the arcs of spirals from the generating-station throughout the territory to be served, both spiral arcs extending from one pole of the generator nearly to the other pole. In both of the loop systems the amount of material required for the conducting system is considerably increased, with, however, the advantage of much greater constancy in the electrical pressure delivered to the receivers.

569. The Tree System. — Nearly all of the earlier installations upon the parallel system were laid out upon the so-called "Tree System," indicated in Fig. 296. The origin of the name is made quite evident by the illustration, from which it will appear that the main conductors in the system resemble a tree trunk, from which the auxiliary leads branch in various directions, quite after the fashion of a spreading tree, the receivers occupying the places of the twigs, leaves, and fruit. As the fall of pressure throughout the installation is augmented by the varying electrical distance of the receivers from the source of supply, the plan is, in this respect, defective.

570. The Closet System. — The "Closet System" was an attempt to minimize the effect of electrical distance by collecting the various receivers into groups, each one of which was supplied with a separate and independent circuit back to the generating-station. This design is indicated in Fig. 297, the receivers being collected into four groups, those of each bunch equally placed in a circle around a center of distribution. From each distributing center, a set of leads is carried back to the generating-station, thus rendering each group independent of the other groups. This method is chiefly used in interior wiring, and may have formed the basis for the development of the famous "Feeder and Main System," to which detailed refer-

ence will be shortly made. The detail of a single group, in the Closet System, is given in Fig. 298. Here the receivers are placed in a circle around two circular mains, which receive their special circuit to the generating-station at two points diametrically opposite each other. A little consideration of the diagram indicates that the electrical distance of all of the receivers, with reference to the attachments of the feeding circuits, is the same.

571. Conical Conductors. — Referring to Figs. 292 and 293, it is evident that the greatest current in the conductors occurs in the section immediately between the generator and that of the first consumer; and as the distance from the station increases, thus placing more and more consumers between the station and the point of the mains under consideration, the current in the conductors decreases in direct proportion to the number of receivers that lie towards the station. It needs but little consideration to perceive that, if the cross-section of the mains is kept constant throughout the entire system, the conducting material in the circuit is not disposed to the best advantage. Either the current density near the station is too great, and the mains are in danger of becoming overheated, or, at the more remote portions of the systems, the current density is too small, and the conducting material is wasted. To proportion the mains to attain a *constant current density* throughout the entire system is evidently the remedy. Such an arrangement, when carried to the limit, would produce a conical conductor having the greatest cross-section at the station, and gradually tapering to zero at the extremity of the system. Under such a design, the *rate* of fall of the potential, due to the resistance of the conductors, evidently becomes much more uniform throughout the entire length of the circuit. Such an arrangement of conductors is indicated in Fig. 299.

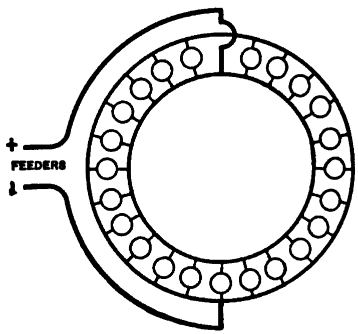


Fig. 298.
The Closet System, Detail of Group.

572. Another attempt to equalize pressure throughout the system has resulted in the employment of two conical conductors, so placed that the apex of one of the conductors is connected to one

of the poles of the generator, while the base of the other conductor is connected to the other pole. Such an arrangement is shown in Fig. 300.

Though this plan tends to equalize the *total* resistance of the conductors to the receivers from pole to pole of the generator, it produces quite an unequal variation in the "drop" to which the various receivers are subjected. The resistance of $\overline{AB}$ is less than

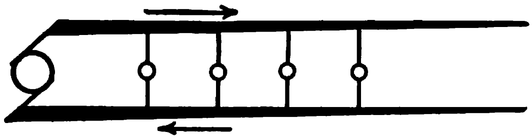


Fig. 299. Conical Conductors.

$\overline{CD}$, though the current is the same in both. Hence there will be more drop in $\overline{CD}$ than in $\overline{AB}$, and of the total drop due to conductor resistance a greater proportion will occur in $\overline{CD}$ than in $\overline{AB}$.

573. Anti-Parallel Feeding. — In the diagram of the loop system, Fig. 294, one conductor, in extending away from the station, ran to the extremity of the line and then returned upon itself. A modification of the loop system is shown in Fig. 301, in which this extension of the conductor is split between both mains. In this

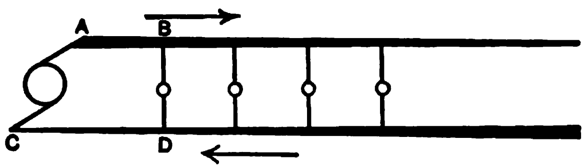


Fig. 300. Anti-Conical Conductors.

illustration the current enters the mains at opposite extremities, flowing in reverse directions through the two conductors. Such a method is termed "Anti-Parallel Feeding," and, as is shortly to be shown, is attended with some considerable advantage.

574. The Distribution of Potential. — Satisfactory service in all systems operating under the parallel method can only be accomplished by preserving, under all conditions of loading, an essentially constant pressure throughout the entire circuit of conductors. Theoretically, the pressure at the terminals of all the translating devices,

be they what they may, should be perfectly uniform at all times and under all conditions of loading, whether the load be that on the translating device in question or that of the entire system. Practically, under no circumstances is it possible to attain an exact equality in the electrical pressure under all conditions. To reach this result in even a manner to secure satisfactory service, requires, with the best exercise of the greatest skill in proportioning, an expenditure of enormous amounts of copper in the conducting system. All of the forms of wiring may be reduced to four elementary forms; and, therefore, a very careful consideration of the distribution of potential through each of these elementary forms, becomes a matter of prime

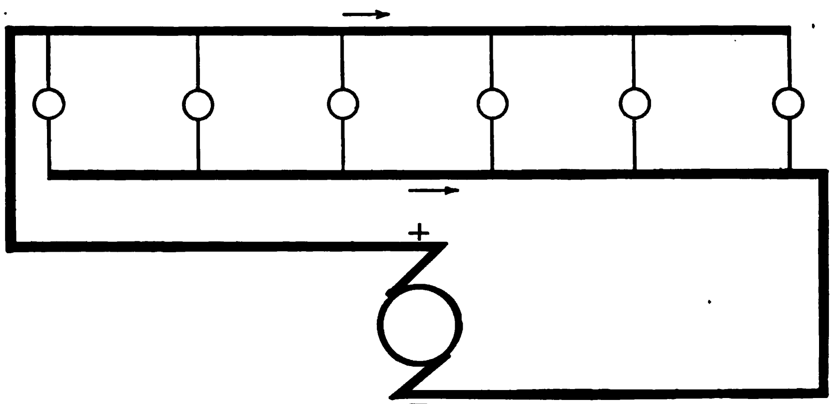


Fig. 301. Anti-Parallel Feeding.

importance to the successful designer of a parallel system. In order to simplify this investigation, the four primary forms of wiring may be classified as follows:—

First. **Cylindrical conductors, parallel feeding.**

Second. **Conical conductors, parallel feeding.**

Third. **Cylindrical conductors, anti-parallel feeding.**

Fourth. **Conical conductors, anti-parallel feeding.**

To further simplify investigation, let it be assumed in all of the four cases now to be considered, that the conductors are two straight lines, supplied with an indefinite number of receivers, uniformly and equally distributed along the entire length of the conductors, each receiver taking the same amount of current, which is equivalent to assuming that the current supplied by the station flows between the

two mains in a thin, uniform sheet, extending from end to end of the conductors. Such an assumption has a mechanical analogy in the replacement of a set of steps by an inclined plane having the same pitch. The load on the mains, in this connection, is supposed to be a constant one and uniform.

CASE I. — *Cylindrical Conductors — Parallel Feeding.*

575. In Fig. 302, let $\overline{AB}$ and $\overline{CD}$ be the two parallel cylindrical conductors connected to the source of supply at A and C, the current flowing in the direction of the arrows.

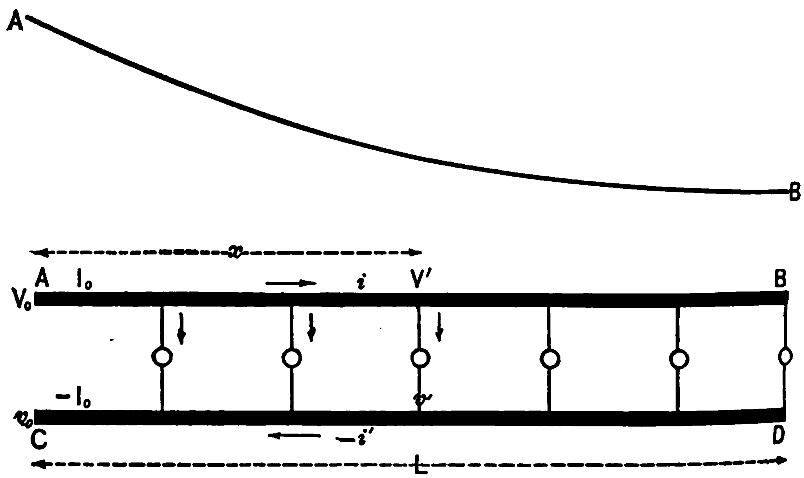


Fig. 302. Diagram of Potential Distribution in Case I.

Let L be the length of the mains in any desired units;

I_0 and $-I_0$ the currents at the station in each main,

i and $-i'$ the currents at any point x in each main;

V_0 and v_0 be the potentials at the station assumed to be constant:

V' and v' be the potentials at any point distant x units from the station, then,

$V_0 - v_0 = u_0$, the difference of potential between the mains at the station.

Also $V' - v' = u'$, the difference at any point x; then,

$u_0 - u' =$ the *fall* of potential or *drop* between the station and point x.

Let $R =$ the resistance of each main per unit of length.

Consider an element of the conductor dx at the point x. By hypothesis, the current decreases regularly from I_0 at the point A, to 0 at the point B, the extremity of the mains. Hence, in each element of the conductor AB, along its entire length, an elementary amount of current will pass from this conductor to the other main. As, by hypothesis, the rate of flow between the mains is uniform, I/L will be the *rate* of flow from one conductor to the other conductor. At any point x, the current in the mains will be the total current at the station, minus all the current which has been transferred across from one main to the other, between the station and the point x under consideration.

The resistance of the element of conductor dx is Rdx. By Ohm's Law, the variation of potential in any conductor is $E = RI$; hence, in the two mains —

$$d(u_0 - u') = Rdx \times 2\left(I_0 - \frac{I_0 x}{L}\right); \tag{195}$$

arranging, —

$$d(u_0 - u') = 2\,RI_0\left(1 - \frac{x}{L}\right)dx.$$

Integrating between $x = 0$ and $x = L$, —

$$u_0 - u' = 2\,RI_0 \int_0^L \left(1 - \frac{x}{L}\right)dx;$$

$$u_0 - u' = \frac{RI_0 x}{L}(2\,L - x). \tag{196}$$

This equation represents a branch of a parabola to which the conductor is an asymptote. When $x = 0$, $u_0 - u' = 0$, showing no drop at the origin; when $x = L$, $u_0 - u' = RI_0L$. To find the maximum drop, —

$$\frac{d(u_0 - u')}{dx} = 2\,RI_0\left(1 - \frac{x}{L}\right) = 0. \tag{197}$$

$$\frac{d^2(u_0 - u')}{dx^2} = -\,2\,RI_0; \tag{198}$$

or $u_0 - u'$ is a maximum when $x = L$ (198), with a value of RI_0L.

576. Take an example. Suppose, in Fig. 302, —

$I_0 = 12$ amperes.
$L = 60$ feet.
$R = .02\ \omega$ per foot.

$V_0 - v_0 = 40.$

$$u_0 - u' = \frac{RI_0 x}{L}(2\,L - x).$$

$$u_0 - u' = \frac{.02 \times 12x}{60} \times (120 - x).$$

Let x be successively 10, 20, 30, 40, 50, and 60, then —

$$\frac{.02 \times 12}{60} \times 10 \times (120 - 10) = 4.4;$$

$$\frac{.02 \times 12}{60} \times 20 \times (120 - 20) = 8.0;$$

$$\frac{.02 \times 12}{60} \times 30 \times (120 - 30) = 10.8;$$

$$\frac{.02 \times 12}{60} \times 40 \times (120 - 40) = 12.8;$$

$$\frac{.02 \times 12}{60} \times 50 \times (120 - 50) = 14.0;$$

$$\frac{.02 \times 12}{60} \times 60 \times (120 - 60) = 14.4.$$

577. From these values, the curve AB in Fig. 302 is plotted. A very slight consideration of this curve indicates a very unequal drop along the conductors, evidently due to varying current density per unit of cross-section in the mains. For incandescent lighting circuits, it is possible to compensate to some extent for this inequality, by placing lamps of different voltage across the main, the higher voltage lamps being located nearer the source of supply. While almost any desired voltage of lamp may be quite readily obtained, yet to assume this method of compensation introduces a very undesirable maintenance complexity into the service. Furthermore, the slightest inspection indicates that the conducting material is badly disposed in reference to the load on the mains. Either that portion of the conductors nearest the station is too heavily loaded, and dangerously near the heating limit, or at the extremities of the mains there is too much copper, and economy may be introduced in original capital outlay by a reduction in the cross-section. To assume a safe current density per unit of cross-section, and then to construct the main to realize at all points this density, leads to a much more effective disposition of the conducting material. This is accomplished by a tapering conductor, the cross-section of which varies directly with the current.

Case II. — *Conical Conductors — Parallel Feeding.*

578. In Fig. 303, let $\overline{AB}$ and $\overline{CD}$ be two parallel conical conductors connected to the station at A and C, and having a cross-section constantly decreasing in proportion to the diminution of the current,

so that the current density shall be constant at all cross-sections. Assume the notation as indicated in Case I., with the exception of the symbol R, which in Case I. had a constant value per unit of length, while in the present case the value of R will evidently vary with the distance from the generating-station. In Case I., R was the resistance per unit of length of each main; in the present conditions

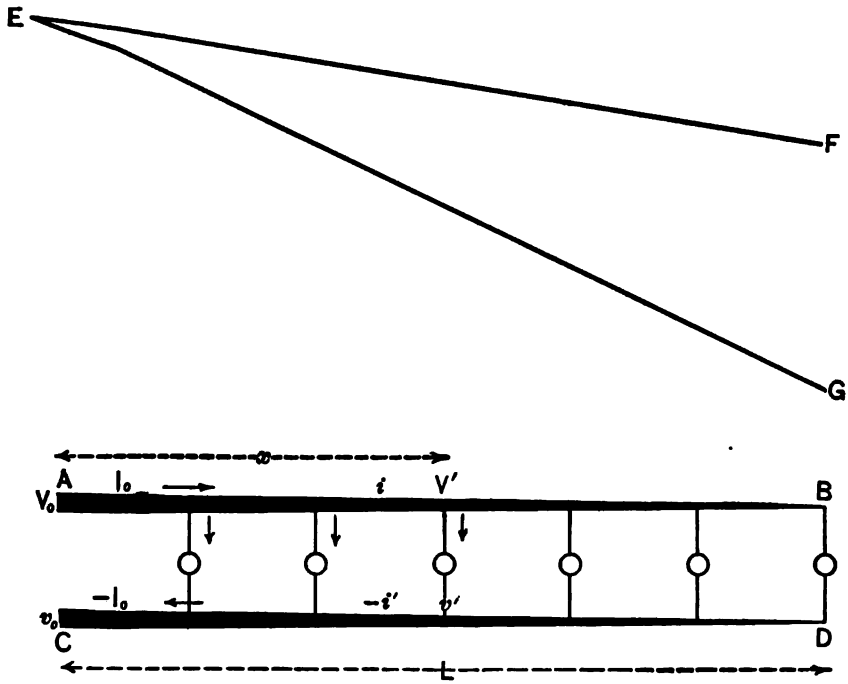


Fig. 303. Diagram of Potential Distribution in Case II.

R will vary with x, and is the resistance of a unit of length at the point x *only;* therefore, for R substitute r, denoting a variable resistance, —

$$d(u_0 - u') = rdx \times 2\left(I_0 - \frac{I_0 x}{L}\right), \qquad (199)$$

in which r is the resistance per unit of length at x. But $r = \rho / S$ at any point x, ρ being the specific resistance and S the section of the conductor, then, —

$$d(u_0 - u') = \frac{\rho dx}{S} \times 2\left(I_0 - \frac{I_0 x}{L}\right); \qquad (200)$$

arranging,
$$d\,(u_0 - u') = \frac{2\,\rho I_0\left(1 - \frac{x}{L}\right)dx}{S}.$$

but,
$$\frac{\left(1 - \frac{x}{L}\right)}{S}$$

is the current density per unit of cross-section, which by hypothesis is constant; hence, integrating, —

$$u_0 - u' = 2\,R_0 I_0 x, \qquad (201)$$

R_0 being the resistance per unit of length at the origin.

This is the equation of a straight line, indicating a uniform drop from the station to the end of the conducting system, $u_0 - u'$ being a maximum when $x = L$.

579. Assuming the data in the example given in Case I., $u_0 - u' = 28.8$ volts, and the curve $\overline{EG}$ (see Fig. 303) is obtained, showing that, with a conical conductor having the same *unit resistance* at the origin as a cylindrical one, there is *twice* the *drop;* but, however, the *weight* of copper used in each main is only *one-third* of that employed in the cylindrical mains. This is evident from the fact that, in both systems the diameter at the origin and the length are the same, while the weights are in the same proportion as the volume of a cylinder and cone having the same base and altitude, or as one to three. If in the conical system the same *weight* of copper is allowed as in the cylindrical, the relative drop in the two systems is reduced in the proportion of two to three, as indicated in Fig. 303 by the curve $\overline{EF}$. The section of the conductors at the origin is then three times as great as in the cylindrical system. Thus, for the same cost of conducting system, the variation in potential may be decreased and the drop rendered more uniform by this method.

Case III. — *Cylindrical Conductors — Anti-Parallel Feeding.*

580. In Cases I. and II., the adjacent ends of the mains A and C are connected to the generator, the path of the current being outward away from the stations along the main AB, and backward toward the station through CD. Thus the direction of the current in AB is opposite to the direction of the current in CD. It is sometimes feasible to connect the *opposite* ends of the main to the station

instead of the adjacent ends. Such a disposition is shown in Fig. 304, A being connected to one pole of the generator, and D to the other, the path of the current, as indicated by the arrows, being in the same direction in both mains. In this case an examination of the diagram shows that no receiver can enjoy the full difference of potential supplied by the generating-station, for the reason that V_0 is at *one end* of one main, and v_0 at the *opposite end* of the other. In this case $u_0 - u'$ is *not* the fall of potential throughout the entire length of both conductors, but is the *variation* between the different receivers, and is *less* than the total fall for the two mains by the amount lost in either one of the conductors.

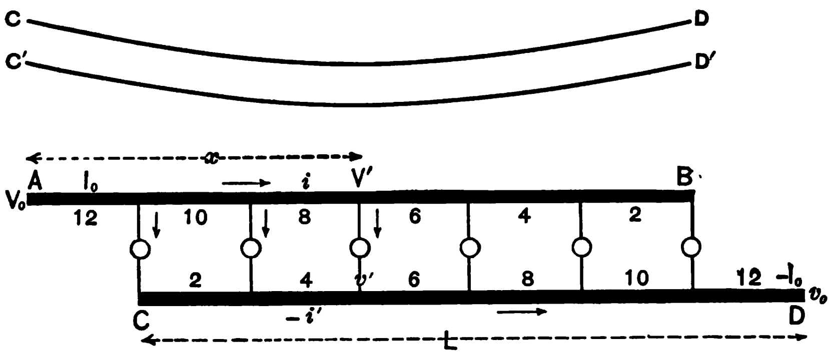


Fig. 304. Diagram of Distribution of Potential in Case III.

A further study of the diagram will show that —

$$d(u_0 - u') = R(i - i')\,dx, \tag{202}$$

i and i' being the current in the respective mains at the point x.

$$i = I_0 - \frac{I_0}{L}x;$$

$$-i' = \frac{-I_0}{L}x;$$

$$i - i' = I_0 - \frac{2I_0}{L}x;$$

$$d(u_0 - u') = RI_0\left(1 - \frac{2x}{L}\right)dx. \tag{203}$$

Integrating $$u_0 - u' = RI_0\int_0^L\left(1 - \frac{2x}{L}\right)dx;$$

$$u_0 - u' = \frac{RI_0x}{L}(L - x). \tag{204}$$

This equation is also that of a parabola; but the vertex is at the center of the mains, and the maximum variation is one-half that of Case I. When $x = 0$ and $x = L$, $u_0 - u'$ is 0, showing that at each end the receivers are operating under the same potential. To locate the maximum difference of potential between the receivers, —

$$d\left(\frac{u_0 - u'}{dx}\right) = RI_0\left(1 - \frac{2x}{L}\right) = 0;$$

$$RI_0 - \frac{2RI_0}{L}x = 0;$$

$$x = L/2, \tag{205}$$

a maximum; hence, the greatest drop is at the center of the main, and has the value $RI_0L/4$.

Taking the same value for the constants and variables as was assumed in the example in Case I., the curve CD plotted in Fig. 236 is obtained.

This curve shows the *rate of variation* in pressure between the various receivers along the mains, and does not indicate the *entire drop* between the potential of the generator and the point of least pressure in the conductor; for, as has already been indicated, the value of $u_0 - u'$ is less than the total difference of potential by a quantity equal to the fall in pressure in one-half of the conducting system. The curve of total fall of potential may be obtained by decreasing the ordinates of $\overline{\text{CD}}$ by a quantity equal to $RI_0L/2$, which is the resistance of half the conducting system, and is represented in Fig. 304 by the curve $\overline{\text{C'D'}}$. Comparing these curves with Cases I. and II., a much more uniform and regular service to the customers is indicated, demonstrating the advantageousness of this method of wiring in cases where it is possible to employ it.

CASE IV. — *Conical Conductors — Anti-Parallel Feeding.*

581. The plan of feeding from the opposite ends of the mains may be applied in conical conductors with equal advantage. The arrangement is shown in Fig. 305. From the equations in Cases II. and III., —

$$d(u_0 - u') = (ri - r'i')\,dx, \tag{206}$$

r and r' and i and i' being the resistances and currents in either main, respectively, at the point x. Hence, by a similar train of reasoning as in the previous cases, $r = \rho i/S$, and $r' = \rho i'/S'$; but

i/S and pi'/S' are constants for each main, by hypothesis; hence, —

$$d\frac{(u_0 - u')}{dx} = 0; \tag{207}$$

$u_0 - u' =$ a constant. But $u_0 - u'$ is here, as in Case III., the *difference* in *pressure* between the receivers; hence, —

$$V' - v' = V_0 - v_n = V_0 - v_0 - R_0 I_0 L; \tag{208}$$

integrating (207) $\qquad u_0 - u' = R_0 I_0 L.$ (209)

582. This is the equation of a straight parallel to the axis of x; hence, by this method of wiring, there is no pressure *variation* between the different receivers, all being submitted to precisely the same difference of potential. This method, then, presents an ideal

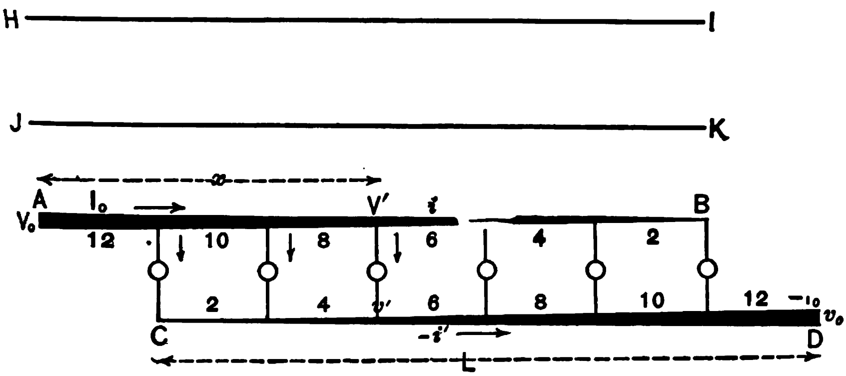


Fig. 305. Diagram of Distribution of Potential in Case IV.

solution of constant pressure distribution. The only variation to which the receivers are subjected is that due to a change in the loading of the entire system, which may be compensated for by the methods of regulation to be described later on. The same conclusions as to the relations of the amount of drop and weight of copper in the conducting system may be applied in this case as in Case II.

583. Collecting the curves indicated by these equations, and referring them to a single set of axes, a diagram is obtained as indicated in Fig. 306, from which a glance will show the relative potential distribution occurring in the four elementary systems. The four equations from which these curves are deduced are also here collected in a group, in order that their properties may be readily scanned. The salient deductions from these equations are collected in Table No. 80, in order to render them more conspicuous.

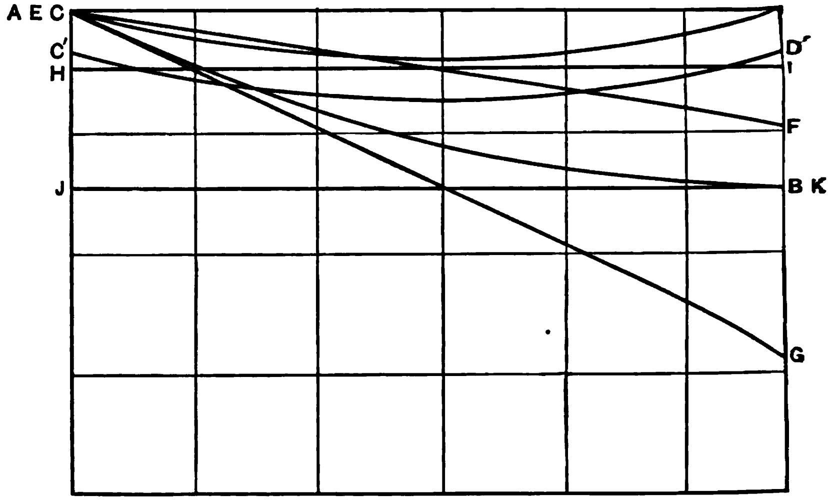


Fig. 306. Curves of Potential Distribution, Cases I., II., III., and IV.

584. CASE I. $u_0 - u' = \frac{RI_0 x}{L}(2L - x).$ (210)

CASE II. $u_0 - u' = 2 R_0 I_0 x.$ (211)

CASE III. $u_0 - u' = \frac{RI_0 x}{L}(L - x).$ (212)

CASE IV. $u_0 - u' = R_0 I_0 L.$ (213)

TABLE No. 80.

Relations between Cases I., II., III., and IV.

CASE NO.	DESIGN.	DROP BETWEEN RECEIVERS.	RELATIVE WEIGHT.	RELATIVE ENERGY EXPENDED.	GREATEST DROP BETWEEN RECEIVERS.	DROP BETWEEN GENERATOR AND RECEIVER AT HIGHEST VOLTAGE.	DROP BETWEEN GENERATOR AND RECEIVER AT LOWEST VOLTAGE.
1	2	3	4	5	6	7	8
I.	Parallel Feeding . . . . Cylindrical Conductors . .	2	3	2	RLI	0	RLI
II.	Parallel Feeding . . . . Conical Conductors . . .	4	1	2	$2RLI$	0	$2RLI$
III.	Anti-Parallel Feeding . . Cylindrical Conductors . .	1	3	3	$\frac{RIL}{4}$	$\frac{RLI}{2}$	$\frac{3RLI}{4}$
IV.	Anti-Parallel Feeding . . Conical Conductors . . .	0	1	3	0	RLI	RLI

585. In this Table the diameter of the conductors at the generator is the same in all cases. If the same *weight* of copper be allowed in all cases, the value in Cols. 6 and 8, Case II., and Cols. 7 and 8, Case IV., must be multiplied by 3 / 2; and the values in Col. 5, Cases II. and IV., divided by 2.

In the fifth column of this Table, figures are given showing the amount of energy which is lost by transformation into heat, due to the resistance of the conductor under a condition of maximum loading. This column is calculated from the following equation: —

$$U = \int r dx i^2, \tag{214}$$

in which r is the resistance per unit of length at the point where the current in the main is i. Integrating, the quantity of energy lost in the cylindrical conductors is found to be —

$$\tfrac{2}{3} RIL, \tag{215}$$

and for conical conductors —

$$RIL. \tag{216}$$

586. This corresponds with the relation shown in the TABLE No. 80. If it is allowable to use the same amount of metal in both conical and cylindrical conductors, the section nearest the station in conical mains may be made three times as large as that in the cylindrical conductors. Under these circumstances, the receivers are subjected to a much less difference of potential, and, at the same time, the energy wasted in the conductors is reduced by one-half of the amount that would be lost in cylindrical conductors having the same amount of copper. From the preceding considerations, it is evident that wherever it is practicable, the conical conductor fed upon the anti-parallel system gives the most uniform and regular service, wastes the least amount of energy, and subjects the receivers to the smallest potential variation. Wherever practicable, therefore, this method should be adopted.

MULTIPLE WIRE SYSTEMS.

587. The Three-Wire System. — If it were feasible to successfully manufacture incandescent lamps capable of operating under any desired voltage, it would be possible, by increasing the resistance of the lamps, to work central stations at higher potentials, and economize in the material employed in the conducting system. It has been

shown that, if the available potential remains constant, the amount of energy distributed will vary directly as the current, and the amount of energy lost in the circuit as the square of the current. By increasing the pressure, more energy may be delivered, or a greater territory served, without increasing the losses in the circuit. So far, attempts to make incandescent lamps of much more than 400 ohms resistance have not been commercially successful; and for the standard 16 candle-power lamp, an available difference of pressure at the lamp terminals of about 200 volts is all that can be rendered useful. Experience has shown that a maximum variation in pressure throughout the conducting system of more than 10 per cent is not compatible with good service. Thus, the greatest difference in pressure at the generators, in the methods of wiring so far described, is usually about 110 volts. Any plan which will render available a greater dif-

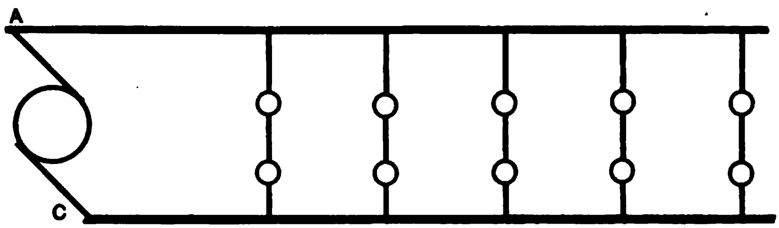


Fig. 307. Multiple Series System.

ference of potential will introduce a notable economy in the cost of the conducting system.

588. Suppose two conductors A and C, Fig. 307, between which is maintained a difference of potential double that which is necessary for any one of the receivers; for example, 220 volts in an incandescent lighting system of 110 volt lamps. It is then feasible to place lamps in series of two between the mains A and C. Therefore, for a given number of lamps, the necessary quantity of current is halved, and the admissible fall of potential may be doubled. The resistance of the conductors for a given output along the line may be quadrupled, and, consequently, the price of installation is reduced nearly 75 per cent. This device, however, involves the sacrifice of the independence of each receiver. As the receivers are placed in groups, each one involving a series of two, it is necessary to throw in or out of service an entire group; for if a single receiver be placed

in service, it will receive double the pressure for which it was intended. This defect, however, may be obviated, by so designing the station machinery that the dynamos are operated in groups of two placed in series, to obtain the desired voltage, and then introducing a third wire B, as indicated in Fig. 308, which occupies a position intermediate between the two generators, and extends through the entire system of conductors. Under these circumstances, each *unit* in the station must consist of two generators, connected together in series. By inspection, it is evident that the middle wire is traversed by a current which is only equal to that originated by the *difference* in the number of receivers that are simultaneously in service on the two sides of the system. The principal, or outer conductors, when the

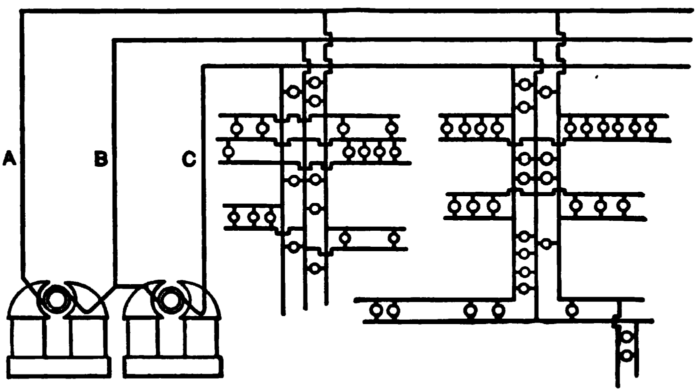


Fig. 308. Three-Wire System.

whole plant is in service, only carry a current equal to one-half that which would be necessary to supply the same number of receivers if installed on the two-wire system; and in this case, as the center wire is not traversed by any current, it has received the name of the "Neutral Wire."

589. Supposing now that a part of the load on one side of the system be thrown off. In the previous instance, both sides were balanced, but now one side needs more current than the other; and to preserve the independence of the individual receivers, the neutral wire acts as an overflow main, permitting the excess of current on the overloaded side of the system to return to the generator without affecting the other side. For example, in Fig. 308, suppose 100 receivers to be in service between A and B, and 80 between B and C,

each requiring one ampere; 100 amperes will be needed on the AB side, and 80 on the BC side. 100 amperes will evidently pass out through A, 80 back through C, and 20 back through B. By this method it is practical to introduce very large economics in the cost of the conductors, to greatly extend the scope of the plant and the distance over which it is possible, from a financial standpoint, to transmit electrical energy. The copper saving which may, in this way, be practiced, is easy to calculate, and depends upon the rules which have already been given. Suppose that, for a given plant, a certain economical density of current has been determined. This value is independent of the method of distribution employed; so if the same current density is to be used in the two systems, it will be observed that, by the three-wire method, the fall of potential is reduced to one-half. As an example, assume a current of 500 amperes under a pressure of 110 volts to be required by the receivers, with a drop of 8 volts, and that the most advantageous current density is found to be 1000 amperes per square inch. It is necessary then to employ for two-wire system conductors .5 in area, each giving rise in the length of the conducting system to a drop of about 7½ per cent of the total available potential. Adopting the three-wire system for the same case, as the potential is doubled, the same amount of energy is delivered with half the current, and so the current of the outer conductors is reduced to 250 amperes, and, at the same current density, the amount of copper is reduced in each conductor to .25 sq. in. in area. It is a common practice to make the third wire equal in section to the principal conductors, then the total cross-section is .75 sq. in., instead of 1.00 sq. in., and the total amount of copper is reduced by one-fourth. The fall of potential, however, remains equal to 8 volts; and, inasmuch as the total voltage is raised to 220, the percentage value of the fall potential is only 3.6 per cent instead of 7½ per cent, as in the preceding example. If, on the contrary, the calculations are based upon an *equal* percentage fall of potential in each case, the economy to be obtained in the amount of copper is evidently increased to five-eighths. Thus, in reality, each of the three conductors of this system may be reduced to one-fourth of the area necessary with the two-wire plan, the third wire being still assumed to be equal to the other two. Therefore, the total amount of copper in the three conductors is only three-eighths of that

which is required under the two-wire system to deliver the same amount of energy with the same percentage of drop. If all the receivers on one side of the conducting system were out of service, while all on the other side were in commission, it is conceivable that the neutral wire would be called on to carry a current as great as that of the outer main. This can only happen by some accident, such as the blowing of a main fuse, which would actually *open* one of the outer conductors; for by no possibility of service condition would half of the customers be out of service while the other half were in action.

590. Good practice indicates the advisability of placing half of the receivers of each customer on one side of the system, and half on the other. Then, in the event of the opening of one conductor, one-half of all of the receivers that are in the circuit are thrown out; and under this condition the neutral wire can only be traversed by

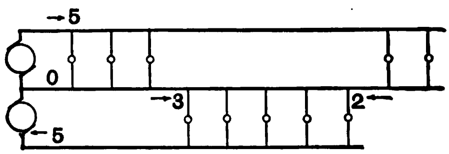


Fig. 309. Balancing Diagram.

one-half the current of the outer main, and the relative amounts of copper for the two and three wire systems under such similar conditions are as sixteen to five. It is also feasible, by a careful study of the various consumers, to place across the outer conductors such of the receivers as are able to accommodate themselves to great changes in voltage, as motors, for example, or groups of lamps that are rarely in service. Moreover, the two halves of every important installation may always be arranged to make the demands of each part sensibly the same. This precaution prevents long portions of the neutral conductor from being traversed by a current of sensible amount that does no useful work, thus economizing the lost energy.

591. The different parts of the third wire may be traversed by various amounts of current, both as to intensity and as to sign, although at the same time a balance of the whole system be carefully preserved. Fig. 309 gives an illustration of this condition, from which it is obvious that on each side of the system the same number

of lamps are in use, and that, while a part of the third wire is traversed by a current equal to three lamps, the system, on the whole, is balanced, and the portion of the conductor nearer the station is entirely neutral. It is advisable to carefully study every important installation, and to arrange the distribution of currents to attain the maximum conductor and energy economy.

Such are the advantages of the three-wire system. On the other hand, there are certain inconveniences, which it is now advisable to consider. It must be noted that it is necessary to maintain in operation two dynamos, instead of a single machine; and a station unit on the three-wire plan must consist of two dynamos, each having half of the power required on the two-wire plan. While the weight of the two dynamos for the same output will not differ sensibly from that of a single machine, the initial cost, expense of operation, and loss of efficiency are increased. The apparatus at the central station becomes somewhat more complicated; but this inconvenience is confined to the dynamos, for the engine, or other motor, may remain in each case the same. The driving machinery, such as shafting, pulleys, etc., must be connected to two generators instead of one. Also, the difference of potential between the system and the ground is doubled, and the chances of accident due to failure of insulation become largely increased. These objections to the three-wire system are by no means comparable with the advantages and economy to be derived; so all central stations of any importance are now, without hesitation, designed and laid out in accordance with the principles enunciated. Indeed, so great are the benefits, that the economical principles outlined have been extended, and similar systems using five and seven wires, with corresponding advantages, are by no means unique.

592. Multiple-Series System and Modifications of the Three-Wire System. — In local installations of small extent, where the full plant load, or at least the greater part of it, is constantly on the circuit, and where the independence of the individual receiver is not essential, it becomes possible to avoid the complexity introduced by the three-wire system, by operating the receivers in groups of two or more, placed upon circuits in parallel with each other, thus giving rise to the multiple-series method — see Fig. 307. The origin of the three-wire system was, doubtless, an effort to secure at once

the independence of each receiver, and the economy of high potential.

In the multiple-series system, all the receivers of each group must be in operation at any one time; as, if any one of the group is thrown out of service, the remainder will be either subjected to an electrical pressure greater than that for which they were designed, or idle resistance must be introduced in the circuit to absorb the energy previously consumed by the now isolated receiver. In this direction the multiple-series circuit labors under the same disadvantages and limitations as the ordinary plain-series circuit.

It is possible in some cases, where the load on the system is a reasonably constant one, to simplify to some extent the three-wire system by avoiding the complexity of two dynamos at the central station. In Fig. 307 such an arrangement is indicated — a single dynamo supplying the circuit having double the electro-motive force of the receiv-

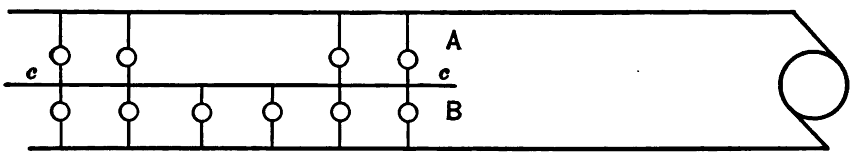


Fig. 310. Three-Wire System with Single Dynamo.

ers on the sides A and B. The third wire still exists in the circuit, but does not return to the station, nor is any connection made with the generator. Under these circumstances, if any one of the receivers in the group A be thrown out of commission, the circuit of none of the others will be actually opened, and, conseqntly, their operation will proceed uninterruptedly. As the generator produces a constant electro-motive force, and as now the resistance of the A group of receivers is increased by the opening of the circuit of one or more of them, and is higher than the resistance of the B group, the fall of potential throughout the system will be the same as when all the receivers were in operation, but the potential upon the A group will now be greater than that upon the B group. While formerly the potential of the central conductor *cc* was precisely midway between the potentials of the external conductors, the potential of *cc* is now not midway between those of the outer conductors, but approximates more nearly to that of the conductor B. In cases where such a

variation in service can be tolerated, as in factories, or other commercial institutions operating their own lighting plants, this method approximates sufficiently toward the best service to be desirable.

593. For very small installations, where the mains extend a considerable distance from the generator, yet where the load is so light as to render two machines inexpedient, the conductor economy of the three-wire system may be rendered available by the device indicated in Fig. 311. Here the generator is supplied with a third brush F′, set midway between the regular brushes, to which the third wire is attached. If the system is well balanced, with rarely any current in the neutral, this scheme is fairly successful; otherwise, there is likely to be destructive sparking at the commutator.

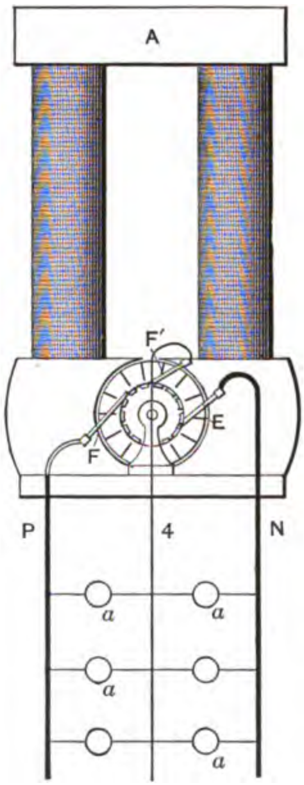


Fig. 311.
Three-Brush Three-Wire System.

556. The three-wire system is susceptible of a very great number of modifications, many of which will readily occur to the fertile designer. Mr. Leonard, in the *Electrical Engineer*, indicates several useful combinations, which are illustrated in Figs. 312, 313, and 314.

In the arrangement outlined in Fig. 312, a single dynamo is connected to the external conductors M and P, having an electro-motive force double that of the receivers to be placed upon the circuit. The third wire, instead of returning to the generator, is connected with the pole of the storage battery S, the other pole of which is in electrical communication with the main M. Under these circumstances, a current thrown upon the third wire N is absorbed by the storage battery, while the extra load upon the M side of the system is cared for by the output of current from the battery in question.

594. In Fig. 313 an arrangement is indicated, consisting of two generators, A and B; the A generator has double the potential of

all of the receivers, while the B dynamo is capable of developing an electrical pressure equal to that required by a single receiver.

When the load on the P side of the system is greater than that on the M side, a current returning through the central conductor N

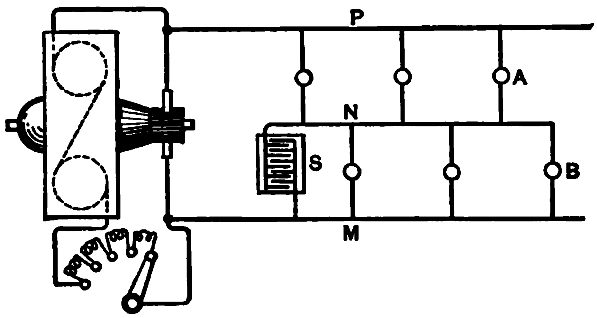


Fig. 312. Three-Wire System with Storage Battery Equalizer.

actuates the dynamo B, causing it to operate as a motor, and thus, by means of the counter shaft E, relieving the prime mover of a part of the load of the dynamo A. When, on the contrary, the M side of the system is overloaded, the dynamo B acts as a generator, supplying the necessary additional current.

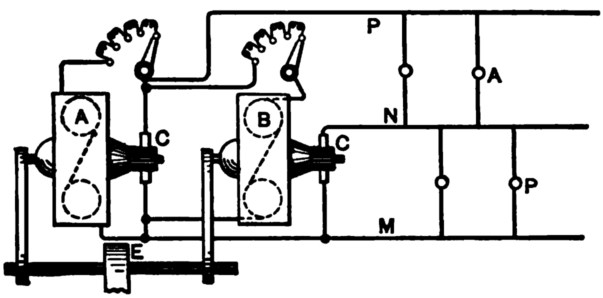


Fig. 313. Three-Wire System with Motor-Dynamo.

595. The design of Fig. 314 is a modification of the preceding arrangement, by means of which the generator A may be located at a considerable distance from the district to be served, while in close proximity to the district a motor R and generator C are located. The generator A may be run at a sufficiently high potential to make the loss between G and R comparatively small, while by means of

the motor R the second generator C may be made to operate in conjunction with the generator A upon the three-wire system, as indicated in the previous design.

596. The Five-Wire System. — By an extension of the principles thus developed, a greater number of circuits may originate from the same station, giving rise to methods of multiple wire distribution, embracing five or even seven wires, operating at correspondingly high potentials, and enabling a corresponding reduction in the expense of the conducting system. All of the advantages previously enumerated are augmented in proportion to the increase in the number of wires, while, on the other hand, the objections inherent in the multiple wire systems present themselves with correspondingly in-

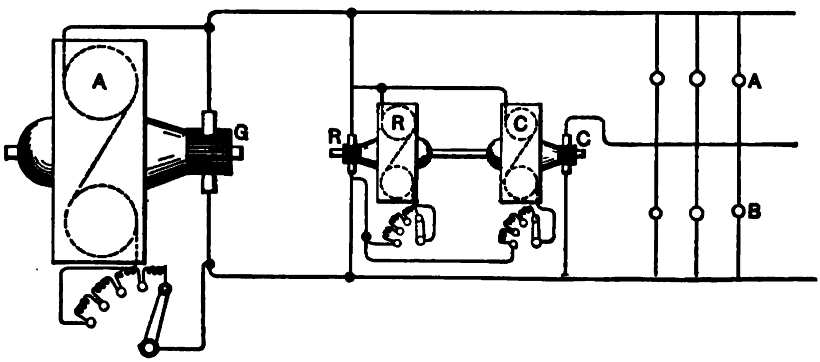


Fig. 314. Three-Wire System with Compensating Dynamo and Motor.

creasing force. The greater the complexity of the station, and the difficulties of obtaining sufficient insulation for the higher potential differences have so far, in this country at least, prevented a very wide introduction of anything but the three-wire system. Nearly all of the plants of the Edison Company in this country are built upon the three-wire plan, and form the most notable, and the most thoroughly designed and executed, examples of this method of distribution. In Europe, on the contrary, where the areas to be covered are perhaps not so great as in America, and where greater care and more thorough work is to be expected from an older and more complete civilization, the five and seven wire systems have attained quite a wide introduction, accompanied with very notable success.

597. In Fig. 315 are given diagrammatically the systems of

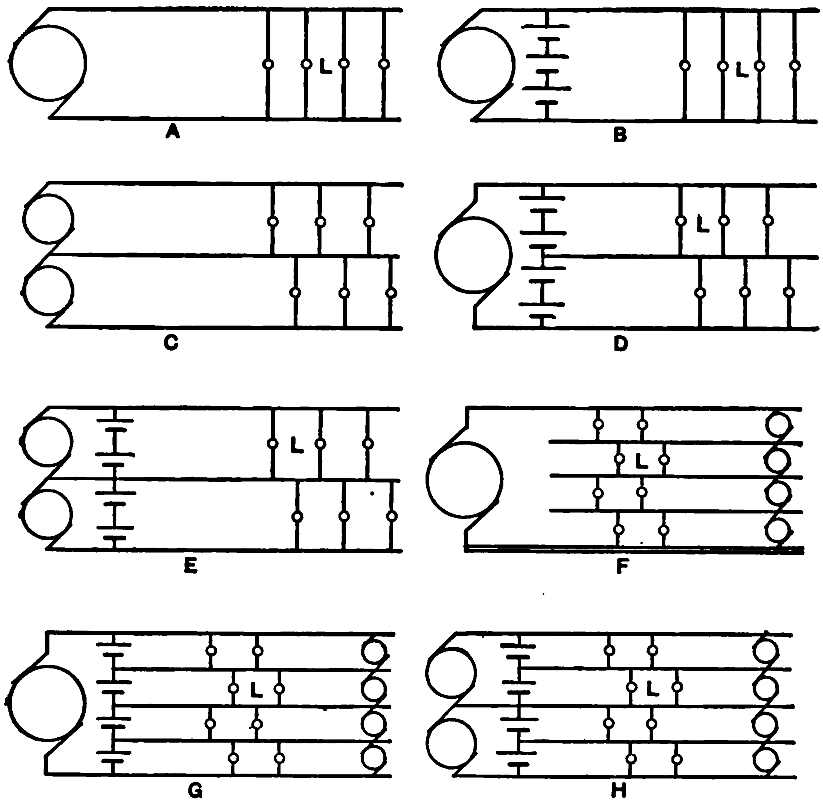


Fig. 315. Designs for Multiple-Wire Systems.

some of the most extensive European systems of direct current distribution, being adopted in the following towns: —

A. Parallel system, Berlin (Markgrafenstrasse Station), La Coruna.

B. Parallel system with secondary batteries, Salzburg, Lyons, Toulon, Montpellier.

C. Simple three-wire system, Berlin (Mauerstrasse Station, Schiffbauerdamm, and Spandauerstrasse), Elberfeld, Helsingborg, Malaga.

D. Three-wire system with one dynamo and secondary batteries, Mülhausen, Stockholm, Sundswall.

E. Three-wire system using two dynamos, Vienna (Mariahilf), Darmstadt, The Hague, Stettin, Breslau, Copenhagen.

F. The five-wire system with equalizing dynamo, Trient.

G. The five-wire system with equalizing dynamo and secondary batteries, Paris (Place Clichy).

H. The five-wire system with two generator dynamos, Vienna (Neubad).

598. The use of auxiliary dynamo machinery in several of these installations will be noticed. The method of employment is similar to that already indicated in Figs. 312, 313, and 314, for the three-wire system, and in Fig. 316, showing in detail the design for a five-wire system. Here the generator G runs at a potential sufficient for four receivers, and is attached to the two external mains of the system. At any desired intermediate point or points, three dynamo machines are introduced, each one operating at a potential equal to a single receiver. So long as the system is entirely balanced, the auxiliary dynamos absorb merely sufficient power to turn their armatures against friction of the bearings and the slight losses due to internal

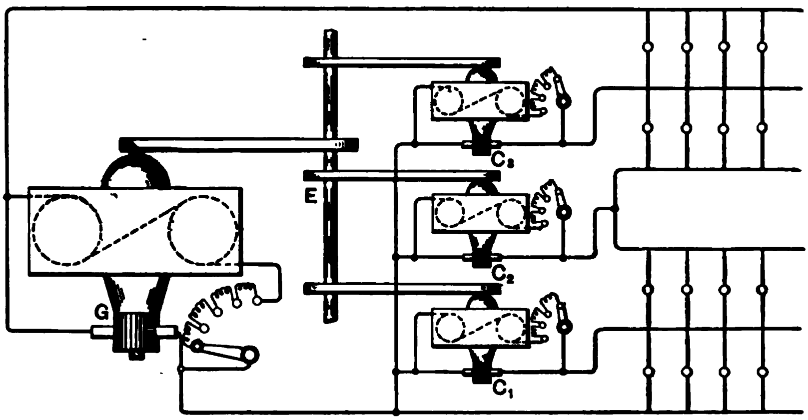


Fig. 316. Five-Wire System with Motor-Dynamo.

currents. In the case of any want of balance in any part of the system, the dynamo connected with that portion acts either as a motor or as a generator, depending upon whether the unbalancing is such as to give rise to a current flowing towards the station through the intermediate wire, or away from the station. In the first instance, the current flowing toward the station passes through the auxiliary dynamo, causing it to run as a motor, thus releasing the station of that amount of load. Contrariwise, should the unbalancing make the current flow away from the station in any intermediate wire, the dynamo acts as a generator, demanding from the station such an amount of power as will enable it to add to the circuit the required current. This subject will be further illustrated in the paragraphs upon motor transformers, in the succeeding chapter.

599. Relative Area Covered by Two, Three, and Five Wire Systems. — The territory that can be served by a central station only, depends upon the amount of copper that is placed in the conducting system in proportion to the number of customers to be served. With a limited drop, the cost of the conducting system may, even in a small territory, rise to such an amount as to prevent the enterprise from being a commercial success. If a given receiver is to operate at a definite distance from the station under a predetermined drop, the weight of the conducting system is readily calculated. If the distance is increased M times, the drop remaining the same, the weight of the conducting system will be proportional to M^2. In general, the weight and, consequently, approximately, the cost of the conducting system, is expressed by the formula —

$$W = Al^2, \tag{216}$$

in which W is the total weight of the circuit, A a constant embracing the drop, the conductor weight per customer, and the length of the system. With a drop of 5 per cent, and an average of $12\frac{1}{2}$ lbs. of copper per lamp in the conducting system per 50 watt lamp, a single station on the two-wire system is limited to a radius of about 1600 ft. To extend the territory to 2300 ft., formula (216) indicates that an expenditure of 25 lbs. of copper per lamp is required. TABLE No. 81 indicates the relative possible areas to be served, and weight of copper per lamp and drop.

TABLE No. 81.

Areas Covered by Multiple-Wire Systems.

KIND OF SYSTEM.	RADIUS OF PROFITABLE DISTRICT WITH 5% DROP.	
	Copper $12\frac{1}{2}$ lbs. per 50 Watt Lamp.	Copper 25 lbs. per 50 Watt Lamp.
2-Wire System	1600 Feet.	2300 Feet.
2-Wire System Feeders and Mains .	2500 Feet.	3500 Feet.
3-Wire System	2300 Feet.	3500 Feet.
3-Wire System Feeders and Mains .	4000 Feet.	6000 Feet.
5-Wire System	5000 Feet.	7200 Feet.
5-Wire System Feeders and Mains .	8200 Feet.	12000 Feet.

600. The Feeder and Main System. — All the methods for circuit design thus far indicated may, analytically, be reduced to one of the four elementary cases. When applied to the distribution of very large amounts of electrical energy, extending over considerable areas,

embracing points widely separated from each other, all of these methods, even including the multiple-wire systems, require the expenditure of so much material in the conducting system, in order to maintain a sufficiently uniform electrical pressure throughout the conducting network, as to make the cost of the system too great to permit of a profitable return upon the capital invested. As a solution of this problem, Mr. Edison, in this country, introduced the Feeder and Main System, which consists in subdividing the territory to be served into a large number of districts, by grouping the customers in proximity to each other into blocks located as near as possible at equal distances around a number of central points. From each of these radiating centers, receiving the name of Centers of Distribution, a pair of conductors, termed Feeders, is carried back to the central station. Upon the feeders no customers whatsoever are, under any circumstances, placed. From each of the centers of distribution there also extend a second set of mains running electrically away from the center of distribution, and so away from the central station, the office of which is to serve the various consumers.

601. This set of conductors has received the name of "Distributing Mains." By this means the entire territory is split up into a number of subdivisions, each in the most direct electrical communication with the station, by means of such an independent pair of conductors as will enable the station to supply the distributing center with the required amount of current, at any desired electrical pressure. Inasmuch as there are no customers upon the feeders, the fall of potential in the feeders is a matter of but little importance so far as the requirements of good service are concerned, these conductors being designed merely to supply to the distributing center the required amount of current under the most economical conditions. The central station may embrace a number of different dynamos, all running at different electrical pressures, each one conveying to its appropriate center the necessary current, so adjusted that on arrival at the distributing center all of the currents will come in under a uniform pressure, maintaining the essential constant electrical potential over the entire district. Such an arrangement is indicated diagrammatically in Figs. 317, 318, and 319.

In Fig. 317 the central station is shown at MN. From this point a pair of feeders MA, MA′ extend to the center of distribution

AA'. Two other sets of feeders also extend to the centers CC' and BB'. From AA', BB', and CC' the distributing mains are extended, across which the receivers are located. From M to A, B, and C any desired fall of potential may be allowed to take place in the feeders without in any way affecting the service of the respective receivers extended upon the distributing mains emanating from these points.

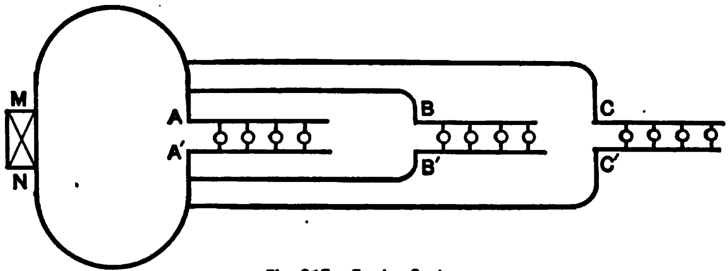


Fig. 317. Feeder System.

The distributing mains, however, must be so designed that, with all variation of load which shall be thrown upon them, the fall of potential shall be kept within such a limit as good service conditions require.

602. A similar design is indicated in Fig. 318, in which the station is located at MN in the center of the district to be served,

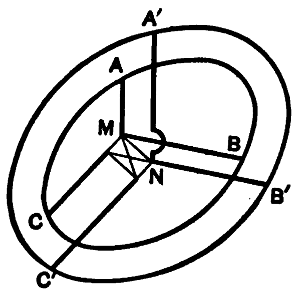


Fig. 318. Feeders and Mains.

from which three sets of feeders, MA, MB, MC, and MA', MB', and MC', extend to the distributing mains, that are located in ellipses around the central station. As in the previous illustration, no receivers are placed upon the feeders, so the variation in pressure in this part of the conducting circuit does not affect the customers, but may be made as great as the rules of maintenance, economy, and

safety dictate. Upon the distributing mains, A, B, C, and A′, B′, and C′, the drop must be restrained within service limits, no matter at what sacrifice of conducting material. This design is much superior to that in the preceding illustration, as it is evident that each feeder can supply the distributing mains in two directions, thus shortening the electrical length of the distributing mains, and obviating accident in case of the rupture of any conductor.

603. A still further improvement is indicated in Fig. 304, in which the distributing mains assume the form of a circle, around

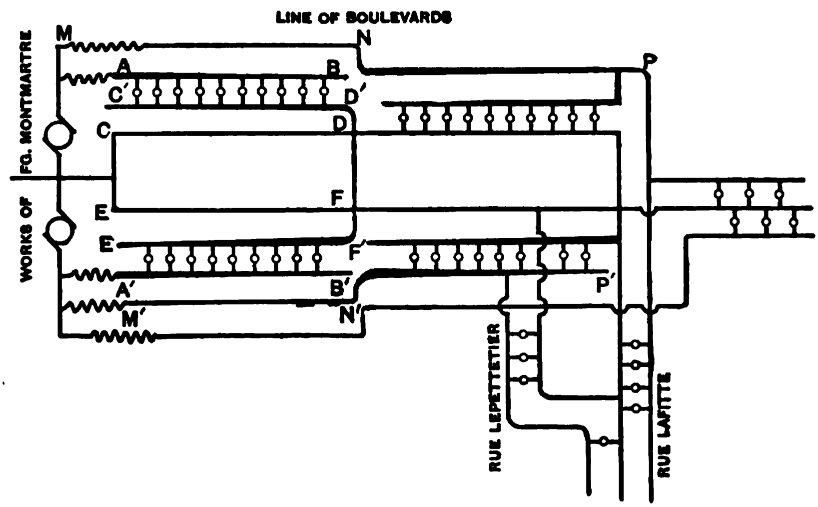


Fig. 319. Feeder System, Paris.

which the receivers are equally spaced, while the feeders are introduced at two diametrically opposite points upon the circumference.

604. For installations upon a large scale, such as are required by urban distributing plants, especially for incandescent lighting, all of the foregoing principles are usually combined in the design of the conducting system. An excellent example of this is indicated in Fig. 319, giving in a skeleton form the mains of the Edison Company along one section of the plant in Paris. A slight inspection of the diagram indicates that the general design of the conducting circuit is that of a three-wire system embracing conical conductors for the distributing mains, with anti-parallel feeding, thus realizing the highest

economy in the conducting material, and the least possible potential variation.

605. Location of the Central Station. — In the case of series distribution, it is shown that the location of the central station is a matter of relatively slight importance, provided the site is chosen on the perimeter of the polygon formed by the line of circuit; that all locations on this line are equally advantageous; and that in case it is necessary to slightly depart from the actual location of the circuit, all points are equally available that are equally distant from the line. Under the parallel system the location of a central station becomes a matter of the most paramount importance; for, under this system, the amount of current, and *not* the electrical pressure, is the governing factor.

The losses entailed by the conducting system vary as the square of the current flowing, and as the resistance of the conducting system; and as the supply of a definite territory requires a definite current, the resistance remains as the only variable at the command of the engineer. The dictates of both maintenance and economy, and the requirements of good service, make it essential to reduce the expense of the conductor system in every direction to the lowest possible amount. As will be now shown, this is best accomplished by locating the central station at the electrical center of gravity of the district to be served, considered in relation to the various points to which energy must be distributed, and the amount of energy to be conveyed to each respective center. In many cases, the very important consideration of coal and water supply, the availability and economy of real estate, and physical causes affecting the ability to obtain the requisite foundations for heavy machinery, must be taken into consideration in the selection of the station site. Leaving, for the present, these conditions out of the question, the location of the site should be determined, in so far as the relation to the conducting system is concerned, at such a point in the district as will place it at the electrical center of the conducting system. To properly locate the central station, a reasonably accurate map should be made of the district, with a careful canvass of all the probable customers, obtaining the amounts of energy that they are likely to demand. An inspection of a good map so prepared will enable the engineer to select a number of points in the district, which will, from the topo-

graphical features of the territory, be made centers of distribution. From these centers of distribution the distributing mains extend, electrically speaking, away from the station, while toward the station from each point the feeders will run. From a canvass of the customers, the amount of energy to be delivered at each of the centers of distribution is determined, thus giving the maximum current flowing through the feeders, and the current to be diffused by the distributing mains. These amounts should be carefully noted opposite each of the centers of distribution, where the feeder joins the distributing main.

606. To determine the proper station site, suppose, in Fig. 320, the irregular outline includes the territory to be served, the black dots scattered throughout indicating the location of each of the centers of distribution, and the amount of current to be delivered at each of the respective points. The problem to be solved, in order to determine the proper location of the station, is to ascertain the electrical center of gravity of the points of distribution, precisely as the center of gravity of an irregular solid would be obtained. Graphically, a solution is obtained by selecting any two centers of distribution, and joining them by a straight line.

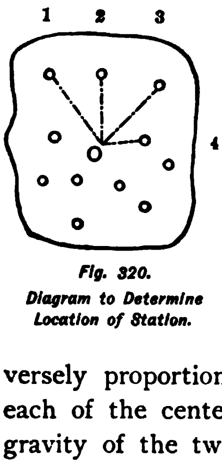


Fig. 320.
Diagram to Determine Location of Station.

This line should be divided into two parts, inversely proportional to the amount of current to be delivered at each of the centers; such a point is then electrically the center of gravity of the two distributing points in question. The process is repeated until the resultant center of gravity of all of the centers of distribution is obtained, indicating the best location for the station, considering simply maximum economy in the cost and maintenance of the distributing system. Analytically, the determination of the station site is as follows: —

Assume that at the points 1, 2, 3, etc., currents represented by i, i', i'', etc., are to be delivered, and that the centers of distribution may be connected by straight lines with the station. The weight of each conductor for a given fall of potential is proportional to the product of a constant depending upon the allowable drop, the current i, and the square of the length of the con-

ıctor l. Hence, the weight of each conductor O1, O2, O3, etc., ill be —

$$w = Al^2i, \tag{217}$$

ı which w is the weight of the conductor, and A the constant above eferred to. The total weight of the feeder system is the sum of all hese equations applied from the point O to the distributing points , 2, 3, etc., —

$$W = A\Sigma l^2i. \tag{218}$$

The study of mechanics presents a similar problem, the solution of which indicates a key to the solution of the above equation.

If, at the points 1, 2, 3, etc., masses of matter are supposed to be concentrated, the magnitude of which may be represented by the current values i, i', i'', etc., the term $\Sigma l^2 i$ represents the moment of inertia of this system with reference to the point O, from which the following conclusions may be drawn: —

607. *First.* The moment of inertia of the system, with reference to the center of gravity, is a minimum.

608. *Second.* The moment of inertia referred to any other point than the center of gravity, depends upon the distance "D" between the center of gravity and the second point of reference. The moment of inertia is equal upon all points of any circumference described with the center of gravity as a center with "D" for a radius, therefore — *The best site for a central station is the electrical center of gravity of the centers of distribution. All locations equally distant from this point are equal in value.* As the station is removed from the electrical center of gravity, the pounds of copper required for the conducting system are rapidly increased.

609. Let W be the weight of copper required for the conductors, when the station is located at the electrical center of gravity of the system, and W' the weight when located at any point distant d feet from the center of gravity.

Let l be the length of any feed.
Let i be the current in this feed.
Let L be the average length of all the feeds.
Let I be the total current.

Let the above symbols apply when the station is located at the electrical center of gravity, then, —

$$\Sigma l^2i = L^2I; \tag{219}$$

and

$$W = AL^2I, \tag{220}$$

A being a constant depending on the allowable drop, as shown in the paragraph on the "Limits of the Three-Wire System." If the station is moved to another site, distant d feet from the center of gravity, then, —

Let l' = the new length of any feed,
Let L' = the average length of all the feeds,
Let W' = the weight of copper now required,
Let i and I remain the amounts of the current, as before;

then, $$\Sigma l'^2 i = L'^2 I, \tag{221}$$
and $$W' = AL'^2 I; \tag{222}$$
from which $$W' - W = L'^2 - L^2.$$

The mean length is analogous to radius of gyration and, therefore, —

$$L^2 - d^2 = L'^2; \quad W' - W = AId^2 \cdot$$

and $$\frac{W' - W}{W} = \frac{d^2}{L^2}. \tag{223}$$

This is the equation of a parabola having its vertex at the origin and branching upward away from the axis of x. So the conductor material increases very rapidly as the station is moved away from the electrical center of gravity of the system. Increasing the weight of the conducting system means not only a much larger initial investment, but also increased maintenance expense, and increased cost due to energy lost in the conducting system. (See "Location of Central Offices," by the same author.)

610. On the other hand, the removal of the station from the center of gravity may permit the utilization of real estate at such an advantage as will more than compensate for the extra capital invested in the line. Furthermore, locations may be chosen permitting the utilization either of water-power directly, or of water to supply condensing engines, or may provide access to transportation facilities for fuel supply, thus cheapening the cost of power production to such an extent as to make the additional conductor investment required by the change in location a most desirable investment. The decision of the location should be determined from the following considerations: Determine the cost of plant, and cost of operation, with station located at the electrical center of gravity. Determine the value of the same items with the station placed at any other location presenting supposed advantages. An equation between these quantities will at once indicate which of the two sites possesses the greater advantages, and the relative value of the merits in both cases.

611. Location of the Feeders and Centers of Distribution. — In the preceding analytical investigation, it has been assumed that the feeders extended to the centers of distribution in straight lines. In practice, this would rarely be the case, and the symbols in the equations must be assigned values obtained from the actual location of the conductors. Many other circumstances contribute to limit the number of feasible selections for the location of the central station, and the design for the main feeds. Regarding the points where the feeds unite and join the network of distributing mains, it is obvious that the possible theoretical locations are very much limited, and must conform to urban geography.

Distributing boxes and other conduit structure must be located on the streets and at street corners; and, therefore, for each block there can only be four possible locations from which to choose for the junction between the main feeds and the distributing mains. Application of the equations, however, are available in determining which, of all possible locations, will be, on the whole, the most advantageous; and the actual placing of the mains should conform, as nearly as possible, to that which is thus obtained.

612. Distributing Mains. — It has been pointed out that the reason for employing the system of feeders and mains, is the necessity for preserving, at all points throughout the distributing mains, a constant difference of potential, in order to secure satisfactory service to the consumers. In illuminating plants, the most brilliant lamps will evidently be those placed nearest the junction between the feeds and distributing mains, while those of the least brilliancy will be such as are located midway between the two feeding-points. Experience has shown that a variation of 2 per cent in the voltage at any lamp is about as large as can be entertained compatibly with reasonably good service. Therefore, the value of 2 per cent of the available voltage, at the center of distribution, is a compulsory constant, and must be applied in the equations for determining the copper cross-section of the distributing mains. At each junction point between a feeder and the network of distributing mains, there flows a current having a maximum value sufficient to supply the territory surrounding the distributing center. This current must now be subdivided among the various distributing mains that terminate at the center of distribution, in proportion to the probable demands of each main. In the

district thus to be served, the total number of lamps and their distance from the center of distribution being determined from the map and canvass of the territory, the copper cross-section to deliver the required current with the specified fall of potential may be readily calculated by the methods already given.

613. It should be carefully noted, however, that all calculations should be made for the *maximum load* that will ever be thrown upon any conductor. It has been customary to calculate a section of the mains for several points in the network where the heaviest and where the lightest loads may reasonably be expected to occur, and proportion the rest of the system between these extremes. Good engineering, however, scarcely sanctions this practice; for while in complicated plants full calculation is exceedingly tedious, satisfactory service, and economy in first cost, always warrant the most careful investigation and calculation of the design of the conducting system. If the junction points between the feeds and network are placed too far apart, the equations will indicate an excessive copper cross-section for the distributing mains, in order to prevent too great a fall of potential. Such a result points to the advisability of introducing a greater number of feeders, in order to reduce the copper cross-section to a minimum. Evidently a relation between the copper to be placed in the feeders and in the mains may be written, which, when differentiated and equated to zero, will give the minimum copper volume to be employed in the entire plant.

614. **Calculation of Feeders.** — As the feeders operate as simple conductors of definite length and carrying a definite maximum current, the calculation of the appropriate section by Ohm's formula, taking into consideration the lines of economy indicated by Lord Kelvin, becomes exceedingly simple. The only constants requiring careful determination are those of the allowable fall of potential in the feeds, the maximum and mean currents, and time of operation. As no customers are connected with the feeders, service limitations have no bearing in the calculations for this part of the conducting system. Here, on the contrary, the rules of economy and safety become of paramount importance. It is first advisable to determine the maximum current to which each feed will be subjected, and then to ascertain the requisite cross-section required by heating limit to carry this current. For this purpose,

formulæ for current density, given in a preceding chapter, are available. Particular care, however, must be taken in the case of conduit or concentric conductors, to allow an ample margin of safety. Having determined the minimum cross-section for the maximum current, it is advisable to apply Lord Kelvin's laws, as indicated in the section for "Minimum Cost of Plant," and "Minimum Cost of Operation and Maintenance," to determine whether the cross-section already found is that indicated by the dictates of economy. All of the necessary constants may be readily valued, excepting the quantities I and T, indicating the mean current and time of operation of the plant. These demand careful study, and can only be estimated by considerable experience in similar plants, operating under equivalent conditions to the one under consideration. In series plants, the determination of T and I presents no difficulty; for I (the current) is a constant, and T is the total annual hours of operation. With parallel plants, however, the current is constantly varying, and to ascertain its mean annual value (the quantity necessary to consider) requires special investigation and precaution.

615. The energy lost in the conducting system, by transformation into heat, is proportional to the square of the current multiplied by time during which it flows. Thus, if i, i', i'', etc., are the respective currents flowing for a time dt, then the energy wasted is proportional to —

$$\Sigma\,(i^2 + i'^2 + i''^2 + \text{etc.})\,dt, \tag{224}$$

which is a quantity which may differ considerably from the square of the mean current multiplied by the time of its flow. To determine the current and time factors, it is advisable to procure a number of load curves from stations probably similar to the one under design. From a careful consideration of these a fair estimate may be made. This value may be checked by a consideration of the probable number of consumers to be obtained for the plant, with the amount and length of time that they are likely to use a current. It is considered that 500 hours per year is a minimum for paying stations. Greater values, 1,500 hours ($4\frac{1}{4}$ hours per day) for ordinary custom lighting, such as restaurants, stores, etc., or 3,600 hours for public lighting, etc., requiring all night service, are the customary averages. From a careful analysis of the probable demands for

current, and a comparison with load diagrams of stations similarly situated, it is possible to deduce the probable load diagram of the plant with a reasonable degree of accuracy. Given the load diagram, the value of the expression, —

$$\Sigma (i^2 + i'^2 + i''^2 + \text{etc.})\, dt,$$

is most easily made by integrating with a planimeter the area of the load diagram. By this method the appropriate values for I and T are readily selected, and a solution of the preceding equations indicates the appropriate economical section for the feeder; a repetition of the process serving to determine the section for all the various feeder mains. Having determined the appropriate current density for the feeder, both with respect to the heating limit and dictates of economy, the fall of potential in each feed is given by the equation $E = \rho li/S$.

616. The question of the best number of feeds to be employed yet remains, and deserves careful consideration. The weight of copper is not increased by augmenting the number of feeds; for, by multiplying the feeds, the weight of distributing mains may be decreased. The expense of installing and the cost of laying the feeders are, however, augmented to some extent by the number introduced. This, however, is largely counterbalanced by the greater saving in copper that can be made in the distributing mains; for the increased number of feeds will render the potential throughout the network more constant and uniform, thereby reducing the amount of copper required in this part of the system.

It is sometimes assumed that the weight of distributing mains in the network varies inversely as the square of the number of feeds; while this ratio is probably too large, it is certainly greater than the first power, and, as will be shown, increasing the number of feeds forms one of the best methods for close regulation. The exact number of feeds to be introduced in any plant is a question of judgment which can only be adequately determined by special consideration of the design of the plant and of the probable number of consumers. Beyond this no fixed rule can be given.

617. Efficiency of the Conductors. — The minimum efficiency of the network of distributing mains is that which corresponds to the maximum current, and may be deduced from the calculations for

current density. A consensus of experience, in distributing plants of this nature, indicates that a permissible fall of potential of 7 per cent may, in the feeders, be allowed, 2 per cent in the network of distributing mains, and 1 per cent in the consumers' wiring, reckoned upon the potential at the terminals of the generators. It may, therefore, be assumed that about 90 per cent of the energy delivered by the station reaches the consumers. The mean annual efficiency may be considerably higher than this, for the instances in which a plant is being constantly worked to its full capacity are very rare. Occasional overloading, even to the extent of causing a loss in the feeds of from 15 to 18 per cent, will not seriously alter the annual efficiency, inasmuch as the time of such overloading is usually extremely short. Even under these circumstances, an annual efficiency of 95 per cent or more may be reached by the conducting system.

618. **Methods of Regulation.** — The prime condition demanded by good service is that the difference of potential at the terminals of the receivers shall remain constant. A network properly calculated does not cause a loss of over 2 per cent between the receivers, the feeds being so designed as to deliver equal pressures to all the centers of distribution. The station should be so arranged that the potential delivered by the generators may be slightly varied to correspond to the demands of the service. Direct current distribution is usually effected by arranging a number of similar generators to operate in parallel, and connecting them at pleasure with the various feeders, to meet the varying demands of the service. The divergence of the current among the various feeders should take place in accordance with the demands of the customers; and to this end it is necessary that the station should be able to control the supply in such a manner that it may take place substantially in accordance with the requirements of each circuit, so a knowledge of the actual potential delivered from time to time at the centers of distribution is essential.

619. For this purpose a series of fine wires called voltmeter wires, or pilot wires, are extended through the conduits from the centers of distribution back to the station along each of the feeder circuits. These wires, running from the centers of distribution, are permanently connected to the voltmeters in the station, and so give a constant indication of the pressure actually delivered at these

points. If there are as many tell-tale wires and voltmeters as there are feeders, it becomes a very easy matter for the station attendants to keep a perfectly constant pressure at the centers of distribution. An improvement over this method, combining a greater sensitiveness with a clearer knowledge of the demands of the circuit, consists in supplying each of the feeders with an ampere meter having a double scale arranged to measure the current flowing in the feed, and the fall of potential thereby occasioned.

620. The most common method of station control consists in introducing in each of the feeders an adequate, adjustable rheostat, either of wire or carbon, by means of which the current delivered to the feeder in question may, from time to time, be adjusted by the station attendant. It has been recently proposed to accomplish regulation by giving the field magnets of the generators a differential winding, placed in series with the pilot wires returning from the centers of distribution, in such a manner that a fall of potential at the center of distribution will be followed by an increase of current through the field coils of the generator, and a proportional increase in the pressure delivered by the machine. A parallel result could evidently be attained by over-compounding the generators, and passing the feeder current through the field coils. The difficulty with both of these methods lies in the necessity of so constructing the generators that their fields may work at a very low degree of saturation, in order to be sensitive to slight variations in field current, and in the inevitable sluggishness with which the magnetic circuits of large dynamos will respond to changes in the field currents, even at low points of saturation.

621. A more hopeful design for automatic regulation lies in arranging the governor of the engine, or other prime mover, to respond to changes in the feeder currents, in a way to vary the speed of the generator. Designs of this kind are reported to be very successful. Regulation may also be accomplished by multiplying the number of feeds, with the notable advantages of a proportionate saving in the energy expended in the conductors, and a much more satisfactory service. Where regulation by a multiplication of feeds is undertaken, they are so arranged that they can be cut in and out of service, in such a manner as to vary the total resistance of the circuit, as nearly as may be, in proportion to the changes of load.

Then, during the hours of minimum service, only a few feeds are in service, and as the load increases, more and more are thrown in service.

622. The Compensator. — The most ingenious, and probably the most successful, method of regulation consists in the employment of compensating dynamos, whereby such an amount of energy as is wasted in any feeder may be restored to the current transmitted from the station. To overcome the ohmic resistance of any conductor requires the expenditure of a certain amount of energy. This expenditure of energy manifests itself in a fall of the electrical pressure. If, by some device, there could be added to the station output precisely the amount of voltage that is expended in transmitting the current through the feeder, the energy would always arrive at the center of distribution under a constant tension. Mr. W. S. Barstow conceived and put in practice the idea of using a small auxiliary dynamo, the office of which should be to add, from time to time, to the station's current the required amount of voltage necessary to overcome the resistance of the feed. As usually arranged, a small dynamo is placed with its brushes in series with the feeder circuit. The station current, passing through the armature of the compensator, receives precisely the additional amount of electrical energy that is to be expended in transmitting the current through the feeds. As the increase of energy is manifested by an elevation of potential, the compensating dynamo is frequently known as "Barstow's Booster." If the compensators are made sufficiently large, that they may normally work along the straight portion of the characteristic, either the whole or any desired fraction of the main current may be passed through the field coils, and the device made self-regulating, the voltage imparted by the compensator automatically varying precisely in proportion to the current output. To accomplish this requires a larger and more expensive dynamo; and it is, therefore, frequently customary merely to pass the current through the brushes, depending for adjustment upon the normal regulation of a rheostat placed in the field circuit. In Fig. 321 is given the curve of pressure at the station end of a feeder in the Brooklyn Edison Station, as obtained from Mr. Barstow's experiments, recorded in the *Electrical Engineer.* It is further shown that the cost of transmitting a current of 800 amperes over a distance of about two miles, by means

of the compensator, averaged about $4.32 per day for the winter months, and $3.00 per day for the summer months. In the diagram, the energy added by the compensator is clearly represented, as indicated by the varying pressure delivered to the feeder. The constant in this diagram is one volt drop to every 7.82 amperes.

623. The compensator method has recently been still further developed by Messrs. Barstow and Mailloux in the Brooklyn Edison station. Here the service conditions are found to be such as to require the station to supply three different voltages. To operate sufficient independent dynamos to supply the three voltages would require too large a plant, and would not be conducive to good station economy. The solution of the problem is diagrammatically indicated in Fig. 322.

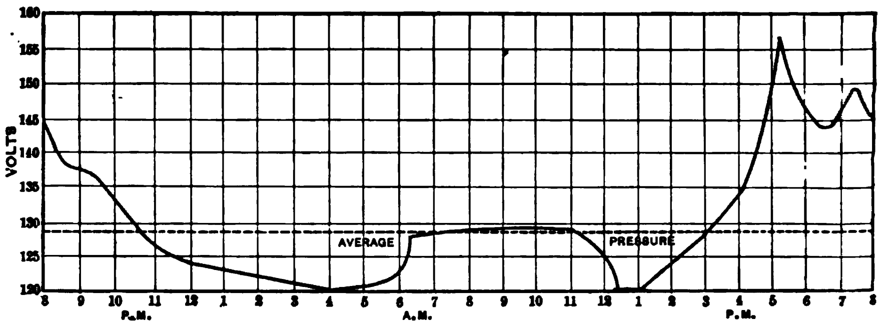


Fig. 321. Curve of Feeder Pressure, Brooklyn Edison Station.

The station is supplied with three sets of omnibus bars, one for high pressure, one for normal pressure, and one for low pressure. By appropriate switches, any feeder or set of feeders may be connected to any set of omnibus bars. Normally, the dynamos of the station represented by BB are connected to the main bus bars, furnishing 220 volts on a three-wire system. For the high and low pressure service, two sets of compensators are provided, C and C^4 being the low pressure compensators, while C^1 and C^3 are the high pressure machines. The entire compensator plant is so mounted as to be driven from the dynamo C^2, that, receiving power from the main omnibus bars, acts as a motor. When it is desired to raise the pressure of any feeder, the machines C^1 and C^3, driven by C^2, operate as dynamos supplying the desired additional energy. When it is

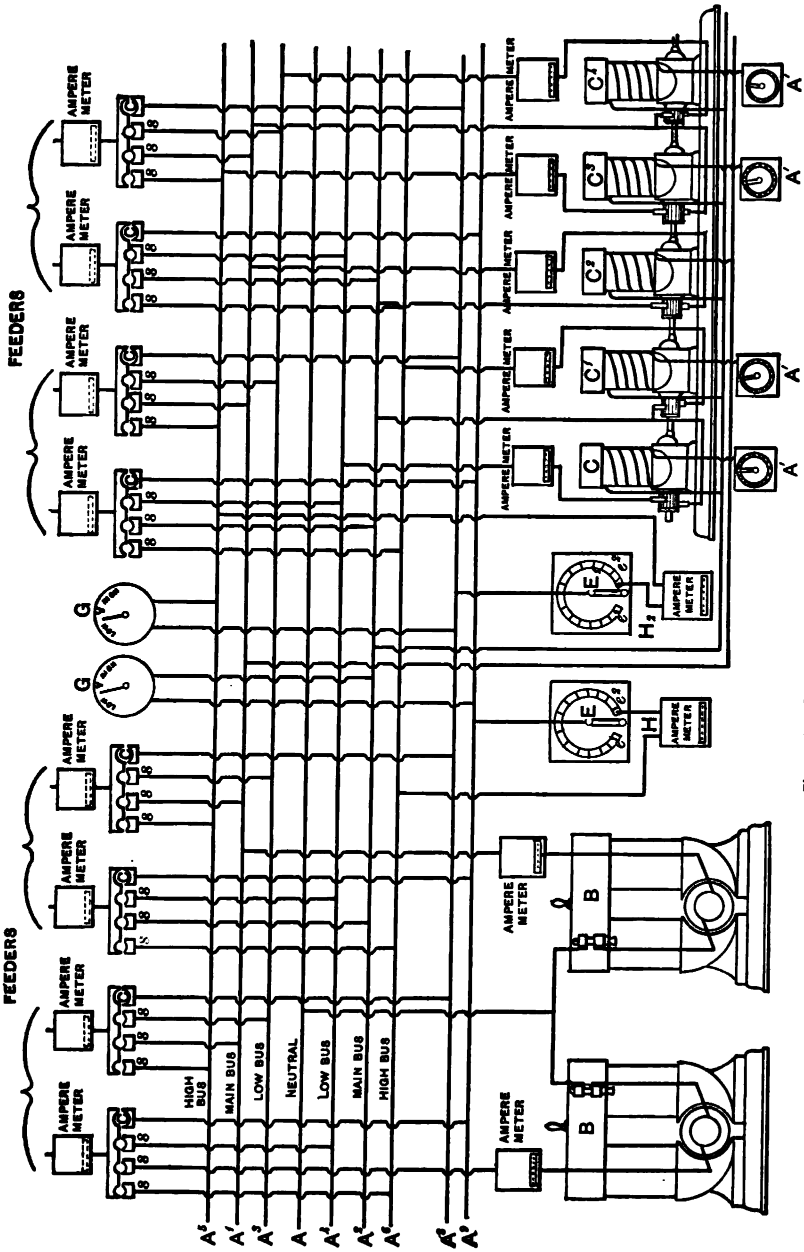


Fig. 822. Compensator for Three Voltages.

desired to reduce the pressure, C and C^4 operate as motors, absorbing a certain amount of energy and depressing the potential. By this means the main generators are run at such a pressure as is found suitable for the majority of distributing centers, while the pressure to the long feeds may be re-enforced, and that in the short feeds diminished. The beauty in the device is the ability to change the pressure in any set of feeds, without interrupting the service. This is accomplished by a pair of auxiliary omnibus bars A^8 and A^9, to which any feeder may, for the time being, be transferred. By means of the rheostats E and E′ the pressure in the feeder after transference is raised or lowered to that corresponding to the high or low omnibus bars. The feeder is then again transferred to either the high or low bar, the whole operation being accomplished without the slightest interruption. The connections in the diagram are so obvious as to render further explanation unnecessary.

624. The Compensator in Electric Railway Work. — The application of the compensator to electric railway circuits has recently been made by J. H. Vail, in the construction of a road from Poughkeepsie to New Hamburg. The power station is at Poughkeepsie, and is located centrally with reference to some ten miles of track extending through the streets of that town. A spur line runs due south connecting the towns of Wappinger Falls and New Hamburg, a distance of over ten miles. To avoid the excessive amount of copper that a line of this length would require, under the usual design of street railway circuits, a compensator is introduced in the station, that, by means of two No. 0000 feed wires, carries the necessary current to a distributing center eight miles south of the power station, thus supplying this section of the system, with the employment of a very small amount of conducting material in the overhead line.

625. Knowing the cost of building the feeder system, the cost of compensator and of operation, it is a simple matter to substitute these values in the equations given for feeder calculations, and ascertain the relative economy. Under ordinary circumstances, Mr. Vail shows that for a plant delivering 200 amperes at 500 volts, the compensator system requires less initial capital investment, when the distance to which the current is transmitted exceeds from two and one-half to three miles, and that the operating expense is decreased when the distance exceeds one and one-half miles.

626. As the capital absorbed by the feeder system designed to aintain a constant potential varies as the square of the distance ver which the current is delivered, while the cost of the compensaor varies directly as the distance, it is evident that great economy ay be effected by this means, in long distance transmission.

627. The compensator also adds great flexibility to the railway ystem. It is usually necessary to introduce a large amount of copper in the feeder system to provide for emergencies, such as excursions, etc., or for unusual bunching of cars at particular points. By the aid of the compensator, the feeder system may be designed for normal traffic only, and by means of a switchboard, the extra pressure delivered by the compensator applied to the various sections of the line, as occasion may from time to time require. A similar advantage appears in the ability to meet the load changes in a distributing system during the daily variation of traffic. If the circuit is calculated for the hours of greatest business, the copper employed is partally idle during a greater proportion of the time; while, if the circuit is arranged for the average business, it will not carry the maximum loading. To build the line for average work, and to put the compensator into service upon the system morning and evening, and on holidays, is an economical solution of the problem.

628. Fall of Pressure and Necessary Section in the Feeders. — The pressure at the centers of distribution, where the feeder joins the network, being maintained constant by some form of regulator placed at the station in the feeder circuit under the control of the station attendants, the loss of pressure in the feeder is not a factor in the supply condition of preserving a constant potential at every consumer.

Let S be the cross-section of one main.
L be the length.
I be the current.
ρ be the specific resistance in any desired units, as the mil-foot, square-inch-mile, square-millimeter-kilometer, etc.
V be the potential at the generator.
v be the potential at the center of distribution.

Then $V - v$ is the loss in the feed, and —

$$V - v = \frac{I \rho L}{S}. \qquad (225)$$

ρ must be given such a value as will be cognizant of the final temperature to be attained by the conductor.

629. Solving now for S—

$$S = \frac{I\rho L}{V - v}. \qquad (226)$$

From this expression the area of either the feeds or the distributing-mains may be calculated, in so far as the variation in pressure is considered to be the governing condition.

For a three-wire system, let Fig. 323 represent the feeders, Vv and $V''v''$ the outer mains, while $V'v'$ is the neutral wire.

Let V, V', and V'' represent the respective pressures at the generators.
v, v', and v'' the pressure at the center of distribution.
I, I', and I'' the respective currents, S the sectional area of the outer main, and S' the area of the neutral, the remaining symbols retaining the preceding meaning.

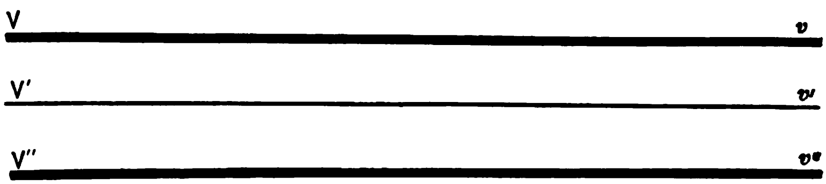


Fig. 323. Diagram for Fall of Pressure in Feeders.

Then $V - V'$ and $V' - V''$ are respectively the differences in pressure between the outer mains and the neutral wire at the generators, and $v - v'$ and $v' - v''$ are the corresponding differences at the center of distribution. Then the drop along Vv is—

$$(V - V') - (v - v') = \rho L\left(\frac{I}{S} + \frac{I - I''}{S'}\right) \qquad (227)$$

and along $V''v''$ —

$$(V' - V'') - (v' - v'') = \rho L\left(\frac{I''}{S} - \frac{I - I''}{S'}\right). \qquad (228)$$

630. In the best designed three-wire systems, it is customary to make the area of the neutral conductor equal to one-half of the area in either of the outer conductors. Assuming, then, that the greatest inequality in the balance between the two sides of a three-wire system is $2p$ per cent of the maximum load, the current in the outer main, having the lightest load, will evidently be $I\,(100 - p)$, and in

the outer main with the heavy load, $I\,(100 + p)$, and the current in the neutral wire will be $p\,I$; p being expressed in percentage. The value to be assigned to p will vary greatly, depending upon the care exercised in the balancing of the load on the two sides of the system. Good practice indicates the advisability of placing one-half the load of each consumer on each side of the mains. If this is skillfully done, it is rarely that the want of balance will rise above 5 or 10 per cent. When this arrangement is conscientiously carried out, it is impossibe for the load on the neutral wire to rise above 50 per cent of the entire load of the whole plant; for, even in the extent of the failure of one of the main conductors, half of the total load on the plant would be thrown off, and therefore the neutral wire could, even under the most extraordinary circumstances, only be called upon to carry one-half of the total current for which the conducting system is designed. From experience p has been found to vary from 5 to 25 per cent for installations skillfully designed. In the St. James station in London, working under a maximum output of some 3,000 amperes, the variation in balance rarely exceeds 7 per cent.

631. To determine the necessary section for the outer conductor of a three-wire system, let q = the relative area of the outer and neutral wires, then, from what precedes, —

$$(V - V') - (v - v') = \rho L\left(\frac{I}{S} + \frac{pI}{S/q}\right);$$

$$(V - V') - (v - v') = \rho L\left(\frac{I(1 + pq)}{S}\right);$$

$$S = \frac{\rho L I (1 + pq)}{(V - V') - (v - v')}. \qquad (229)$$

1+pq
= 1 + 0.15 × 2
= 1 + 0.30
= 1.3

From this expression it may be observed that if, in imagination, the resistance of the outer conductor be increased in the proportion of $1 + pq$ to 1, the neutral wire may be omitted from the calculation, and designs made as if it did not exist.

632. The Laws of Economy in Feeder Design. — In transmission parlance, the word feeder is broadly applied to any conductor in which the current density at any particular time is uniform at each point of the entire length of the conductor, no matter what the variations in density may be that occur between successive time intervals. In other words, a feeder is such a conductor as carries for the time

under consideration a uniform current between two fixed points. The current in the feed may vary from time to time, but it does not vary with the length of the circuit. In the simplest case of a series circuit, a constant current is always maintained throughout the entire length of each conductor, the only variable being the *length* of time during which the current acts.

633. While it is conceivable that a series circuit might operate with currents of different intensity from day to day, yet such conditions have not been put into practice, and the time element is the only variable. Knowing the respective costs of the line, the station, and the production of energy, and the interest and depreciation charged on the plant, the method of ascertaining the most economical cross-section for the conductor has been indicated in Chapter XIV.

634. In the parallel system, conductors are found in which, by reason of the attachment of the receivers at different points, the current density varies from point to point along the circuit. By definition, such conductors are excluded from the class "Feeders," being termed "Distributing-Mains." As a condition of good service, the pressure must be constant within very small limits, along the entire length of each distributing-main. With the feeders, as there are no customers to be supplied, the pressure may vary from point to point to any extent, so long as the desired voltage is given to the distributing main at the point of attachment to the feeder. The current, however, is constant from point to point. Two diametrically opposite conditions prevail in these two classes of conductors. In the feeder the current is constant and the pressure varies. In the distributing main the pressure must be uniform while the current varies. With the feeder, then, there are no service limitations upon the variation in potential, and consequently the dictates of economy may be closely followed in the design of this part of the circuit. By multiplying the number of feeds, the length and size of the distributing-mains may be reduced to a minimum, and thus the greater proportion of the plant brought under the operation of economical law.

Each feeder receives at one end energy from the generating-station in the form of current under a predetermined pressure, and delivers at the other end a less amount of energy, owing to inevitable losses in transmission. Therefore, in every transmission problem the following quantities must be dealt with, any or all of

w[1] may be variable; and it is now necessary, under the circum-s · of each case, to ascertain the most economical disposition of the material employed in the conductor system, due consideration being given to the commercial aspects of both the generating and the receiving station.

Let V = the pressure at the receiving end of the feeder.
v = the pressure at the delivering end of the feeder.
W = the power given to the receiving end of the feeder.
w = the power obtained at the delivering end of the feeder.
I = the current in amperes.
S = the cross-section of the feeder.
L = the length of the feeder in any units.
ρ = the resistance per unit of length (such as the mil-foot).

635. The values of L and ρ are always determined by the geographical conditions and the selection of the conductor, and are then fixed for each plant. Between the other variables the following relations exist: —

$$V - v = \frac{IL\rho}{S}; \qquad W = VI; \qquad w = vI;$$

so, if any two of the above first six variables are given, a single additional relation prescribed by economic law serves to fix the value of the remaining four.

636. Between six variables fifteen combinations, two at a time, can be made. When applied to feeder design, some of these combinations are mere repetitions of each other; others have no practicable bearing, but there are eleven cases which the engineer may be called on to consider. These may be stated as follows: —

Case No.	Given.	Required.	Case No.	Given.	Required.
1	$V, v.$	$I, W, w, S.$	7	$v, S.$	$V, I, W, w.$
2	$V, I.$	$v, W, w, S.$	8	$I, S.$	$V, v, W, w.$
3	$V, w.$	$v, I, W, S.$	9	$W, w.$	$V, v, I, S.$
4	$V, S.$	$v, I, W, w.$	10	$W, S.$	$V, v, I, w.$
5	$v, I.$	$V, W, w, S.$	11	$w, S.$	$V, v, I, W.$
6	$v, W.$	$V, I, w, S.$			

637. Each of these cases is now to be considered in detail, and for convenience the following notation is employed: —

$y \;= a + bS$, equation of cost of line per unit of length.
$y' = a' + b'S$, equation of cost of installing line per unit of length.
$L\,(y + y')$ or $L\,[(a + bS) + (a' + b'S)]$ = total cost of line.

i = rate of interest in per cent charged against entire cost of plant.
d_l = rate of depreciation in per cent charged against cost of line.
d_c = rate of depreciation in per cent charged against cost of conduit or poles.
d_s = rate of depreciation in per cent charged against cost of station.
F = number of hours that the plant operates per annum.
K = cost in dollars of *operating* per watt or $K.W.$ of output.
K' = cost in dollars of station equipment per watt or $K.W.$ of output.
A = price in dollars received per watt or $K.W.$ of energy delivered.
U' = cost of energy expended in the line. The line resistance is $\rho L / S$, I amperes flow for F hours, hence, $\rho LI^2F / S$ watts are expended, and the cost of this is $\rho LI^2FK / S$. The cost of station required to produce this energy is $\rho LI^2K' / S$, and the interest and depreciation on this sum is $\frac{\rho LI^2K'}{S}(i + d_s)$; hence, the total cost of the energy expended in the line, is

$$U' = \frac{\rho LI^2}{S}[FK + K'(i + d_s)].$$

To simplify, let $\lambda = \rho LI^2[FK + K'(i + d_s)]$;
then, $U' = \lambda / S$. Also, let $\omega = \rho L[FK + K'(i + d_s)]$;
then, $U' = \omega I^2 / S$.

U'' = annual charge against the line for interest and depreciation.
$U'' = L[(a + bS)(i + d_l) + (a' + b'S)(i + d_c)]$.
To simplify, let
$\alpha = L[a(i + d_l) + a'(i + d_c)]$ and $\beta = L[b(i + d_l) + b'(i + d_c)]$;
then, $U'' = \alpha + \beta S$.

$U = U' + U'' = \frac{\lambda}{S} + \alpha + \beta S$ = total annual cost of line.
$VIF = W$ = total power produced.
$VI[FK + K'(i + d_s)]$ = total annual cost to produce W; also let
Z = total annual cost to produce W.
To simplify, let $\gamma = [FK + K'(i + d_s)]$;
then, $Z = VI\gamma$.
G = gross annual expense $= Z + \alpha + \beta S$, or $VI\gamma + \alpha + \beta S$, or $Z + U''$, or $VI\gamma + U''$.

$$w = F\left(VI - \frac{I^2\rho L}{S}\right) = \frac{FI(VS - I\rho L)}{S}.$$

To simplify, let $\epsilon = VI$, and $\delta = I^2\rho L$;

$$w = F\left(\epsilon - \frac{\delta}{S}\right).$$

Aw = annual income $= FA\left(\epsilon - \frac{\delta}{S}\right).$

638. CASE I. — *Given V and v, required I, W, w, and S.*

As V and v are the given, the ratio of the energy received by the feeder to that delivered by it is fixed. It is also evident that the cost of the line, station, and operation (per unit of energy transmitted) decreases as the total output increases. Thus, there will be no one value of current and conductor section that will give the maximum economy; but the greater the current and section, the less will be the expense per unit of energy distributed. The size of the conductor will depend upon the demand for current at the receiving-station; and the larger this is, the greater the economy.

The smallest section under the limiting values V and v, consistent with safety, should be employed. As a corollary, it must always be considered whether there is *sufficient demand* at the receiving-end to pay for the transmission of current; for it is evident that the values V and v might be so limited that not enough current could be sold to pay for the cost of production and transportation.

639. CASE II. — *Given V and I, required v, W, w, and S.*

Under the circumstances, $VI = W$, thus fixing one of the desired variables. If it be assumed that there is a market for all the energy the circuit can deliver, it is evident that the most economical section is that which will make the ratio of the gross annual income to the gross annual expense a maximum.

Gross income $= Aw = FA\left(VI - \frac{I^2\rho L}{S}\right) \doteq FA\left(\epsilon - \frac{\delta}{S}\right).$

Gross expense $= Z + \alpha + \beta S$;

and
$$\frac{FA\left(\epsilon - \frac{\delta}{S}\right)}{Z + \alpha + \beta S}$$

must be a maximum; this will occur when —

$$S = \frac{\beta\delta + \sqrt{\beta^2\delta^2 + \beta\epsilon\delta Z + \beta\epsilon\delta\alpha}}{\beta\epsilon}. \qquad (230)$$

640. CASE III. — *Given V and w, required v, I, W, and S.*

The pressure at the receiving-end of the feeder, and the quantity of energy delivered at the delivering-end, are predicated; and

the most economical section is that for which the cost of the energy expended in the line, plus interest and depreciation, is a minimum, or $U' + U''$ must be a minimum, and also —

$$VI - \frac{I^2 \rho L}{S} = w. \tag{231}$$

By the original condition, —

$$U = \frac{\omega I^2}{S} + \alpha + \beta S. \tag{232}$$

From equation (231), —

$$S = \frac{I^2 \rho L}{VI - w}; \tag{233}$$

substituting this value in equation (232), —

$$U = \frac{\omega (VI - w)}{\rho L} + \alpha + \beta \frac{I^2 \rho L}{VI - w}; \tag{234}$$

differentiating and equating to 0, —

$$I = \frac{w}{V}\left[1 + \sqrt{\frac{\beta L^2 \rho^2}{\beta L^2 \rho^2 + \omega V^2}}\right]. \tag{235}$$

Having found the value of I, S is obtained by direct substitution in (233).

641. CASE IV. — *Given V and S, required v, I, W, and w.*

As the pressure and section are given, it is evidently necessary to ascertain I to meet the economic conditions. The ratio of the gross income to gross expense must be a maximum; for if it be attempted to make $U' + U''$ a minimum, U' will become 0 when I is 0, but U'' will remain unchanged. When I is 0, there is no income, and there is expense without income, and the above ratio would be 0, and not a maximum.

Gross income is $\quad Aw = FA\left(VI - \frac{I^2 \rho L}{S}\right).$

Gross expense is $\quad Z + U'';$

and
$$\frac{FA\left(VI - \frac{I^2 \rho L}{S}\right)}{Z + U''}$$

must be a maximum. This will occur when —

$$I = \frac{U''}{V\gamma}\left[\sqrt{\frac{V^2 \gamma S}{U'' \rho L} + 1} - 1\right]. \tag{236}$$

to build a model network of wire of reduced gauge to correspond in resistance to the scale of the map; and then, by supplying the network so designed with a battery current, and measuring the fall of potential in various spots by means of a voltmeter, to make the determination of the location of the central station, the size of the

TABLE NO. 83.

Heating-Limits for Conductors. Sheet II. — Aerial and Paneled Conductors.

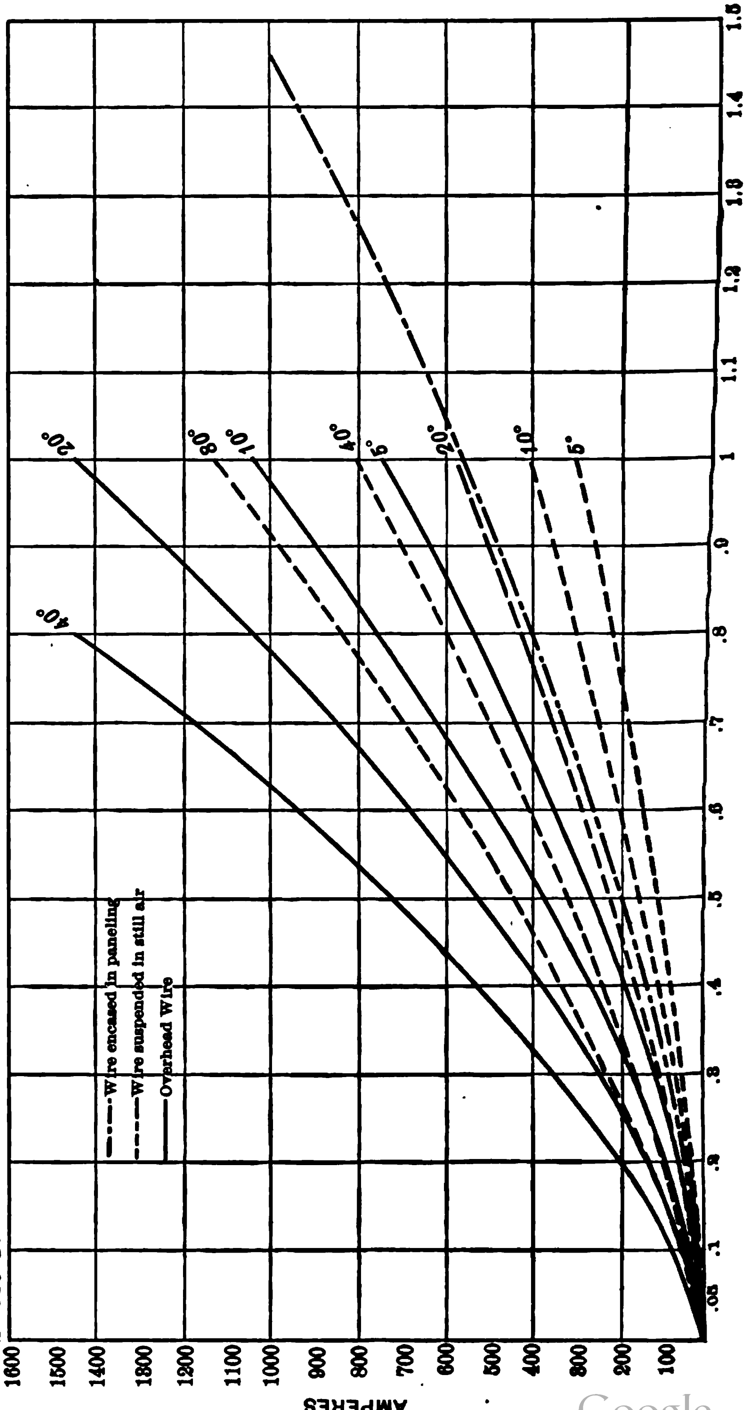
Size of Wire B. & S. G.	Section in Sq. Mils.	Resistance in Ohms Per 1000 Ft.	Current in Amp. for Wire in Panels.	Current in Ampere for Conductors in Still Air.					Current in Ampere for Aerial Conductors.			
				5°	10°	20°	40°	80°	5°	10°	20°	40°
0000	166190	.05012	175	104	146	204	288	403.8	229.6	331.6	461	636
000	131790	.06320	143.5	89	125	177	248	345.9	198.1	280.6	388.6	536
00	104518	.07969	124	77	106	152.7	214.9	299	169.7	237.9	328.9	453
0	82887	.1004	103.5	67	93	132	186.8	257	143.8	200.8	280.8	383.5
1	65732	.1267	87	57	84	115.6	160.9	222.7	121.8	168.7	236.8	326
2	52128	.1598	73	50	69	99	140	192.3	102.5	141.8	198.4	272
3	41329	.2015	62	44	60	86	121.6	166.4	86.7	120.4	169	234
4	32784	.2541	51.5	39	52	74	105.75	143.8	73.1	101.5	143	198.7
5	25999	.3204	43.5	34	47	66	92.5	125.5	62.6	87	122.5	170
6	20618	.4040	36.5	29	42	58	81	109.6	53.5	74.6	104.8	145
7	16351	.5094	30.75	25	37	50.6	71	95.5	45.6	63.7	89.2	123
8	12967	.6424	25.75	22	32	44.4	63.6	83.5	39.3	55	77	106
9	10283	.8100	21.75	19	28	37.5	57.1	72.6	33.6	47.5	66	90.4
10	8155	1.021	18	17	24.3	33.7	50.5	62.8	28.7	41	56.4	77
11	6467	1.253	15	15	22	29.6	45	55.7	24.8	35.4	48.5	66
12	5129	1.624	13	13	19.5	26	39.5	49	21.8	30.8	42	57
13	4067	2.048	11	11	17	22.9	34.5	42.5	19	27	37	49.6
14	3225	2.582	9	9.5	15	20	30.1	37.5	16.5	23.9	32.2	42.6
15	2558	3.256	7	8	13.5	17.5	26	32.7	14.2	20.2	27.5	36.1
16	2029	4.106	6	7	12	15	22	28	12	17	23	30
17	1609	5.178	5.25	. .	. .	. .	. . .	. .	. .	. .	. .	. .
18	1276	6.529	4.6	. .	. .	. .	. . .	. .	. .	. .	. .	. .
19	1012	8.233	4	. .	. .	. .	. . .	. .	. .	. .	. .	. .
20	802.3	10.38	3.5	. .	. .	. .	. . .	. .	. .	.	. .	. .
21	636.3	13.09	3	. .	. .	. .	. . .	. .	. .	. .	. .	. .
22	504.6	16.51	2.5	. .	. .	. .	. . .	. .	. .	. .	. .	. .
23	400.2	20.82	2	. .	. .	. .	. . .	. .	. .	. .	. .	. .
24	317.3	26.25	1.6	. .	. .	. .	. . .	. .	. .	. .	. .	. .
25	251.7	33.10	1.4	. .	. .	. .	. . .	. .	. .	. .	. .	. .

feeds and distributing-mains, and the fall of pressure at various points, entirely in an experimental manner. For very large plants, and those presenting peculiar complexities, a practical method of this kind has certain advantages, especially as the model may afterward be preserved as a *fac-simile* of the conducting system; and may, from time to time in the future, be used to afford means of solving prob-

TABLE No. 83.

Heating-Limits for Conductors. Sheet III.—Minimum Safe Diameter of Copper Wire in Inches.

Sheet 3.



lems relative to the addition or extensions to the conducting system, or the introduction of new customers. Such a method is, however, usually considerably slower and more expensive than the analytical one; and erroneous deductions, due to imperfections in the scale ratios, are likely to be serious. To determine the point of greatest drop, a method embracing a mechanical application of the principle of moments is, however, rapidly and accurately available.

668. Suppose a scale-beam to be arranged as shown in Fig. 328, with the supporting pivot at the center, the length of the scale-beam L and L′ at either side of the pivot being so arranged as to correspond, on any desired scale, to the length of the street between the

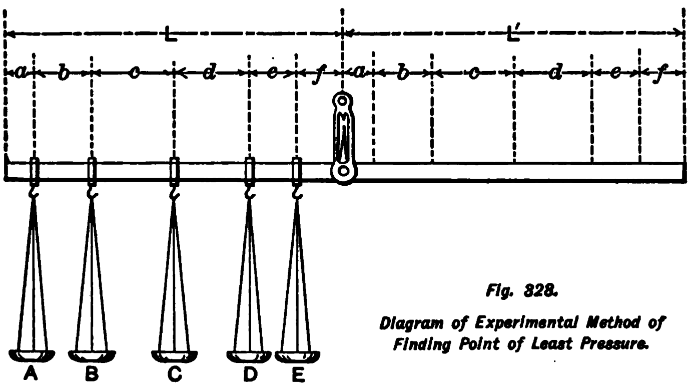


Fig. 328.
Diagram of Experimental Method of Finding Point of Least Pressure.

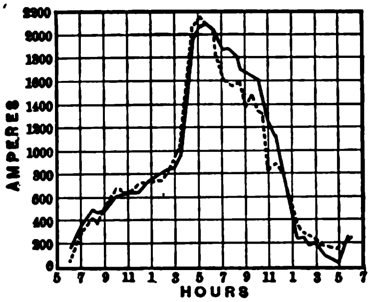
distributing-centers under consideration. Between the points a and f, on the left-hand side of the pivot, arrange scale-pans at distances to correspond to the consumers' frontage along the street. Arrange a corresponding set of scale-pans upon the right-hand side of the pivot in the same order as those indicated in the left-hand side, and balance the beam. In each of the scale-pans on the left-hand side, place weights corresponding to the amounts of current required by the respective customers. Now remove the weights from a, and place them in the corresponding pan on the right hand of the supporting pivot. Continue this operation until the scale-beam again balances about the center pivot. Suppose, when equilibrium is obtained, the weights in the scale-pans A, C, and D have been removed from the left-hand side of the lever, and have been placed upon the right-hand side, the interpretation of this result means that the

point of lowest pressure is located at a distance from *a* equal to *ae*.

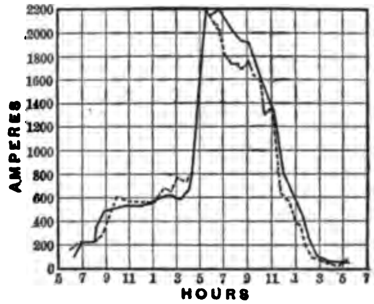
669. It sometimes occurs that equilibrium can be obtained only by *dividing* the weight in one of the scale-pans. Thus, for example, supposing all of the weights in A, C, and D to be removed to the right-hand side of the lever, and one-half of the weight in E. It is evident, under these circumstances, that the point of lowest potential is at the point *e*, and that one-half of the current supplied to *e* comes from *a* and one-half from *f*.

670. Station Loads. — The behavior of a central station under the load thrown upon it, and a study of the variation in the loading due to business emergencies, form one of the most interesting and attractive of investigations for the electrical engineer. This examination is chiefly attractive to the central station designer; the results of the investigation of station-loading being valuable in the solution of transmission problems, only as they afford means of determining the probable loads and variation in loading to which the circuit will be submitted. In the elucidation of this part of the problem, experience is the only guide. As an exponent of the loading to which the central stations in urban districts may reasonably expect to be submitted, the curves given in diagrams Nos. 1 to 17 inclusive, Fig. 329, are presented. The curves numbered from 1 to 12 inclusive are the average monthly curves obtained from the operation of St. James Station in London; they are exhibited by the *London Electrician* as sample curves, giving a fair indication of the monthly output of the St. James Station during the period of a year. In each curve the horizontal axis to a scale of sixteen hours to an inch represents the hours of the day, while the vertical axis represents 1,600 amperes to the inch. The heavy line in each of the diagrams indicates the current output for each hour of the day in one main conductor, while the dotted line gives the current flowing through the other wire, requiring an algebraical summation to give the total station output; the departure of the dotted line from the full line fairly represents the amount of unbalance to which the plant was subjected. An examination of all of the curves reveals a close family resemblance existing between them. In every instance the quantity of current slowly increases from 5 A.M. to about 3 P.M. From this time until from 5 to 9 P.M. the current rises sharply,

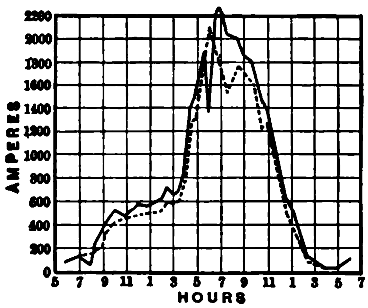
Fig. 329. Curves of Station Output.



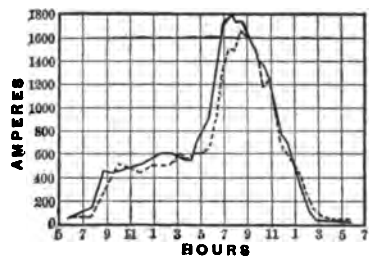
No. 1. Jan. 12, 4,514 Units.



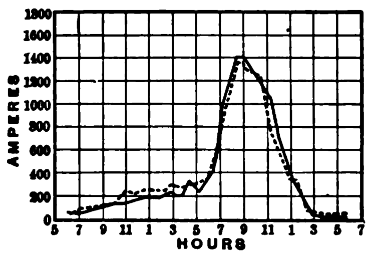
No. 2. Feb. 16, 4,104 Units.



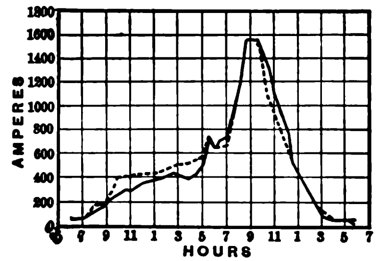
No. 3. March 9, 3,672 Units.



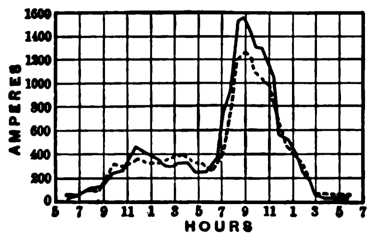
No. 4. April 13, 3,229 Units.



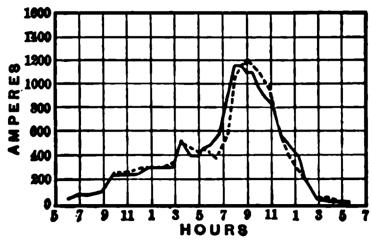
No. 5. May 18, 2,149 Units.



No. 6. June 8, 2,529 Units.

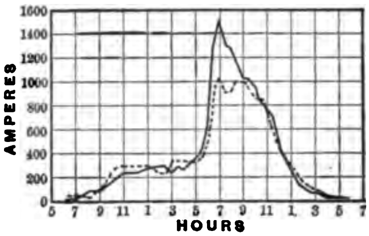


No. 7. July 20, 2,062 Units.

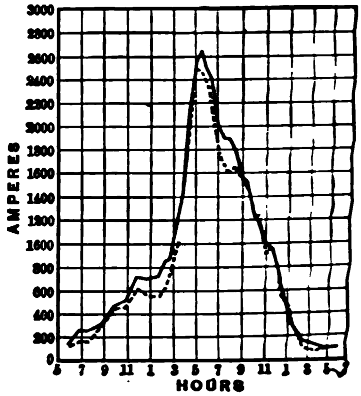


No. 8. Aug. 10, 1,973 Units.

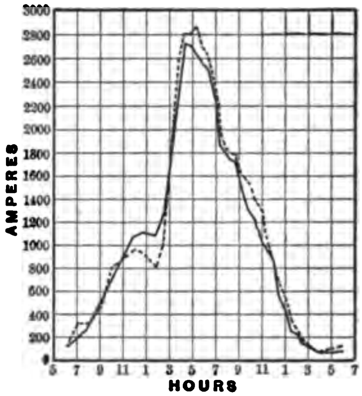
Fig. 329. Curves of Station Output. (Continued.)



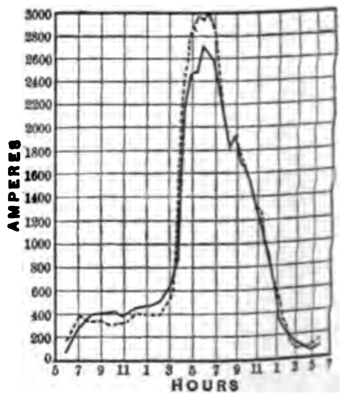
No. 9. Sept. 7, 1,980 Units.



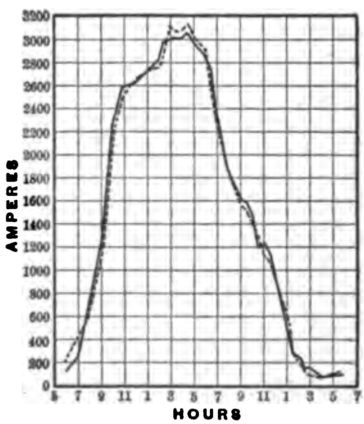
No. 10. Oct. 26, 4,267 Units.



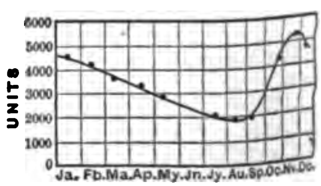
No. 11. Nov. 23, 5,345 Units.



No. 12. Dec. 14, 4,822 Units.

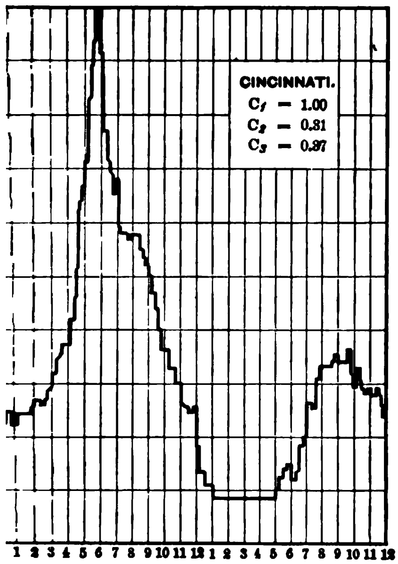


No. 13. Heavy Fog, 7,942 Units.

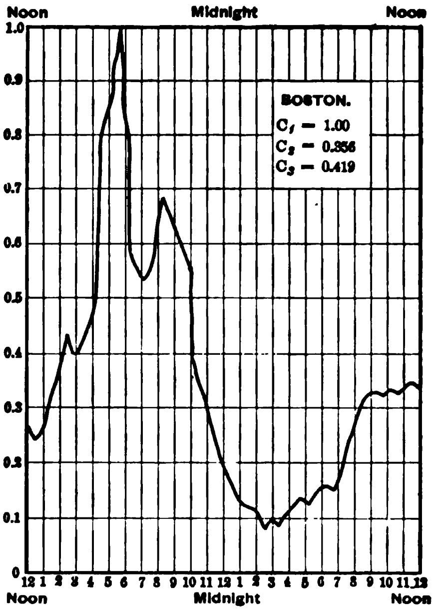


No. 14. Curve of Annual Output.

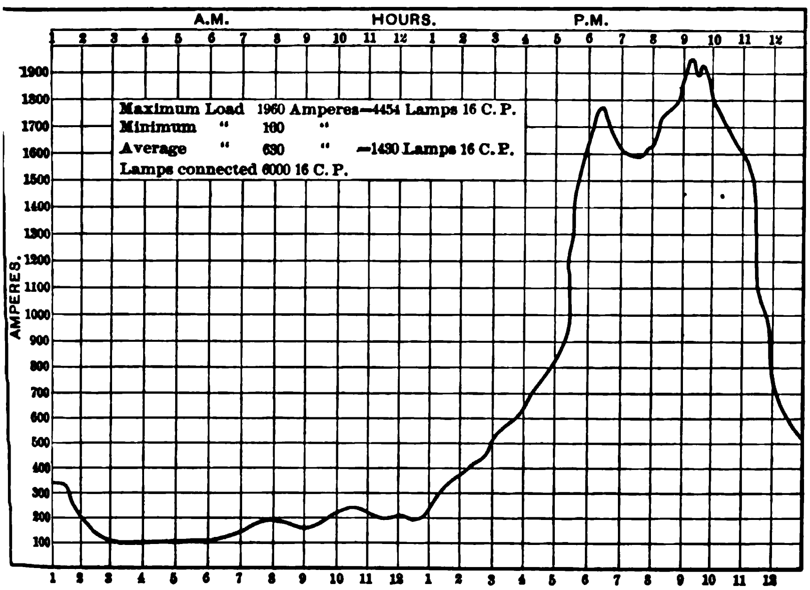
Fig. 329. Curves of Station Output. (Continued.)



No. 15. Output Curve, Cincinnati Edison Station.



No. 16. Output Boston Edison Station.



No. 17. Output Curve, Brooklyn Edison Station.

usually attaining a maximum value between the hours 7 and 9, the variation in the maximum having its origin in the varying length of the days from month to month. The total Board of Trade units delivered by the station is indicated upon each diagram. From January to August the station output regularly decreases, attaining a minimum in August. The minimum current at this time is not only due to the greater length of days, but is also owing to vacation absence during the month of August of the greater part of the business population. If the current curve followed strictly the relation between the lengths of day and night, the minimum would evidently occur in the early part of July, instead of August. In September and October the current output increases toward a maximum more rapidly than the decrease occurred in the earlier part of the season, indicating a resumption of business in September. The curves for November and December are very nearly alike in output, showing that the maximum demand is thrown upon the station a little earlier than the occurrence in the shortest days of the year; for it is a noticeable fact that, although the actual length of the day is a minimum in December, the amount of cloudy weather which occurs in the latter part of October and November makes a greater proportional demand for light in these two months. Diagram No. 13 is exhibited, showing the remarkable fluctuations and extraordinary demand on the station due to exceedingly stormy and foggy weather. Here the maximum current output occurred between the hours of 3 and 5 in the afternoon, instead of between 5 and 7, as in the normal December curve; the output in this case being nearly 8,000 units against the normal December output of 4,800, nearly doubling the demands upon the station.

In diagram No. 14 a summary of the year's work is given by plotting the output of each month. Here the vertical axis indicates the station output at the rate of 8,000 units per inch, while the horizontal axis is at a scale of 8 months to the inch.

671. In diagrams Nos. 15, 16, and 17 are represented the curves obtained from the Cincinnati Edison Station, the Boston Edison Station, and the Brooklyn Edison Station. All of these bear a striking similarity to the London curves, exhibiting, however, some local peculiarities. In the diagrams from Cincinnati and Boston, the

ıeater load brings a noticeable elevation into the curve at eight clock in the evening. In Brooklyn a similar rise of current may so be noted, but this is much smaller in comparison than in the reviously mentioned cities. This is owing to the greater proporon of residence population in the district served by the Brooklyn tation.

672. A study of station load curves affords to the designer the est indication of the amount of load thrown on the conductor sysem, from which a deduction of the amount of current and the time f flow can be most accurately made, for application in the economial formulæ. To determine the mean annual current most accuately, as great a number of diagrams as possible should be procured rom the station under consideration, or from one as nearly similar o it as practicable. If it is possible to ›btain the daily load curve for a year, an ıccurate determination of the constants nay be made. Professor Patterson of Michigan has indicated an extremely ingenious method for the solution of this problem, which may be best illustrated by applying his process to the determination of the amount of mean output of the St. James Station. Assuming the curve of annual output as indicated in diagram No. 14, draw a circle, Fig. 330, having a radius equal to the greatest ordinate of this curve, and subdivide the circle into twelve equal parts, by drawing radii to the circumference at the points Ja, Fb, Ma, Ap, My, Jn, Jy, Au, Sp, Oc, Nv, and Dc, each corresponding to a monthly ordinate in No. 14. Upon each of the radii lay off, from the center of the circle outward, a distance equal to the ordinates of the curve in diagram No. 14. By connecting all of these points an area will be obtained, which is indicated by the shading in Fig. 330. Find the area of this shaded curve by means of a planimeter, and then find the radius of a circle corresponding in area to the area of the shaded portion. The radius of this circle will, to the scale selected for the original circle, and corresponding to the monthly load diagram, be the amount of the mean annual output; for, evidently, each elementary area of the

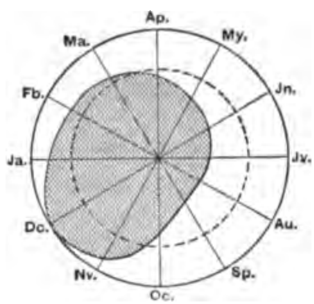


Fig. 330.
Diagram to Determine Mean Annual Current from Station Load Curves.

shaded figure is proportional to the square of the radius vector. In the illustration given, the circle corresponding in area to the shaded figure is given by a dotted line, the radius of which is .42″. Diagram 14, Fig. 329, is 8,000 units per inch, consequently the mean annual output of the St. James Station would be 8,000 × .42 = 3,360 units.

673. Algebraically the same result may be arrived at by finding from the load diagrams the maximum current output, and then ascertaining the relative lengths of time that the plant operates under different fractions of maximum load, say at intervals of 5 per cent, from 5 per cent to 100 per cent. Then, by multiplying the number of hours by the square of the percentage time operation, and extracting the square root of the sum, the square root of the mean square of the annual current is obtained.

674. Arc-Lamps on Constant Potential Circuits. — For every purpose excepting that of arc-lighting, the constant potential circuit, from its greater economy, lower pressures, and greater flexibility, has received a greater development. Many attempts have been made to operate arc-lamps upon constant potential circuits; but the variation in the voltage of the circuit, together with the necessity of the introduction of large, wasteful resistances, has prevented, until recently, the wide adoption of this practice. Successful lamps now, however, are constructed to operate upon constant potential circuits, and the number of these installations is now very rapidly increasing. Lamps are arranged across the constant potential mains of a 110 volt circuit, by placing either two or three lamps in series. In the case of two lamps in series, each lamp would operate at from 40 to 45 volts, thus consuming from 80 to 90 volts out of the possible 110 volts. Under these circumstances, from 20 to 30 volts would be lost in the necessary wasteful resistance, to be inserted in series with the lamps. In many cases, this amount of waste energy is not a serious factor, in a consideration of the other advantages to be derived from the constant potential circuits. The same lamps, however, may be adjusted to run on a 35 volt arc, by means of which three lamps may be placed in series, thus taking 105 volts, and necessitating only a loss of 5 volts in extra resistance. In a similar manner, six lamps may be placed in series across the outer mains of a three-wire circuit. The lamps, also, may be adjusted to take from 3 to 20 amperes, by this means permitting a great variation in the amount of candle-power delivered to the consumer.

675. Electrical Railway Wiring. — So far the distribution of electrical energy under the parallel system has been considered only for the case in which the receivers and the station were placed at constant and fixed distances from each other. A very important application of the system has arisen in the construction of electrical railways, in which the receivers are constantly varying the distance between themselves and the station. The electrical railway problem is also further complicated by the fact that the load thrown upon the station is rapidly varying throughout very wide limits. Take the case of a small road operating a single car. It is evident that at each stop and start of the car the entire station load will be thrown off and on, thus causing the station output to vary from zero to a

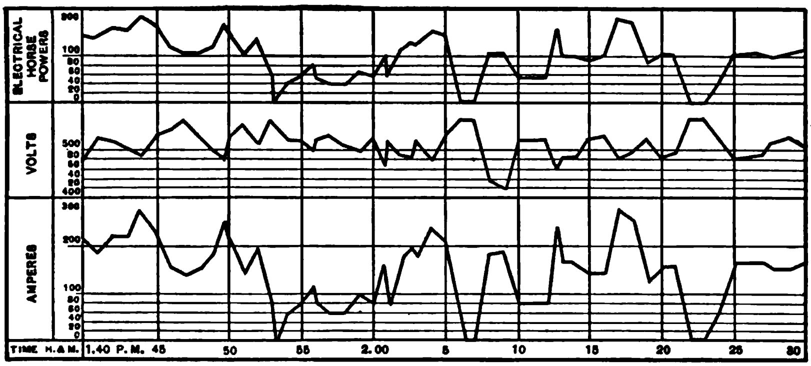


Fig. 331. Station Diagram, Navesink Mountain Railway.

maximum many times an hour. As more and more cars are operated, the station load becomes more nearly constant; but even with roads of the largest capacity, the variation in the station load is large in comparison with that thrown upon ordinary lighting plants.

In Fig. 331 is shown the variation of station load upon a road carrying four cars. The curves here given are by Messrs. Herring & Aldrich, from a test made upon the Navesink Mountain Road. The time during which the measurements are taken covers a period of fifty minutes; and during this short interval of time, with four cars in operation, the load on the station has rapidly varied from zero nearly to 300 amperes.

676. In Fig. 332 the load diagram of the Minneapolis Street Railway is shown. Here 142 motors were in operation, requiring an

average current of 1,168 amperes. Even with this large number of cars in service, the station load varied from 600 to nearly 1,800 amperes within three hours. Many variations of 400 to 500 amperes

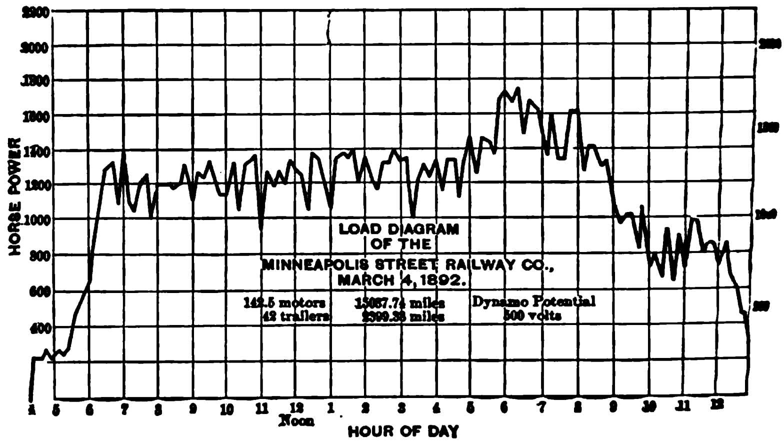


Fig. 332.

occurred within a few minutes of each other. It is plain, from an inspection of these diagrams, that the demands on the conducting system of an electrical street railway are exceptionally severe, and great precautions must be taken to proportion the wiring in such a

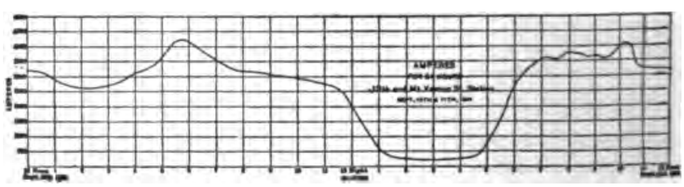


Fig. 333. Load Diagram, Philadelphia Traction Company.

manner as to readily respond to the severest calls that may be made upon it.

677. In Fig. 333 the load diagram from one of the stations of the Philadelphia Traction Company is given as an example from a large and heavily loaded road. Here the variation between the night and day load is striking, but the effect of a large number of cars to smooth out and even the general line is still more noticeable. From

these examples the designer can gather a close approximation to the probable load line of the plant under consideration.

678. The electrical railway system should be constructed upon the feeder and main system; but it is impossible to secure the advantage of connecting the distributing-mains and feeders together, forming a network, for the reason that, in order to avoid interruptions of traffic upon the entire road, it is essential that the trolley wire shall be split into a great number of sections, each one of which is independent of any and all other sections. This precaution is necessary in order to avoid shutting down the whole line in case of a short circuit at any point. Should the wiring be a continuous network, it is evident that the grounding of any portion will throw the whole road out of commission; while, if the trolley wire be subdivided into numerous separately insulated sections, the accidental disarrangement of any one will in no wise interfere with the traffic of the road, excepting in the section injured. For this reason, it is customary to subdivide the trolley wire into a number of parts, each one of which is entirely insulated, and provided with a separate circuit to the station. Such a form of wiring is evidently the feeder and main system in the simplest form. Each section of the trolley wire forms a distributing-main that is connected to the station by means of its appropriate and special conductor. To determine the load diagram of an electrical railway, which is an essential consideration in the calculation of a conducting system, it is necessary to ascertain the number of cars which will at any one time be concentrated on any section of trolley wire, and the maximum amount of current to be taken by each car. To this end, it is essential to secure a plan and profile of the road, showing where the grades and curves occur, and where the travel is likely to be a maximum, requiring the greatest number of stops and starts, also where the cars will be most heavily loaded. Of all the factors entering into the problem of railway wiring, the live load in the car plays the part of least importance. The difference in the current required by an empty car and a fully loaded car is but a small proportion of the current required to start a car, or move it upon curves and grades. For an ordinary car carrying two thirty to fifty horse-power motors, an average running current of from twenty to sixty amperes is required. To start the same car upon curves or grades will cause the starting current to rise to two or even

three hundred amperes for a few moments. The severest stress to which the conducting system of an electrical railway circuit can be subjected occurs in the case of a blockade, when a large number of heavily loaded cars may be expected to start almost simultaneously. It is for cases of this kind that the wiring of the road should be most especially planned, for nothing is so detrimental to street-car motors as to subject them to the requirements of starting under an excessive fall of potential. Such a stress as this almost invariably injures the insulation of the armature, causing the motor to sooner or later burn out. It is, therefore, advisable to prepare the load diagram of each section of the trolley wire with the maximum possible current in view, bearing in mind particularly, that experience has shown that traffic on electrical railway lines immediately increases upon the introduction of the electrical system, usually indicating it advisable to double or triple the load which is, or has been, carried by other forms of traction on the same line. Having obtained the load diagram for each section of the trolley wire, the calculation of the fall of potential in a particular section of trolley wire is an exceedingly simple matter. The calculation of feeders for each section of wire may be made according to the formulæ already given. It is usually advisable to extend the feeders to the center of each trolley wire section, and then to branch the feeders longitudinally along the trolley wire for such a distance as will enable the feeds to supply the required current under the given fall of potential clear to both ends of the section. While railway motors are sensitive to changes in the potential, they will bear a much greater variation than incandescent lamps, so it is customary to design the conducting system of an electrical road to work under a difference of potential for an average maximum current of from 50 to 75 volts on a 500-volt circuit. The mean annual current for a road is so difficult to predict, previous to the operation of the line, that it is exceedingly hard to apply the formulæ for maximum economy. Experience has also shown that in most cases the most economical current density requires the conducting system to vary over too great a difference of potential to enable the motors to safely and successfully operate. The conditions, therefore, which usually limit electric railway wiring are those of the maximum allowable fall of pressure, which should never exceed 10 to 12 per cent of the available voltage at the station. To com-

sate for the drop in the line, it is advisable to over-compound the erators at the station, that, with an increased demand upon the ducting systems, the voltage of the generators shall rise in pro-:ion to the demand upon the line, thus enabling a very notable .ng in the cost of the conducting system to be made.

CHAPTER XVI.

MISCELLANEOUS METHODS.

MOTOR TRANSFORMERS, ACCUMULATORS, TRANSFORMERS.

679. In the description of multiple-wire systems, it has been shown that economy in distribution can be effected by raising the potential of the generator station, and decreasing the current through the conducting system; but, at least in lighting-circuits, a practical limit is soon reached to the possible elevation of potential, by the limited resistance of incandescent lamps, and the independence of the various customers is seriously interfered with in the attempt to run several lamps in series, in order to render elevated potential available. Many attempts have been made to render feasible distribution at high voltages, in order to cover large areas without too serious loss in the conducting system, and without too great an expenditure of capital for the conductors, by means of auxiliary pieces of apparatus, whereby a high voltage and small current supplied by the station could be transformed and changed into a lower voltage and greater current for the consumer.

Devices of this kind have been more or less successful, and already have attained so wide an introduction in distributing systems that, while the consideration of the various appliances used in this connection more strictly belongs to a discussion of station machinery, yet no treatise upon electrical distribution would be complete without, at least, a limited reference to the various systems that have thus been inaugurated.

680. **Motor Transformers.** — The modern motor transformer is a dynamo machine, the armature of which contains two circuits and two commutators. These commutators are usually arranged upon opposite ends of the shaft extending through the armature, so that essentially the machine may be said to be two dynamo machines, excited by the same field magnets. The high potential line from the station is brought to the brushes of one of the commutators,

to which the fine wire windings of the armature are attached, and the machine acts as a motor, the armature rotating rapidly between the poles of the fields. The field magnets are in a similar manner excited by the line current. Now, it is evident that the other set of windings of the armature, being rotated in a powerful magnet field, will behave as a dynamo generating a current, that, flowing out through the remaining commutator, may be used in precisely the same manner as a current from any other source of electrical energy. From a study of the principles governing dynamo electric machinery, it is known that the voltage produced by a dynamo acting as a generator, or absorbed by one acting as a motor, is proportional to the rate at which the armature conductors cut the magnetic lines of the field. In the case of the motor generator, inasmuch as the armature runs in a constant field, the voltages on the two commutators will evidently be proportional to the number of turns in the respective halves of the armature. Thus, by making the windings of that portion of the armatures connected to the line of fine wire and a great number of turns, while those on the generator side are of coarse wire with a small number of turns, it is perfectly feasible to transform the high potential and small current supplied by the station line to a low potential and large current for consumers' use. So, in cases where distribution is required over a very large area, it is possible to design the main source of supply to distribute a small current of a few amperes at a high potential (say several thousand volts), running the primary conducting system to a number of sub-stations, which approximately take the place of centers of distribution in the feeder and main system.

681. At each of the subordinate stations, by means of a motor transformer, the small current and high potential of the original supply is changed to a large current at low voltage, giving a safe and practical supply for all consumers. Such an arrangement leads to marked economy in the primary conducting system, especially if the geographical circumstances are such as to necessitate the expansion of the system over a large territory, and also renders all of the customers entirely independent of each other. The various motor transformers may either be operated in series or in parallel; and, furthermore, additional advantage may be obtained by arranging either the primary or secondary system, or both, to operate upon the multiple-wire system.

682. Installations of this kind have already reached considerable development, especially in Europe, where by their aid widely extended distribution is made a commercial success. Little or no difficulty is experienced with the motor transformers, as, by careful proportioning, these machines may be made exceedingly permanent and durable, requiring for annual maintenance only the renewal of the commutators and the brushes; and when suitably calculated for the loads placed upon them, they can be made exceedingly economical, having an efficiency of over ninety per cent at full load. As, however, they are dynamic machines, a certain amount of supervision is essential; and, therefore, usually an attendant is constantly required at each of the sub-stations during such times of the day as the motor dynamos are in operation. The expense of maintenance

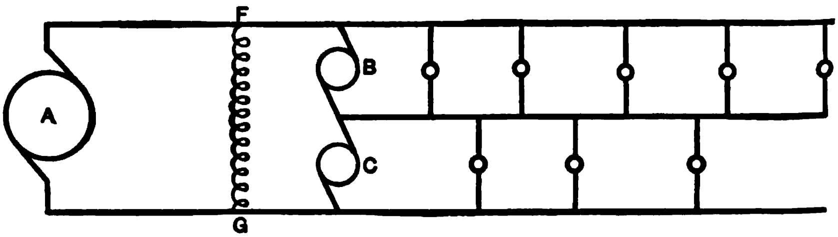


Fig. 334, Compensator on Three-Wire System.

and lack of efficiency of the motor generators are comparatively small items, being usually much less than the cost of interest, depreciation, and loss of energy in the ordinary conducting systems. The cost of attendants, however, is quite a serious item, and has so far limited to a notable extent the expansion of this system. The system just outlined for the employment of motor transformers is one that has received, perhaps, the largest sanction by experience. There are, however, many special methods, of which the following are perhaps the most important.

683. Compensators. — The use of motor generators for compensator has already been alluded to. The design of a compensator plant will be more clearly understood by reference to Figs. 334 and 335.

The simplest case is the employment of a compensator upon the three-wire system, the outline of the connections being shown in

Fig. 334. A is the generating-station, B and C being the two compensators. These compensators are two dyamos, shunt wound, the fields being placed across the outer mains, as shown at F and G. The armatures of the two compensators are wound upon the same shaft, in order to rotate exactly in unison. When the two outer

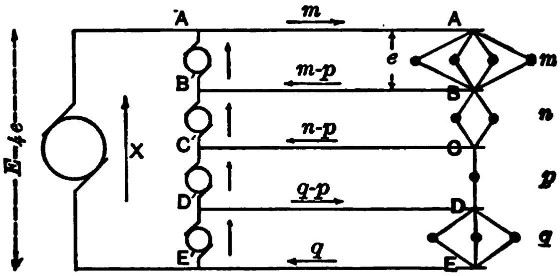


Fig. 335. Compensator on Five-Wire System.

conductors of the line are equally loaded, a very small current flows through the compensators, simply sufficient to turn the armatures, overcoming the frictional resistance. As soon as the system becomes unbalanced, the armature connected with the main carrying the least current becomes a motor, while the other armature plays

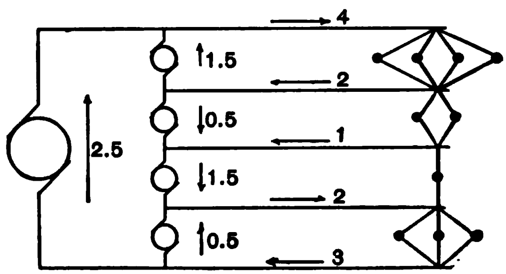


Fig. 336. Diagram of Currents in Five-Wire Compensator.

the part of a dynamo, the balance of the system being restored; for one of the compensators, acting as a motor, drives the other armature as a generator, furnishing the excess current required upon the overloaded main.

684. The calculations for the amount of current required in the various compensators may be made by the application of Kirchhoff's laws. Fig. 335 gives an illustration of the compensator

method, as applied to a five-wire system, and will form a sample to elucidate the application of the calculations.

Let X be the current from the station;
m, n, p, q the currents in the different receivers;
e the voltage of the receivers; and
$E = 4e$, the voltage of the station.

685. Let A, B, C, D, and E, and A′, B′, C′, D′, and E′ be the respective mains, with the compensators located between A′B′, B′C′, and D′E′. By Kirchhoff's laws, the values of the currents in the intermediate wires may be readily found, as indicated in the illustration.

If X is the current supplied by the station, and x, y, z, and t the currents in the armatures of the compensators, the following equations are readily deduced: —

$$X + x - m = 0. \tag{241}$$
$$y + m - n - x = 0. \tag{242}$$
$$z + n - p - y = 0. \tag{243}$$
$$q - t - X = 0. \tag{244}$$

From which $m - x = n - y = p - z = q - t = X.$

686. Moreover, if it is assumed that the compensators have an equal output, whether acting either as generators or motors, and neglecting the small amount of energy expended to overcome their ohmic resistances, and assuming e to be the electro-motive force required for the lamps, the following equation obtains: —

$$4\,eX = e\,(m + n + p + q); \tag{245}$$

From which $$X = \overline{m + n + p + q}\,/\,4. \tag{246}$$

687. Solving, the various values for the currents in the compensators and in the separate wires, as indicated on Fig. 336, are obtained. As all of the compensators are shunt wound machines, having their fields excited between the outer conductors, they always revolve in the same direction, no matter whether acting as generators or motors; and, under the present state of perfection in dynamo construction, they require an exceedingly small amount of attention. The independence of the system is further preserved by the ability to insert compensators at any number of points across the primary mains. This method renders it feasible to operate a station by an

available water-power, situated at some distance from the center of gravity of the consumers, delivering the energy thus obtained at a high potential through a small pair of conductors, and placing the compensators in parallel across the mains at the various centers of distribution. To determine the size of the compensators, it is necessary to establish the greatest possible difference in loading between the outer conductors and any of the intermediate ones, and proportion the compensator to deliver the current thus indicated. By designing the system according to the observance of precautions indicated for the multiple-wire systems, so that the load of the various consumers shall be equally subdivided among the intermediate conductors, it is practical to reduce the probable lack of balance to a small percentage, thereby reducing in a corresponding proportion the size of the compensators required to maintain the balance.

688. Motor Transformers Running and Feeding in Series. — An ingenious method of utilizing motor transformers has been devised by Mr. Bernstein. While usually applied to straight currents, it is also applicable to alternating currents. The method consists in laying from the central station a series circuit receiving power from a single generator, including in the circuit any desired number of motor transformers. One striking peculiarity of this system is the arrangement of the engine, or other prime mover, without any speed regulator for controlling changes in load; the engine being allowed to run at any desired speed up to a certain predetermined maximum, which will correspond to a delivery of the highest voltage to be obtained from the generator.

The motor transformers are those having double windings on the armatures, the motor side of each transformer being connected in series with the primary line, so that the whole number forms a group of series motors operating upon the station circuit. The other windings of the armature are arranged to be in series with the receivers of each separate circuit, and are proportioned to supply the requisite number of receivers that may be expected upon each of the secondary circuits. The number of customers upon the secondary circuits may be increased or diminished at pleasure, by simply short-circuiting the apparatus of the customer, which is for the time being thrown out of service, by this means rendering the various receivers independent of each other. The general outline of this circuit is shown in Fig. 337.

In this design it is apparent that neither the motor transformers nor the generators and engine at the station require any special supervision, the whole plant being entirely self-regulating. The efficiency of the design is low as is common in series circuits, unless the plant can be arranged to operate essentially under a constant full load. It is also noteworthy that there is no need of automatically

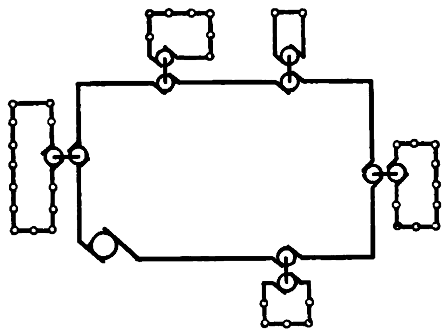


Fig. 337. Series Motor Transformers.

regulating the position of the brushes on the series machines, as is customary; for inasmuch as the current is constant in both armature circuits, regulation is accomplished entirely by a variation in the speed of rotation of the various armatures of the motor transformers.

689. High and Low Potential Distribution from the Same Station. — M. Rechniewski has devised the following ingenious

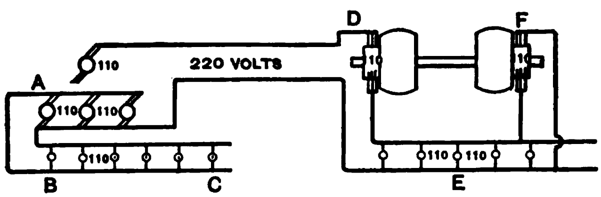


Fig. 338. High and Low Potential from One Station.

method for delivering two different potential values at one station. The generators are arranged at A, Fig. 338, in such a manner that part of the dynamos may operate in parallel upon the circuit BC, which may be supposed to feed a group of 110 volt lamps placed close to the station. The remaining dynamos are placed in series with the first machines, giving a 220 volt circuit to be used, for exam-

ple, at a considerable distance from the plant. At D a motor transformer is placed, so designed that in connection with the circuit D E it shall absorb 110 volts, leaving 110 from the 220 of the circuit to pass the lamps at E. From the other side of the motor transformer at F, a 110 volt current is obtained, which is also passed to the lamps at E. By this ingenious device, only half of the energy undergoes transformation, and the advantages of a high potential with a minimum loss of efficiency is secured.

690. Leonard's System of Motor Regulation. — For the special case of electric motors, operating under wide variations of load and speed, particularly in instances where the direction of rotation is frequently reversed, Mr. H. W. Leonard proposes the following method of regulation, depending upon the principle that the potential at the motor terminals should be proportional to the desired speed, while the current should vary as the torque or twisting moment required. The method is diagrammatically shown in Fig. 339, in which B is the motor, and A and C two generators, the office of the generator C being simply to excite the fields of the motor B and the generator A. The field currents are controlled by the rheostats D and E. The armatures of the generator and motor are directly connected. So long as the generator A is operated at a constant speed, the electro-motive force generated will be proportional to the strength of its field, which is controlled by the rheostat D. The field of the motor being constant, the speed of its amature will depend upon the applied electro-motive force; and, hence, this speed is also controlled by the rheostat D. The current will automatically vary proportionally to the torque exercised by the motor armature,

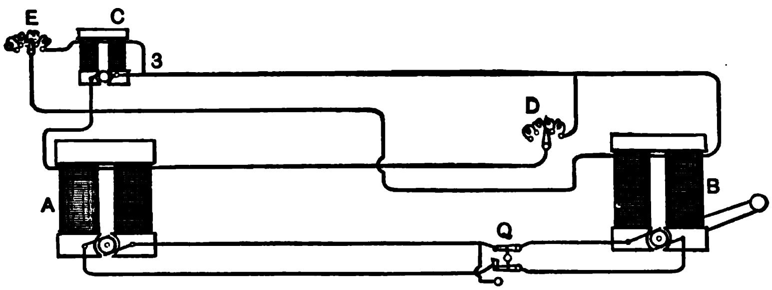


Fig. 339. Leonard's System.

and the efficiency will be constant and independent of both speed and torque. Reversal of rotation is readily accomplished by a reversal of the field current. This method has been very successfully applied to the operation of elevators and to similar pieces of machinery.

691. Accumulators.—A discussion of the properties of the storage battery and its place in electric stations belongs to a treatise on central-station design, and here can receive only a mere mention. All electric plants are handicapped by severe fluctuation of load which prevents the generating station from operating steadily at its point of maximum efficiency. The battery offers a means whereby the electricity produced by the dynamos during hours of light load can be stored and delivered to the line at moments of heavy demand. It may also act as a transformer, for cells may be charged from main line in series at relatively high potential and discharged multiple into a distributing system, but otherwise the battery does not affect the design of the transmission system. In both of these rôles the battery plays an important and successful part, and is rapidly becoming a most valuable central-station accessory.

692. The electrical railway problem would seem to receive from the storage battery an exceedingly happy solution, for, from the station curves already given in Chapter XV., the irregularity of load upon a railway station is seen to be exceedingly severe and irregular. It may be almost confidently stated that no electrical railway plant can operate at its point of maximum efficiency, on account of sudden and extreme variations of load to which the station is subjected. To, therefore, supply railway stations with a suitable accumulator plant, which should allow engines to run at a reasonably constant and uniform load at the point of maximum efficiency, and allow the battery to make up in the line deficiencies of the current supplied by the dynamos, would certainly be attended with the happiest results. Already railway station managers in this country are seriously considering the increasing of station capacity by means of accumulators, and the wide adoption of this method in the near future seems to be a foregone conclusion.

693. Sub-Station Accumulators. — The accumulator may be

used in a manner similar to the motor transformer by being located at a number of sub-stations which may correspond to centers of distribution. The employment of accumulators in this manner forms one solution of high potential transmission, by allowing the generating-station to work at high pressure, the batteries in the sub-stations being arranged to be charged in series, while they are discharged into the consumers' circuits in parallel. In this way the batteries at the sub-stations may be charged under a

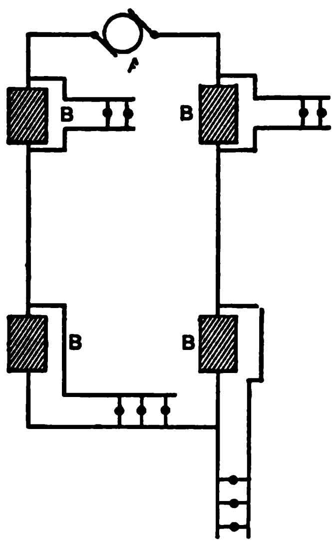


Fig. 340. Accumulator Distribution.

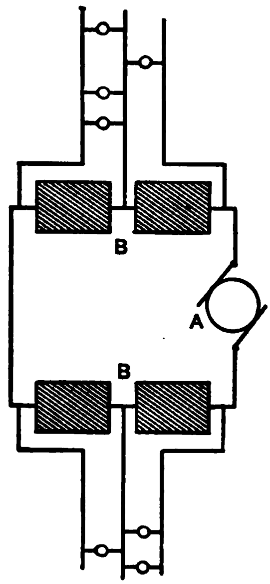


Fig. 341. Accumulators on Three-Wire Mains.

very high potential with a small quantity of current, and yet serve a large territory at the ordinary lighting voltage with a large current. Inasmuch as the batteries do not need constant attention, it is practical to place the care of a number of sub-stations in the hands of a single assistant, who may visit each station at different periods of the day, giving each of the separate batteries such supervision and maintenance as may be necessary. In this respect, the accumulator is an improvement over the motor dynamo, for the latter almost necessitates the constant presence of an attendant. On the other hand, however, the cost of battery maintenance,

including the deterioration of the plates, the losses of exciting fluid, etc., are considerably larger than the maintenance expense entailed by motor generators; for, while the present battery manufacturers are prepared to give five or even ten years' guarantee for the permanence of their goods, the life of a motor dynamo, the commutators and brushes alone excepted, is practically without limit. Accumulator stations may be designed either on single or multiple wire systems, for either the primary or secondary conducting systems, or both.

694. Accumulator Distribution. — The more customary methods of distribution by means of accumulators are indicated in Figs. 340 and 341. In the first illustration, the batteries B B B B are arranged in series along the entire line, the generator feeding all of the sets. The receivers are taken off in four parallel circuits, each one of the batteries being of sufficient voltage to adequately supply all of the customers. A similar arrangement is indicated in the latter illustration, with the simple modification that in this case the secondary consumers' circuits are laid out upon the three-wire system.

695. Regulation by Means of Accumulators. — A very convenient application of the accumulator system is in the accomplishment of the regulation of voltage to be delivered at the end of the feeders, by planning the battery so that there are a sufficient number of cells to make up for the fall of pressure in the feeds, and arranging the extra cells in such a manner as to be readily thrown in and out of service by means of an appropriate switching apparatus.

Thus the accumulator forms an exceedingly valuable and simple method of regulating the potential delivered at the ends of the feeder system. As the load is thrown on the plant, the increasing current causing a drop at the ends of the feeds, additional cells may be switched into the circuits in series, thus increasing the potential precisely in accordance with the demands of the line.

696. Transformers. — The devices for rendering high potential circuits available, that have so far been considered, are those applicable to straight currents. The alternating current presents a solution of the problem, at least so far as lighting-circuits are concerned, in an exceptionally beautiful manner. The motor transformer and the

accumulator are dynamic pieces of apparatus, which constantly require more or less supervision, and from this cause are sources of considerable expense. With the alternating system, the principles of induction may be so utilized as to enable the plant to distribute electrical energy over wide areas with the greatest economy, without the interposition of machinery needing supervision. In the case of the motor transformer, a rotating armature is supplied with a high potential current through the fine windings, and distributes a low potential current through the coarse windings. In this case, the cutting of the magnetic lines is accomplished by the *rotation* of the armature. In the case of the alternating currents, no dynamic rotation is necessary, as the wave form of the current itself supplies the necessary changes in the magnetic field. The transformer in its essential parts consists merely of an iron core surrounded with two coils of wire, a fine coil and a coarse coil. The fine coil is connected to the primary line, receiving electrical energy at a high potential, while the coarse wire coil is in communication with the lines of the consumers. The alternations of the primary current cause magnetic alternations in the core, thus inducing a secondary current in the coarse wire coil. Without serious error, it may be stated that the transformation thus effected is in proportion to the ratio of the number of turns of wire in the coarse coil to the number in the fine coil. If the primary wire is operated under 2,000 volts, and 100 volts is required in the consumer's circuit, the turns on the two coils in the transformer will be in the proportion of twenty to one. The advantage of the transformer lies chiefly in the fact that it needs absolutely *no* supervision. Once built and placed in position, it needs no further attention, unless injured from some exterior cause, but will go on performing its part of the service for an indefinite time.

697. Economy in the Conductor. — It is easy to calculate the economy that may be made in the circuits. Let W be the energy in watts to be transmitted between the generating station and the receivers. Supposing I to be the current, and E to be the difference of potential, and w the energy which is lost in the line, equal to a certain fraction m of W. If direct current distribution is used, the resistance of the line becomes, —

$$R = m\frac{E}{I}; \tag{247}$$

and also,
$$w = mEI = RI^2. \tag{248}$$

Suppose, on the contrary, that the same energy be transmitted under a difference of potential K times more elevated and a current K times more feeble. Under these circumstances —

$$w = R\frac{I^2}{K^2}; \tag{249}$$

from which it follows that $R = K^2R$, and therefore, as the length of line remains the same in the two cases, the same relations exist between the relative amounts of copper that are necessary for the appropriate circuit. For example: energy transmitted under a potential of two thousand volts requires four hundred times less copper for the same line losses than is needed under a transmission of one hundred volts; and conversely, with a tension of two thousand volts, it is possible to make the circuit four hundred times as long, with the same loss, as would be required for a potential of one hundred volts. Thus it is evident that raising the pressure of the energy will permit a distribution over a very much greater space. For an aerial line, the economy indicated is usually attainable; but when the circuit is placed underground, the full saving can very rarely be realized, for in the latter case a large proportion of the expense of line is required in the construction of the subway. Under these circumstances, a saving in the weight of the conductors will only decrease the total cost of the circuit by a smaller proportion, inasmuch as the cost of the subway will remain the same. It should also be recollected that the saving in the cost of the circuit is also offset by the expense made necessary by the use of transformers — an outlay of capital that is necessary to incur when the plant is first established.

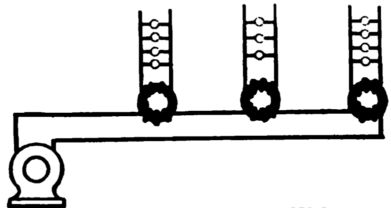
698. There are three possible ways in which transformers may be operated.

First. All of the primaries may be in series with the receivers on the secondaries in parallel.

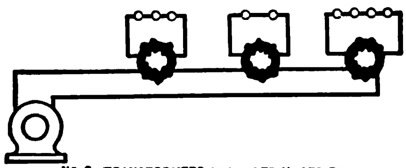
Second. Both the primaries and the receivers on the secondaries may be in series.

Third. The primaries and the receivers on the seconda ies may both be in parallel. These methods are indicated diagrammatically in Fig. 342, Nos. 1, 2, and 3.

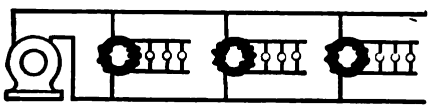
According to the first system, if any receiver be put into or out of commission, the resistance of the secondary circuit will be correspondingly diminished or increased. This will proportionally vary the impedance of the primary circuit; and the current therein will be, in



NO. 1. TRANSFORMERS ARRANGED IN SERIES, WITH LAMPS IN PARALLEL.



NO. 2. TRANSFORMERS ARRANGED IN SERIES, WITH LAMPS IN SERIES.



NO. 3. TRANSFORMERS ARRANGED IN PARALLEL, WITH LAMPS IN PARALLEL.

Fig. 342. Transformer Circuits.

a like manner, varied in quantity. Such a system cannot be made self-regulating; and, if used, must depend entirely upon manual regulation at the station.

699. In the second system, with the receivers arranged in series, a practical working arrangement is obtained, if the primary current is derived from a constant-current alternator having the ability (by compound winding or automatic regulators) of maintaining a constant current for considerable variations in the impedance of the primary circuit. This arrangement has received quite a wide development in arc-lighting plants operated by alternating currents, the customary design being indicated in Fig. 343.

700. The third method is the one most usually employed; for, if the transformers are designed with sufficient impedance in the

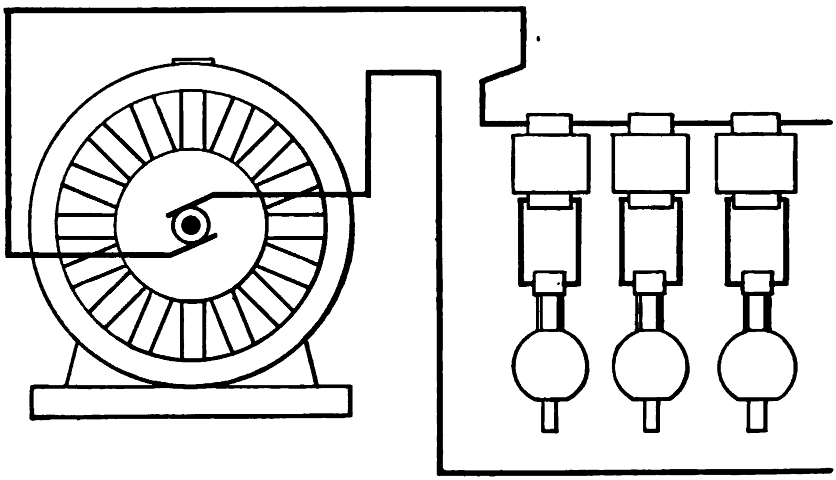


Fig. 343. Transformer for Arc Lamps.

primary circuit to practically block out all current, when the secondary is open, the system then becomes almost perfectly self-regulating.

701. Usually, however, the transformer service is installed as indicated in Fig. 344. With this arrangement, the transformers may

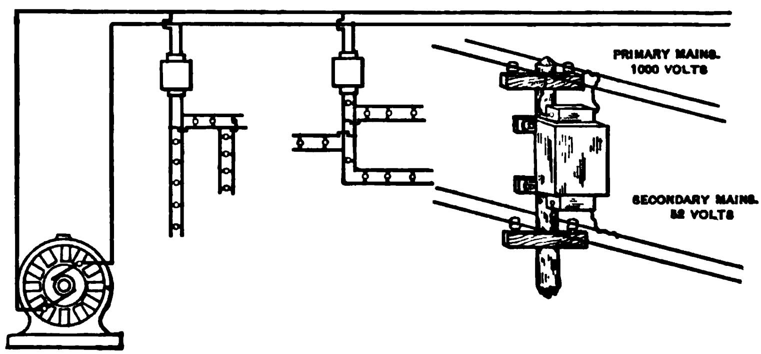


Fig. 344. Common Transformer System.

be regarded, in their relation to the central station, precisely as if they were the receivers themselves, and the distribution studied and designed in accordance with the principles for direct currents.

Where the area to be covered is very large, or the amount of energy transmitted great, the feeder and main system finds economical application, as indicated in Fig. 345.

702. The transformer system further presents great flexibility in the distribution from the secondary circuit. Where a large number of receivers are to be supplied at a single location, the transformers may be banked, with their secondaries in parallel, as shown in Fig. 346. Contrariwise, if higher potential be desired to overcome the resistance of long interior leads, the secondaries may be placed in series, as in Fig. 347, thus doubling the potential of the

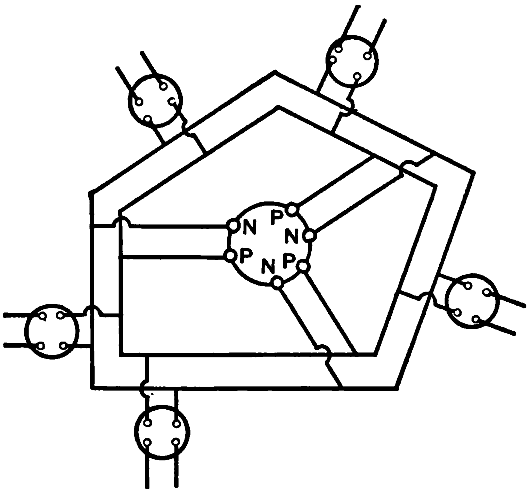


Fig. 345. Feeder and Main System with Transformer.

individual transformer. Finally, the service leads may be arranged upon a multiple-wire system, the transformer secondaries being arranged in series, and appropriately connected with the intermediate wires. For three-wire distribution the arrangement is indicated in Fig. 348. Even with the best proportional transformers, there is a small unavailable consumption of energy due to I^2R losses and hysteresis. When operating under a large load, the percentage of energy thus wasted becomes insignificant; but, during the hours of light loading, these wastes rise to formidable proportions. From this aspect, the common method of installing a separate transformer to serve the wants of each consumer is exceedingly

detrimental to the attainment of high service efficiency. The transformer supplied to each customer must have sufficient capacity to carry the maximum load ever desired. Necessarily, even during the

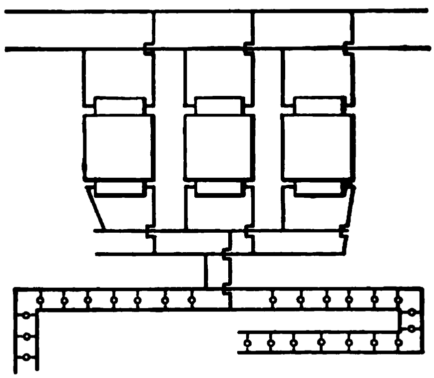


Fig. 346. Wiring for Transformer Secondary Circuits. Secondaries in Parallel.

daily hours of maximum loading, the transformer will be operating uneconomically at a light load. This is particularly the case in residence districts, where each individual house must have a possible transformer capacity sufficient to provide for occasional *fêtes*, while

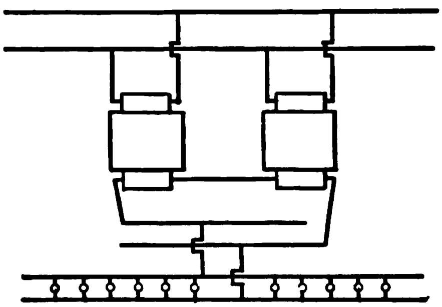


Fig. 347.
Wiring for Transformer Secondary Circuits.
Secondaries in Series.

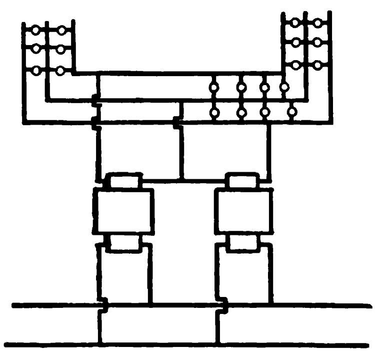


Fig. 348.
Wiring for Transformer Secondary Circuits.
Secondaries on Three-Wire System.

the daily load is but a fraction of the occasional demand. A great improvement in efficiency may be attained by designing the transformers to serve groups of buildings, as it is evident that special loading will rarely simultaneously occur to more than one customer

in such a group. Thus the average load will much more nearly approach a full load.

703. If the transformer is located to feed a subscriber, or group of subscribers, it is most appropriately placed at the center of gravity of the system of receivers that it is expected to feed. Inasmuch as it is not advisable to allow a high-tension circuit to enter the houses of the subscribers, this condition cannot always be strictly followed.

Theoretically, the transformer system becomes the most economical under the following conditions: —

First. A supply to the transformers by a primary circuit served by a system of feeders.

Second. A secondary circuit from the transformers supplying the receivers by a multiple-wire system.

704. The Efficiency of Distribution by Isolated Transformers. — The greatest offset to the use of the transformers lies in the low efficiency which is to be obtained when the instruments are operated for a greater part of the time under a light load. No matter to what extent the perfection of the transformer may be carried, the output is never quite equal to the total amount of energy which is supplied to it. For example, supposing the output of a transformer under full load to be 95 per cent of the energy supplied to it, there remains 3 per cent to be lost by hysteresis and Foucault currents, and 2 per cent due to heating of the circuits. The loss by hysteresis is continual, and is entirely independent of the loading placed upon the transformer. The heating-losses, however, diminish in proportion to the load. Now, with all the transformers at work during twenty-four hours of each day, assuming, as a fair estimate, that they will operate for two hours under a full load, four hours under a half load, and during the remainder of the time under no load, the mean daily efficiency then becomes easy to calculate. Assuming the efficiency of the transformer to be 95 per cent at full load, the demand on the station during the twenty-four hours is as follows: —

3 per cent during 18 hours	= .54
50 per cent during 4 hours	= 2.00
100 per cent during 2 hours	= 2.00
Total	4.54

The output which the transformer gives to the secondary circuit is —

0.00 during 18 hours	= 0.00
0.46 during 4 hours	= 1.84
0.95 during 2 hours	= 1.90
Total	3.74

The total efficiency is —

$$3.74 / 4.54 = 0.825.$$

705. Now, admitting the above excellent conditions, and allowing an annual operation covering 1,500 hours, the employment of the transformers reduces, in the above proportion, the amount of electrical energy which can be utilized in the secondary circuits. If, on the other hand, the transformers operate for two hours under full load, and only two hours under a half load, the efficiency is lowered 4 per cent. From these figures, it is evident that great care should be exercised in the selection and design of a plant. If, for example, it is practical, in a mountainous region, to take advantage of a waterfall, where the power costs little or nothing, and where the expense of installation is moderate, it is evident that the transformer system may be used with great economy and advantage. On the other hand, in cases where natural power is not available, as, for instance, in the center of a crowded city, and it becomes essential to use steam power, the losses experienced in the transformer may entail a coal expense which is equal to, and often far greater than, the interest and depreciation of a large conducting system.

706. Transformers Arranged as Sub-Stations. — In order to avoid the loss of energy due to light loading of the transformers, they may be grouped in sufficient numbers at a single spot, thus forming an auxiliary station. It is a simple matter here to install appropriate switches, by means of which the transformers may be thrown out of circuit during the idle hours of the day, when the load is comparatively light, and may be successively thrown into action as the load increases. This operation is usually a manual one, thus requiring the presence of an attendant during a part of each day. If, however, the sub-stations are so arranged that the attendant can proceed from one to the other successively, to cut in or out the various instruments, a single attendant will be sufficient. Many pieces

of apparatus have been proposed to throw the transformers in and out of circuit automatically; and while, on the one hand, it is doubtless possible to accomplish this, it will be necessary to have much greater experience with automatic machines of this description before full confidence and reliance can be placed upon them. The location of the auxiliary stations should be studied with equal care to that which is devoted to the selection of the site of the main plant. The secondary system of mains and feeders also becomes a matter of care in design, and requires even greater attention than in the case of house-to-house transformers. The three-wire system now becomes particularly applicable, in view of the economy to be derived in the conductors, especially as the auxiliary stations are designed to feed a much larger territory. With the auxiliary station arrangement, it becomes practically advisable to feed each sub-station by means of a single pair of mains extending from the main generating-plant. Under these circumstances, particular care should be taken to connect the auxiliary stations among themselves in such a manner that, in case of any accident to a pair of mains from the central station to a particular auxiliary station, the service may not be interrupted, but that the incapacitated sub-station may be fed by a roundabout circuit through the other auxiliary plants from the central station. A separate dynamo may be arranged to connect each set of mains to its corresponding transformer, or set of transformers. However, from an economical standpoint, it is usually preferable to unite the generators among themselves, in order to make them operate under the best possible conditions of loading. It should be noted here that one of the largest English central stations has preferred to employ a small number of machines, graded in size in such a way that one after the other may be thrown in or out of service, so as to keep the machines that are at work constantly under full load, and, therefore, operating at their point of maximum economy.

CHAPTER XVII.

POLYPHASE TRANSMISSION.

707. It has been shown that the losses in the conducting system are equal to the product of the resistance and the square of the current, while the energy transmitted is proportional to the product of the *E.M.F.* and current. Hence it follows that where a given amount of energy is to be transmitted if the voltage be increased, the current may be correspondingly decreased. By this process the line losses will be diminished with a given conductor, in proportion to the square of the voltage, or with a given line loss the quantity of conductor material may be decreased in the same ratio. Consequently as the pressure is raised either the cost of the energy lost in the line may be decreased or the cost of conductor material economized. Thus there is a constant tendency to increase the pressure at which transmission lines are operated. It has so far been found impracticable to build direct-current dynamos that will successfully develop a pressure of more than a few thousand volts; and even if such construction were possible, there is no convenient way of changing the voltage of a direct-current circuit, and it would be utterly impracticable to operate electrical-supply systems as they are now known at much greater pressures than are at present commonly used. With alternating currents the transformer presents a method of changing the mutual relations of current and pressure with a simplicity and economy that is almost ideal. The dynamo-builder may therefore construct his machine to deliver energy at any pressure and current that may be convenient. At the generating end transformers may be used to raise the voltage to that point which is most economical for transmission, and at the receiving end other transformers may lower the pressure to that which is desirable for distribution. So long as the supply of light was the chief object of the electrical installation, such a system filled all demands, but when a power market arose the plain alternating current found itself at a serious disadvantage, for as yet the alternating-current motor lacks many desirable qualities of the

direct-current machine. Thus the common alternating-current motor must be started by auxiliary apparatus, and brought into step with the generator, before the load is applied, and then can only operate under almost constant conditions of loading, nor is its efficiency equal to that of the direct-current machine. In the search for the ideal alternating-current motor, electricians have evolved the polyphase system, which, particularly for power transmission, presents especial advantages.

708. For a proper comprehension of polyphase systems, some consideration of the way that electric currents are generated is necessary.

Fig. 349 is a diagrammatic representation of a dynamo machine supplied with a gramme ring armature upon the periphery of which the coils are wound. As the ring revolves in the magnetic field *E.M.F.s* are

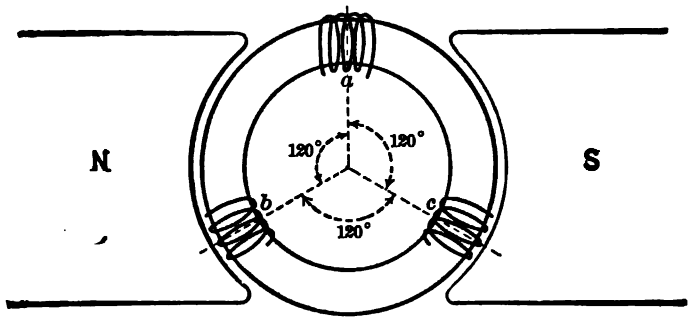


Fig. 349. Diagram of Polyphase Armature.

set up in the coils, and by differently combining the coils a great variety of circuits may be produced, but to avoid too great complexity practice has chiefly sanctioned three kinds—the Diphase, the Triphase, and the Fourphase systems. Consider now more in detail the *E.M.F.s* and currents of these systems. In the discussion of the voltage and current relations which follow, it is assumed that each phase is equally loaded, both as to current, *E.M.F.*, and phase, so that the systems are balanced. The effective current will be represented by I, and the effective *E.M.F.* by E, while the *E.M.F.* between any coil and the neutral point or common center will be represented by e.

709. **Diphase Systems.**—CASE I. *Four-wire Circuits.*—Suppose the winding of an armature to consist of only two coils or the equivalent. Such a condition is diagrammatically presented in Fig. 350, in which a represents one coil, and b another coil 90° behind the first one.

From the ends of each coil wires extend to slip-rings to which the four wires of the line circuits are respectively attached, which are represented by AA' and DD', forming the circuit of one phase, and CC' and BB' the circuit of the other phase. By inspection it is obvious that the pressure between AA' and DD' and also between CC' and BB' is the effective $E.M.F.$ which is developed in one coil, represented by E. This is the pressure measured by a voltmeter connected to the two wires of either circuit. The current in each phase is I amperes. There is no electrical connection between the two phases, each of which is entirely independent of the other and behaves as if it were generated by a separate machine. or even by a different plant.

710. CASE II. *Three-wire System.*—It is possible to join the leads DD' and CC' or BB' and AA' outside of the armature, as is shown in

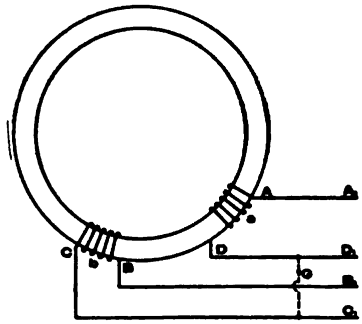


Fig. 350. Diagram of Diphase Armature.

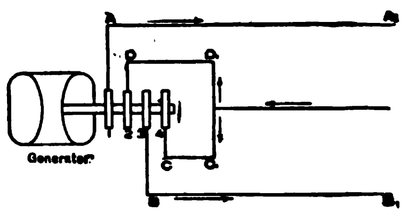


Fig. 351. Diagram of Diphase Circuit with Common Return.

Fig. 350 by dotted line, and in Fig. 351 by full lines, and thus do away with one of the four wires that form the two circuits, using the remaining wire as a common return. Under this condition the pressure between AA' and the common return and BB' and the common return is the same, and is equal to the $E.M.F.$ of one coil or E, and the $E.M.F.$ between AA' and BB' will be the resultant of the $E.M.F.$ developed in the two coils a and b. To determine this $E.M.F.$ let $ACDB$, Fig. 352, represent the path of the armature coils, OA the pressure e developed in the coil a or e, and OB the pressure developed in the coil b, also e. Complete the parallelogram $AFBO$, then OF is the resultant of OA and OB. If OA be taken as unity, then OF is $\sqrt{2}$; hence the pressure AA' to BB' is $e\sqrt{2}$. It also follows that the current

in the outgoing wires BB' and AA' will be I amperes, while the current of the common return will be $I\sqrt{2}$.

711. **Fourphase or Quarterphase.**—CASE III. *Four-wire System.*—The diagram of Fig. 350 represents a single pair of coils upon the

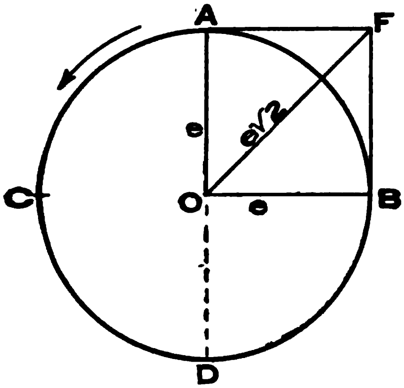


Fig. 352. Diagram of Resultant E.M.F. in Diphase Armature with Common Return.

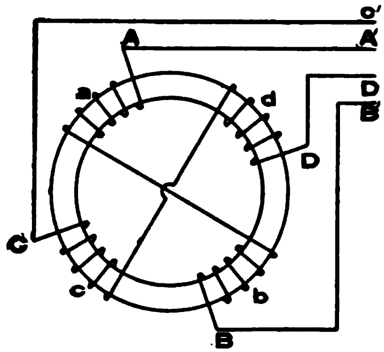


Fig. 353. Diagram of Quarterphase Armature.

armature, but, of course, its entire circumference is completely filled. It is possible to connect diametrically opposite coils, as shown in Fig. 353. The line wires of one phase AA' and BB' are connected to the outside ends of the coils a and b, while the inner ends of these coils are diametrically connected through the center of the armature. The other phase is arranged in a similar manner. With this method of wiring the pressure between AA' and BB' is equal to the $E.M.F.$ developed in two coils, or to $2E$, and is the same in both phases. There is no electrical connection between the coils and consequently no pressure between the wires of different phases. Each phase behaves like a separate and independent single-phase alternating-current circuit, having an $E.M.F.$ of $2E$. The current in each phase is I amperes.

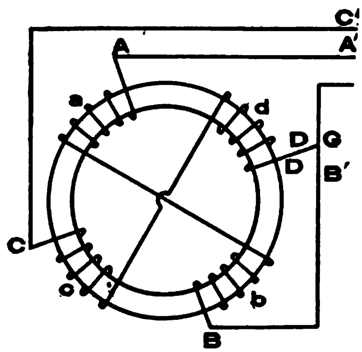


Fig. 354. Diagram of Quarterphase Armature with Common Return.

712. CASE IV. *Three-wire Circuit.*—The preceding circuit may be operated by a common return precisely as described for Case II. The wiring is shown in the diagram Fig. 354, in which coils b and d are connected to a common wire at the point G.

The relations of $E.M.F.$ and current which now exist may be demonstrated by the same train of reasoning as used in Case II, that is to say, the pressure between AA' or CC' and the common return is equal to the pressure of two coils, or $2E$, while the pressure between AA' and CC' is equal to $E\sqrt{2}$. This may be proved in another manner by referring to Fig. 355, in which AAA and BBB represent either the $E.M.F.$ or current curves in the coils a and d, separated by 90° in phase. The curve CCC is the algebraic sum of the ordinates of AAA and BBB and is consequently

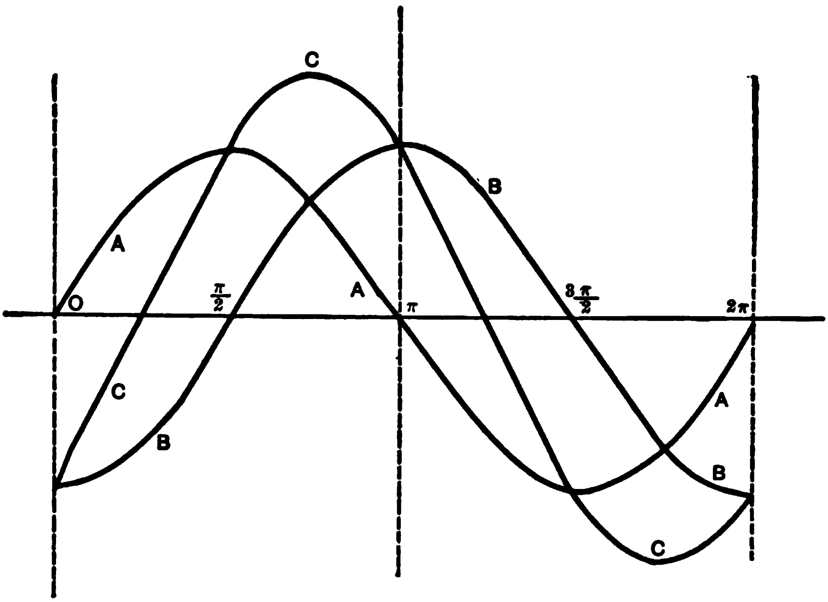


Fig. 355. Diagram of Quarterphase E.M.F. or Current.

the resultant pressure between the wires AA' and CC' or the current of the common return. From the well-known properties of the sine curve this is $E\sqrt{2}$. As in Case III, the current in the outgoing wires is I amperes, and in the common return $I\sqrt{2}$ amperes.

713. CASE V. *Mesh or Star Winding.*—In Fig. 353 the ends of the diametrically opposite coils are shown electrically connected through the center of the armature. Suppose where the leads from different phases intersect they be electrically connected, as shown in Fig. 356. As in Case III, the pressure between the wires of the same phase will be $2E$, but now the interior ends of all the coils are joined to a common

point and consequently there is an *E.M.F.* between the wires of the different phases, which, by the same reasoning as used in Case II, is equal to $E\sqrt{2}$.

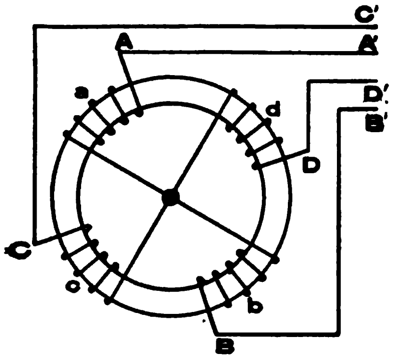


Fig. 356. Diagram of Quarterphase Armature, Star-Connected.

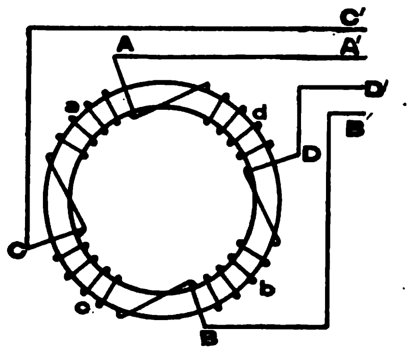


Fig. 357. Diagram of Quarterphase Armature, Ring-Connected.

CASE VI. *Ring Winding.*—In Fig. 357 all of the coils around the circumference of the armature are connected together in series, and line wires are attached at four points, each 90° in advance of the other, the connection of each line being made between two coils. Under this arrangement the pressure between each wire and the one which immediately precedes or succeeds it (the leads of different phases) is due to the *E.M.F.* of one coil, or E, while the pressure between the wires of the same phase is equal to $E\sqrt{2}$; the current consequently in each phase is $I\sqrt{2}$.

714. Triphase Circuits.—CASE VII. *Star or Y Connection.*—In the previous circuits the line conductors have been supposed to be connected to the armature coils at intervals of 90°. If the connections are made at intervals of 120° the so-called triphase circuit is produced, the coils of which may be connected either in star or in mesh, as already described for the quarterphase circuit. Fig. 358 is a diagrammatic representation of the triphase star-connected armature, or, as it is usually called, the Y connection, from the similarity of the connecting conductors to a capital Y. To determine the relations of the *E.M.F.s* consider diagram Fig. 359, in which the radii a b c of the circular path of the coils represent the pressure e between each coil and the neutral point d. As each coil is in advance of the succeeding one, the resultant *E.M.F.* or pressure between the line wires (which

is the pressure given by a voltmeter) must evidently be the difference between the $E.M.F.$s of the coil in advance and that of the one behind, and is easily obtained by subtracting one $E.M.F.$ from the other. This subtraction is most conveniently performed vectorially, as shown in

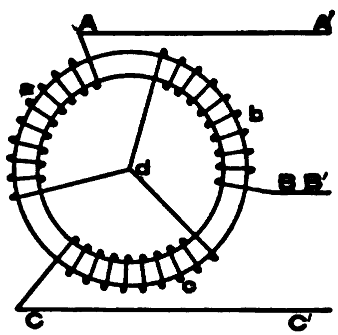


Fig. 358. Diagram of Triphase Armature, Y-Connected.

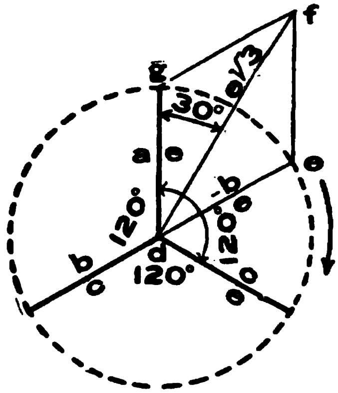


Fig. 359. Diagram of Composition of E.M.F. in Y-Connected Triphase Armature.

Fig. 359: prolong b, making de equal to b, and complete the parallelogram $defg$, then the diagonal df is equal to the resultant $E.M.F.$, which according to the well-known properties of the circle, is $e\sqrt{3}$. Hence in a Y-connected triphase circuit if the pressure E between the line wires be measured, then the pressure between any wire and the neutral point will be $\frac{E}{\sqrt{3}}$. Or if the pressure e between the neutral and any line wire be ascertained, the pressure between any two phases will be $e\sqrt{3}$. In the Y-connected circuit the current is I amperes in each wire.

715. It is easy in imagination to resolve the circuit into three separate and independent circuits. Let Fig. 360 represent the currents in the lines. The algebraic sum of the current is zero, hence no current will flow in the return wires and they may be dispensed with. This is on the assumption that all three phases are balanced; if this is not the case a return must be provided over the unbalanced portion. Fig. 361 illustrates this condition.

716. CASE VIII. *Mesh or Δ Connection.*—When all the coils on a triphase armature are connected in series, and each line wire tapped between two coils as shown in Fig. 362, it is said to be mesh-connected or

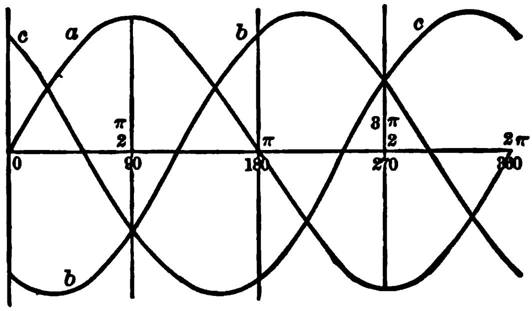


Fig. 360. Diagram of Triphase Currents.

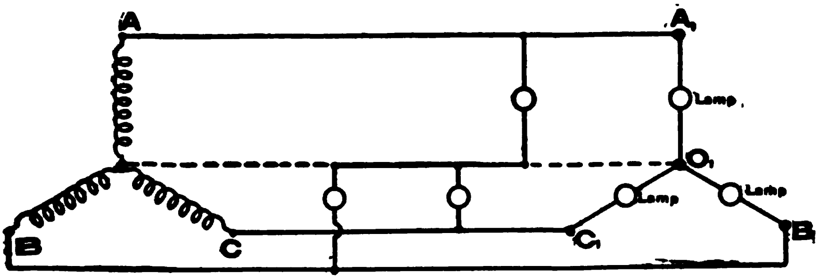


Fig. 361. Diagram of Balanced and Unbalanced Circuit.

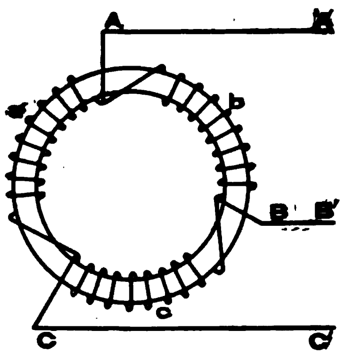


Fig. 362. Diagram of Δ-Connected Triphase Armature.

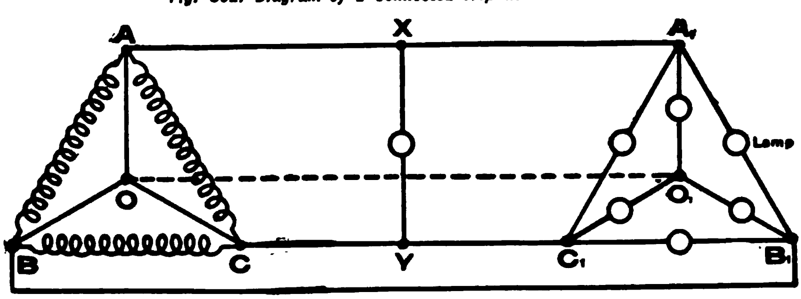


Fig. 363. Triphase Circuit with Double Connection.

Δ-connected, from a resemblance to the Greek letter of that name. The pressure between any two line wires is that developed by one coil or E, but the current in each wire, by the same reasoning as in Case VII, is $I\sqrt{3}$.

The power delivered by a triphase circuit is the same with either method of wiring because, by Case VII, $P=IE\sqrt{3}$ and, by Case VIII, $P=EI\sqrt{3}$.

717. CASE IX. *Double Wiring.*—The triphase circuit is sometimes connected up as is shown in Fig. 363, in which both the Y and the Δ connections are provided. With such an arrangement it is practical to operate six different circuits, namely, three circuits between the three line wires and the common centre, and three circuits, one between each of the line wires. Under such conditions if the pressure between any wire in the common centre is e, the pressure between each wire is $e\sqrt{3}$.

718. Taking all things into consideration, experience now tends to the selection of the triphase system as on the whole best adapted to

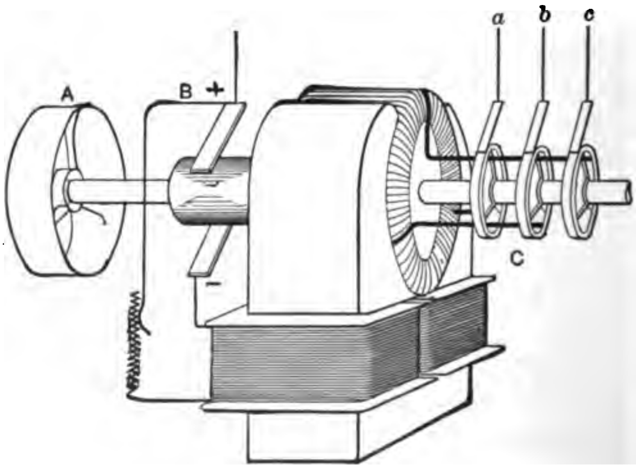


Fig. 364. Triphase Generator.

power transmission under the condition which usually prevails. To illustrate the flexibility of the polyphase system refer to Fig. 364, showing a typical triphase dynamo.

Here the machine consists of a shaft carrying a gramme ring armature, placed so as to rotate between the two field magnets. The shaft

carries a commutator at B, and three collecting-rings, *a*, *b*, and *c*, the connections to the armature being separated from each other by 120 degrees, each part being, as indicated in the illustration, carried to its appropriate collecting-ring. Such a machine as this is extremely flexible.

First. By applying power to pulley A, making the armature turn mechanically, a continuous current may be obtained from the commutator B.

Second. By supplying a continuous current to the commutator B, the dynamo is operated as a motor, and mechanical power may be obtained from the pulley A.

Third. By applying mechanical power to the pulley A, turning the armature, and, instead of collecting the current at the commutator *B*, collecting it by the rings *a*, *b*, and *c*, a triphase alternating-current generator is obtained, being self-excited by a current derived from the armature by the commutator at B.

Fourth. By supplying triphase alternating currents to the collecting-rings *a*, *b*, and *c*, a synchronous alternating-current motor is obtained, and mechanical power may be derived from the pulley A.

Fifth. If a continuous current be supplied to the commutator B, triphase alternating currents may be obtained from the collectors *a*, *b*, and *c*; thus, in this case the dynamo acts as a motor transformer, transforming a continuous current to a triphase alternating current.

Sixth. If a triphase alternating current be supplied to the collectors *a*, *b*, and *c*, a continuous current may be obtained from the commutator B, the dynamo now operating as a motor transformer, changing a triphase alternating current into a continuous current.

719. Transmission systems may be divided into three classes, depending upon the employment of transformers for changing the line pressure.

A plant of the first class is shown diagrammatically in Fig. 365. The generator is wound for, say, 250 volts, and motors and lights are operated from it directly without intermediate transformers. It corresponds to the system commonly employed with continuous current, and is well adapted to plants in which the distances are not great.

Fig. 366 shows a plant of the second class. The generator is wound for, say, 2300 volts. Large motors may be made for this potential, but lights and small motors must take their power through lowering transformers.

Generators of large size can be wound for pressures up to 13,000 volts, so that this system covers the greater portion of the power transmission field and is that used generally by lighting and power stations.

When the distances over which power must be carried are so great as to require a higher potential than the generator can be wound for, the system must be of the third class, shown in Fig. 367.

The generator is wound for any convenient low potential, and the pressure is raised by step-up transformers to a higher potential. At

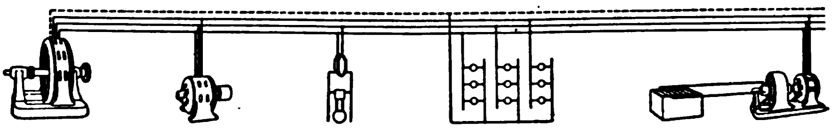


Fig. 365.

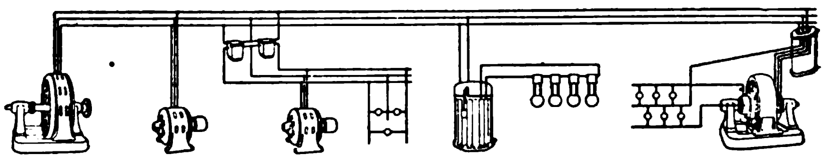


Fig. 366.

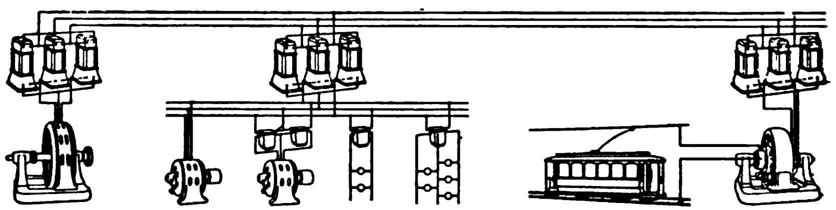


Fig. 367.

the distribution end the pressure is reduced to a potential suitable for distribution.

The diagram Fig. 367 indicates two substations. In one the power is distributed at an intermediate potential, a second set of step-down transformers being employed to supply most of the motors and lights. In the other the load—a rotary converter—is supplied with low potential current through a single bank of transformers.

Power systems do not necessarily confine themselves to one of the methods, but often combine two and sometimes all three.

720. The Relative Amounts of Conductor Material in Different Systems.—In long transmission lines the cost of conductor material becomes a large fraction of the total installation cost; hence the relative quantities of conductor material form an important consideration in choosing the system. There are two aspects from which this question can be viewed. Every circuit must be insulated, and the amount of insulation must be sufficient to withstand the *maximum* pressure to which the conductors are subjected. It has been shown that in an alternating circuit the effective pressure is .707 of the maximum pressure. Evidently the power delivered is the effective pressure multiplied by the current, while the insulation is stressed by the maximum pressure. So the relative conductor material required by different systems may be compared either on the basis of *maximum pressures*, thus viewing each system from the *insulation* standpoint, or *danger* aspect. Or comparison may be made on the relative *effective pressures*, that is, on the ability to transmit power. As the relative weights of conductor material are directly proportional to the squares of the respective pressures which have already been calculated, it is easy to tabulate the relative weights of conductor material. These are as shown in TABLES NOS. 84 and 85, first published by Mr. Steinmetz.*

TABLE NO. 84.

Relative Amounts of Conductor Material on Basis of Effective Pressure.

System.	Number of Wires.	Conductor Material, Per Cent.
Single phase	2	100.0
Edison three-wire neutral, full section	3	37.5
" " " half "	3	31.25
Inverted three phase	3	56.25
Quarter phase, common return	3	72.9
Three phase	3	75.0
Three phase, with neutral	4	33.3
Three phase neutral, half section	4	29.17
Independent quarter phase	4	100.0
Edison, full neutral	5	15.625
" , half neutral	5	10.93
Quarter phase, common neutral, full section	4	31.25
" " " " half section	4	28.125

* Alternating-Current Phenomena, p. 478.

Table No. 85.

Relative Amounts of Conductor Material on Basis of Maximum Pressure.

System.	Number of of Wires.	Conductor Material, Per Cent.
Single phase	2	100
Two phase, with common return	3	145.7
Two phase	4	100
Three phase	3	75
Direct current	2	50

721. Commercial Pressure.—Within the past few years the high-potential transmission plant has multiplied enormously. Railway transmission lines from 10,000 to 20,000 volts are sprinkled all over the country. In the West, particularly in California, where coal is expensive and water power plenty, electrical engineers have been bolder and 40,000-, 50,000-, and 60,000-volt lines are in regular commercial operation, and it is reported that one line has been operated successfully at 80,000 volts. But as the pressure rises the cost of installation and operation, owing to augmented danger, rapidly increases, and for the present it seems as if by commercial limitations the probable maximum pressure is about 50,000 to 60,000 volts.

722. Determination of Conductors for Transmission Lines.—The object of the transmission line is to carry energy, in the form of an electric current, from one point to another, and the problem presented in the design of the line is that of finding the most desirable size of wire under the conditions given. The economic laws which govern the selection of the site of the generating station with reference to the consumers, and the kind of system to be employed, have already been discussed; consequently when the time arrives to calculate the line, the engineer finds the following four conditions imposed:

1st. The amount of power to be delivered at the *end* of the line in $KW = P$.

2d. The distance between the generating and receiving station in miles $= L$.

3d. The nature of service to be supplied.

4th. The permissible loss in per cent of $P = p$.

Consider first transmission by continuous currents; then

$$P=EI, \quad \ldots \ldots \ldots \quad (250)$$

$$pP=pEI, \quad \ldots \ldots \ldots \quad (251)$$

also

$$RI^2=pEI; \quad \ldots \ldots \ldots \quad (252)$$

hence

$$R=\frac{pE}{I}, \quad \ldots \ldots \ldots \quad (253)$$

and the resistance of the line per mile is

$$\frac{R}{L}=\frac{pE}{LI}. \quad \ldots \ldots \ldots \quad (254)$$

This expression gives the resistance of one mile of two wires of metallic circuit; the resistance of one wire is evidently $\frac{pE}{2LI}$, and the proper size wire can at once be found by seeking the value of $\frac{pE}{2LI}$ in any wire table showing resistance per mile opposite which will be found the proper wire-gauge number. In case multiple-wire distribution is used the conductor sizes can be reproportioned by the factors for relative weights of conductor for different systems previously given.

723. In the preceding formulæ R is the ohmic resistance of the entire line, but it has been shown that in alternating-current circuits the relations of *E.M.F.*, current, and power delivered are affected by other factors than simply ohmic resistances, which must be reckoned with in determining the proper size wire. Every transmission line possesses inductance and capacity whose mutual relations determine the reactance and the impedance of the circuit. The various receiver circuits also possess inductance and capacity, the effect of which is to cause the current in the transmission line to lead or lag behind its *E.M.F.* by varying amounts, producing so-called wattless currents, which, while they do not demand power from the generating station, must be recognized in the heating effect which they produce. With every reversal of phase the iron cores of all transformers, and other inductances, and the dielectrics of all capacities absorb some energy in molecular changes which tend to alter the phase relations of the

line current and *E.M.F.* Finally the skin effect in the line wire must not be forgotten.

724. The Power Factor.—The ratio between the true watts in any alternating-current circuit, as ascertained by an indicating watt-meter, and the apparent watts, or the product of the volts and amperes, is termed the "power factor" and is shown to be numerically equal to the cosine of the lag angle (see page 417). The so-called wattless currents are displaced approximately 90° from the energy current and so will be numerically equal to the sines of the lag angles. As this displacement is chiefly produced by various inductances, the sines of the lag angles may be called "inductance factors."

Table 86 gives the value of the power and inductance factors for various lag angles. The power factor is used to determine the true energy when the volt amperes are known, the resistance when the impedance is given, and the component of the impressed *E.M.F.* which is in phase with the energy current. The inductance factor is an aid to a determination of the quantities which are in quadrature with the energy component.

TABLE 86.

Power and Inductance Factors.

Angle of Lag or Lead.	Power Factor.	Induction Factor.	Angle of Lag or Lead.	Power Factor.	Induction Factor.	Angle of Lag or Lead.	Power Factor.	Induction Factor.	Angle of Lag or Lead.	Power Factor.	Induction Factor.
Deg.	Cos.	Sin.	Deg.	Cos.	Sin.	Deg.	Cos.	Sin.	Deg.	Cos.	Sin.
1	.9998	.0174	24	.9135	.4067	46	.6946	.7193	69	.3584	.9336
2	.9994	.0349	25	.9063	.4226	47	.6820	.7313	70	.3420	.9397
3	.9986	.0523	26	.8988	.4384	48	.6691	.7431	71	.3256	.9455
4	.9976	.0698	27	.8910	.4540	49	.6561	.7547	72	.3090	.9511
5	.9962	.0872	28	.8829	.4695	50	.6428	.7660	73	.2924	.9563
6	.9945	.1045	29	.8746	.4848	51	.6293	.7771	74	.2756	.9613
7	.9925	.1219	30	.8660	.5000	52	.6156	.7880	75	.2588	.9659
8	.9903	.1392	31	.8572	.5150	53	.6018	.7986	76	.2419	.9703
9	.9877	.1564	32	.8480	.5299	54	.5878	.8090	77	.2249	.9744
10	.9848	.1736	33	.8387	.5446	55	.5736	.8191	78	.2079	.9781
11	.9816	.1908	34	.8290	.5592	56	.5592	.8290	79	.1908	.9816
12	.9781	.2079	35	.8191	.5736	57	.5446	.8387	80	.1736	.9848
13	.9744	.2249	36	.8090	.5878	58	.5299	.8480	81	.1564	.9877
14	.9703	.2419	37	.7986	.6018	59	.5150	.8572	82	.1392	.9903
15	.9659	.2588	38	.7880	.6156	60	.5000	.8660	83	.1219	.9925
16	.9613	.2756	39	.7771	.6293	61	.4848	.8746	84	.1045	.9945
17	.9563	.2924	40	.7660	.6428	62	.4695	.8829	85	.0872	.9962
18	.9511	.3090	41	.7547	.6561	63	.4540	.8910	86	.0698	.9976
19	.9455	.3256	42	.7431	.6691	64	.4384	.8988	87	.0523	.9986
20	.9397	.3420	43	.7313	.6820	65	.4226	.9063	88	.0349	.9994
21	.9336	.3584	44	.7193	.6946	66	.4067	.9135	89	.0174	.9998
22	.9272	.3746	45	.7071	.7071	67	.3907	.9205			
23	.9205	.3907				68	.3746	.9272			

Commercial Power Factors.—The power factor usually shows the phase relations which are produced between the current and *E.M.F.* by the kinds of load which are placed upon the distributing system. If an alternator supplies a bank of lamps close at hand, the reactance of the circuit is infinitesimal and the power factor is 100% (approximately). If there is a long line with raising and lowering transformers, with lamps near the secondary terminals, the power factor at the low-tension side of the lowering transformers will be 100%, but at the generator it will be decreased by the reactance of the line and the transformers and the hysteresis of their cores. For fair average practice, constants at from ½ to 1¼ load may be assumed to average as follows:

Reactance	3½	per cent.
Hysteresis	1½	"
Copper loss	1	"
Efficiency	97½	"
Magnetizing current	4	"

The addition of motors of any kind to a distributing circuit reacts seriously upon the power factor. It is possible to wind alternating-current motors so that they will give about 100 per cent. power factor at any predetermined load, but as the load decreases or increases the power factor falls rapidly. From ¼ to ½ load the power factor is rarely as much as 50 per cent., but from ¾ to 1¼ load the power factor will vary from 75 to 90 per cent. It is possible to add a condenser to a motor circuit, so that the reactance of the condenser may counterbalance that of the motor and thus vastly improve the power factor. The only objection to this plan is the expense of this apparatus. Rotary converters react upon the circuit very much after the fashion of motors, excepting that their power factors are lower, as it is rare that the converter is so uniformly loaded as the motor.

725. Impedance.—The impedance of a circuit has been shown to be numerically equal $\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}$.

The data for obtaining resistance R have been fully given, as well as general explanations for the value of $\left(L\omega-\frac{1}{C\omega}\right)$.

A transmission line forms a special case, and as practically it must

be designed for one of three or four commercial frequencies, and must use commercial sizes of wire, some short cuts in calculation are available. The inductance L in henrys per mile for one wire of a parallel circuit of two copper wires in a non-ferric environment is given by the following formula, in which D is the distance between the centers of the wires, and r the radius of each, both in the same units:

$$L=\left(80.5+740\log\frac{D}{r}\right)10^{-6}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (255)$$

Authorities differ slightly as to the coefficients of this formula, some giving the value of 85 to the first term inside the parenthesis and 742 to the coefficient of the logarithm, but the balance of opinion seems to favor the expression as written above. To illustrate the use of this formula take the following example:

726. Example.—Find inductance of 5 miles of a circuit of two parallel copper No. 000 wires set 48 ins. apart. Here $r=.2048$, $D=48$ ins.

For 1 mile of one wire $L=\left(80.5+740\frac{48}{.2048}\right)10^{-6}$.

Solutions by logarithms:

Log.	48	= 1.6812412	
"	.2048	= −1.3113300	
"	$\frac{D}{r}$	= 2.3699112	
"	2.3699112		= 0.3747300
"	740.		= 2.8692317
"	$740\frac{D}{r}$		= 3.2439617
Corresponding number			= 1753.8
Add	80.5		80.5
			1834.3
Multiply by 10^{-6}			= .0018343 henrys per mile

For five miles one wire........................ .001843 × 5 = .009215

" " " of circuit........................ .009215 × 2 = .018430

As inductance is proportional to current, it is easy to deduce it for any conductor arrangement. Thus for a Δ triphase circuit each wire acts successively as a return for the other two, and compared with a single-phase circuit the current is $I\sqrt{3}$. Hence the inductance per circuit mile is $\sqrt{3}$ times the inductance per wire mile of a single-phase line.

727. **Methods of Reducing Induction.**—An examination of Eq. 255 shows that the interaxial distance is a direct function, hence induction may be decreased by diminishing the distance between the wires. Obviously in open-wire lines at high pressure it is possible to carry this out but to a very limited extent, but in a cable where the wires can be placed close together and twisted about each other the induction becomes insignificant, and if a concentric cable be used the effect of one conductor entirely annuls that of the other. The drop due to inductance may be reduced by subdividing the conductor, that is, by using several wires whose combined area and weight are the same as that of the required single conductor. This is shown by the following example:

728. Assume a circuit 1 mile long of two No. 0000 wires set 72 ins. apart. Frequency 100; amperes 50. The constants of such a mile of No. 0000 are as follows:

	Per Mile of Wire.	Per Mile of Circuit.
Resistance..............	.258 ohms	.516 ohms
Inductance..............	1.930 millihenrys	.00386 henrys.
Circular mileage..........	211600	

Impedance..... $\sqrt{.516^2+(2\times3.1415\times100\times.00386)^2}=$ 2.44 ohms

Drop due to resistance.................... $50\times\ .516=$ 25.8 volts

Total drop.......................... $50\times2.44\ =122.0$ "

Substitute eight No. 6 wires in parallel. Constants for one No. 6 wire:

	Per Mile of Wire.	Per Mile of Circuit.
Resistance..............	2.083 ohms	4.166 ohms
Inductance..............	2.266 millihenrys	.004532 henrys
Circular mileage..........	26251	

Resistance of eight No. 6 wires in parallel........ $\frac{4.166}{8}=.5207$ ohms

Total circular mileage........................ $26251\times8=210008$

So eight No. 6 wires have about the same weight and resistance as one No. 0000.

Impedance of one No. 8 $=\sqrt{4.166^2+(2\times 3.1415\times 100\times .004532)^2}=$ 5.044 ohms.

The current in each wire is $\frac{50}{8}=6.5$ amperes.

Resistance drop per wire................ $4.166\times 6.5=27.079$ volts
Total drop............................ $5.044\times 6.5=32.786$ "

In both cases the resistance drop is about the same; while the total drop is more than four times as great with the single large wire.

729. Mutual Induction.—By the principles of mutual induction the lines of different alternating circuits may if placed on the same pole line for long distances react on each other sufficiently to seriously interfere with satisfactory operation. The effects of mutual induction usually manifest themselves by causing objectionable flickering of lamps and other abnormal functioning of the receivers. The best remedy is to transpose the wires of the perturbing circuits. The theory of the transposition has been fully explained in Chap. III. The application to alternating-current power circuits differs from the methods there outlined only in the mechanical details necessary to handle large and heavy wires, and that, owing to the decreased number of circuits on each pole and the insensitiveness of power circuit receivers, it is sufficient to transpose each circuit two to four times in its entire length, unless this be very great.

730. Capacity.—The capacity of one wire of a circuit of two parallel conductors may be calculated either with reference to the other conductor or with reference to the earth. For underground cables both capacities are approximately the same, but for aerial lines there is a sensible difference. The following formulæ are generally accepted as giving capacity with practical accuracy.

CASE I.—*Underground Cables. Capacity Conductor to Conductor or Conductor to Earth.*

$$C \text{ (mf. per mile)} = \frac{.03883K}{\log \frac{D}{d}}. \quad \ldots\ldots \quad (256)$$

where D = diameter of cable (internal diameter of sheath);
d = " " conductor, both in the same units;
K = specific inductive capacity of insulation (Table 74).

CASE II.—*Aerial Lines. Capacity to Earth* (one wire).

$$C \text{ (mf. per mile)} = \frac{.03883}{\log \frac{4h}{d}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (257)$$

h = height above ground;
d = diameter of conductor, both in the same units.

CASE III.—*Aerial Lines. Capacity Wire to Wire.*

As most transmission lines are of open wire, the capacity effect between the various conductors becomes of much importance. Various authorities, who have treated this subject, are not as explicit in their language as could be desired, nor do their results perfectly harmonize among themselves. Probably the discrepancies are largely due to point of view, and on the whole it appears advantageous to consider the question of capacity something as follows:

Take a system of two wires. One conductor will be positive and the other negative. If a line be drawn through the centers of these conductors, the potential, starting at positive conductor, will decrease to zero at the middle point between them and then increase to corresponding negative potential at the other wire, and thus there will be a neutral point of zero potential midway between the conductors. Heaviside* shows that the capacity of *one mile* of *one wire* referred to *the neutral point* is given by the expression

$$C \text{ (mf. per mile)} \frac{0.0388}{\log \frac{2D}{d}},$$

in which D is the distance between centers of the conductors and d the diameter of each one, both expressed in the same units. Obviously there will be a similar capacity between the other wire and neutral point. Now it is easy to imagine these two capacities replaced by a pair of little condensers, so the total capacity between the wires will be the sum of these little condensers. As they are in series, this is equivalent to one-half the sum of the capacities of

* Electrical Papers, vol. I, page 48.

each wire referred to the neutral point, and thus the capacity of one mile of a two-wire circuit will be half the capacity of each wire taken singly to neutral. Dr. Perrine * shows that the capacity of a network of conductors is the same as the capacities between the wires and the neutral point, and confirms this mathematical deduction by a series of exhaustive experiments. Hence it is easy to see that the capacity per mile of a three-phase circuit is the capacity of one wire to neutral.

731. EXAMPLE.—Find the capacity of a two-wire circuit of No. 000 set 18 ins. apart, five miles long.

Solution by logarithms:

Wire 000; $d = .4096$; $D = 18$ inches; $2D = 36$ inches.

$$c = \frac{.0388}{\log \dfrac{2 \times 18}{.4096}}$$

$$c = \log .0388 - \log (\log 36 - \log .4096)$$

$$\begin{array}{lrl} \text{Log } 36 & = & 1.5563025 \\ \text{“} \quad 0.4096 & = - & 1.6123599 \\ \hline & & 1.9439426 \\ \text{“} \quad .0388 & = - & 2.58883 \\ \text{“} \quad 1.9439426 & = & 0.2886739 \\ \hline & & .3001578 = .01996 \end{array}$$

$c = .01996$ mf. per mile one wire to neutral

$$\text{Capacity per circuit mile} = \frac{.01996}{2} = .00998$$

$$\text{Capacity for five miles} = .00998 \times 5 = .0499$$

732. The preceding formulæ enable the designer to calculate the inductance and capacity of any circuit of two parallel lines. For the inductance of a circuit mile of two wires multiply the respective inductances given by the formula for one wire by 2. For the inductance of a triphase circuit multiply the inductance of one wire by $\sqrt{3}$. The capacities given by the formula are capacities of one wire. The capacities of two wires are found by dividing by 2. For a triphase circuit the formula gives the capacity of each wire, and this is the same as the capacity per circuit mile.

* Transactions of American Society of E. E., May 18, 1900.

TABLE No. 87.

Inductance and Capacity of Transmission Lines.

Size of Wire.	Diameter in Inches.	Inter-axial Distance.	Inductance L in Millihenrys per Mile, 1 Wire.	Capacity in Mf. per. Mile, 1 Wire.	Size of Wire.	Diameter in Inches.	Inter-axial Distance.	Inductance L in Millihenrys per Mile, 1 Wire.	Capacity in Mf. per Mile, 1 Wire.
0000	.460	12	1.353	.0226	4	2043	12	1.614	.0187
		18	1.484	.0210			18	1.744	.0173
		24	1.576	.0192			24	1.838	.0162
		36	1.707	.0177			36	1.968	.0152
		48	1.799	.0167			48	2.061	.0145
		60	1.871	.0161			60	2.132	.0140
		72	1.930	.0155			72	2.192	.0136
000	.4096	12	1.391	.0220	5	1819	12	1.652	.0183
		18	1.521	.0198			18	1.781	.0169
		24	1.614	.0188			24	1.875	.0160
		36	1.745	.0173			36	2.005	.0148
		48	1.836	.0164			48	2.099	.0143
		60	1.909	.0156			60	2.169	.0137
		72	1.968	.0152			72	2.229	.0134
00	3648	12	1.498	.021	6	1620	12	1.689	.0179
		18	1.558	.0195			18	1.820	.0165
		24	1.651	.0183			24	1.912	.0156
		36	1.784	.0169			36	2.043	.0147
		48	1.874	.0160			48	2.135	.0140
		60	1.946	.0154			60	2.208	.0135
		72	2.005	.0148			72	2.266	.0130
0	3248	12	1.465	.0206	7	1442	12	1.727	.0175
		18	1.596	.0189			18	1.857	.0162
		24	1.688	.0178			24	1.949	.0154
		36	1.818	.0166			36	2.079	.0142
		48	1.911	.0157			48	2.172	.0137
		60	1.982	.0151			60	2.245	.0132
		72	2.042	.0146			72	2.303	.0128
1	2898	12	1.502	.0200	8	1284	12	1.764	.0171
		18	1.633	.0186			18	1.894	.0158
		24	1.725	.0175			24	1.986	.0151
		36	1.856	.0160			36	2.117	.0141
		48	1.949	.0154			48	2.209	.0135
		60	2.023	.0146			60	2.282	.0131
		72	2.079	.0142			72	2.340	.0127
2	2576	12	1.540	.0197	9	1144	12	1.801	.0166
		18	1.671	.0180			18	1.931	.0155
		24	1.764	.0171			24	2.025	.0148
		36	1.893	.0159			36	2.155	.0138
		48	1.986	.0151			48	2.248	.0133
		60	2.058	.0144			60	2.319	.0128
		72	2.116	.0141			72	2.376	.0125
3	2294	12	1.577	.0190	10	1018	12	1.838	.0162
		18	1.708	.0176			18	1.968	.0152
		24	1.800	.0164			24	2.061	.0145
		36	1.931	.0155			36	2.192	.0136
		48	2.033	.0146			48	2.285	.0130
		60	2.095	.0143			60	2.356	.0126
		72	2.154	.0138			72	2.415	.0123

733. To obviate labor of calculation TABLE NO. 87 gives inductances and capacities calculated by above formulæ for Nos. 0000 to 8 wire, inclusive, and for interaxial distances from 12 ins. to 72 ins.

Knowing the inductance and capacity, the impedance can be calculated for any frequency. But in making such calculations it must be remembered that the inductance and resistance are in series with each other, while the capacity acts as if it were a shunt between the conductors, and is not in series with the resistance and inductance. Therefore the true impedance of a transmission line is the vector sum of the impedance due to resistance and inductance in series, and that of the capacity in parallel therewith. Theoretically each element of the length of the line has an element each of resistance, inductance, and capacity, and hence all three should be taken as uniformly distributed over the whole length of the circuit. This greatly complicates the mathematical formulæ for calculating line properties. An approximate method sufficiently accurate for most practical work is that of assuming the entire capacity concentrated at the center of the line. The practical effect is threefold: 1st, the inductance increases the apparent resistance, causing a greater drop than that called for by the ohmic resistance; 2d, the current is made to lag and the power factor to decrease; 3d, the capacity demands current with every alternation to charge the line as a condenser, and while little power is lost thereby, this charging current must never be overlooked in estimating heating effects. Finally, in the design of all switching apparatus the liability of the line to discharge must be reckoned with. The impedance per unit of length of line is obtained at once from the expression $\sqrt{R^2+(2\pi nL)^2}$; the condensance is $\frac{1}{2\pi nC}$; hence the charging current, $I = E2\pi nC$.

734. A distance of 18 ins. between wires is found sufficient for a pressure of 10,000 volts, and has been widely adopted in electric-railway lines. To save calculation TABLE No. 88 is calculated from the preceding formulæ (originally prepared by the General Electric Co.), containing data for wires from Nos. 0000 to 8 for an interaxial distance of 18 ins.

735. The way is now prepared to exhibit the complete calculation of a transmission line, which may be best accomplished by an example.

TABLE No. 88.

Constants for Transmission Lines.

Wire.				Constants for 1 Mile of Wire.				Reactance r in Ohms at			
Number, B. & S.	Diameter, D, Mils.	Radius, r, Mils.	Area, Circular Mils.	Commercial Resistance, R, Ohms.	Inductance, L, Millihenrys.	Capacity, C, Microfarads.	Charging Current, I, Amps., 100 cycles.	25 Cycles.	40 Cycles.	60 Cycles.	125 Cycles.
0000	.460	.230	211600	.266	1.484	.0210	.1320	.2331	.3735	.5603	1.166
000	.410	.205	167805	.335	1.521	.0198	.1244	.2400	.3835	.5750	1.200
00	.365	.183	133079	.422	1.558	.0195	.1225	.2453	.3920	.5880	1.224
0	.325	.163	105592	.533	1.596	.0189	.1186	.2512	.4030	.6025	1.225
1	.289	.145	83694	.671	1.633	.0186	.1167	.2563	.4090	.6160	1.282
2	.258	.129	66373	.845	1.671	.0180	.1130	.2629	.4200	.6300	1.314
3	.229	.115	52633	1.067	1.708	.0176	.1105	.2695	.4285	.6440	1.340
4	.204	.102	41742	1.364	1.744	.0173	.1086	.2750	.4390	.6585	1.376
5	.182	.091	33102	1.700	1.781	.0169	.1061	.2808	.4485	.6740	1.404
6	.162	.081	26250	2.138	1.820	.0165	.1036	.2865	.4575	.6875	1.430
7	.144	.072	20816	2.689	1.857	.0162	.1017	.2918	.4670	.7000	1.456
8	.128	.064	16509	3.406	1.894	.0158	.0992	.2980	.4770	.7150	1.468

Basis of Table.

Interaxial distance, 18 ins.

$L = .805 + .741 \log \frac{18}{r}$;

$C = \frac{.0388}{\log \frac{18}{r}}$;

$I = 2 \times 3.1416 \times n \times CE \times 10^{-6}$, when $n = 100$ and $E = 10000$;

$I = 6.28 \times 100 \times C \times 10000 = 6.28C$;

E = e.m.f. between line and neutral; that is,
20000 volts single phase;
17300 volts triphase;

$r = 2 \times 3.1416 \times n \times L \times 10^{-3}$;

$r = \frac{6.28 \times n \times L}{1000}$.

Assume

(1) Amount of power to be transmitted 4,000 H.P.—3,000 kw. (approximately).

(2) Distance $L = 15$ miles.

(3) Permissible total loss 25 per cent.

(4) Character of service mixed load, lights and motors. Power factor, say 85 per cent.

(5) System triphase. Δ connected. 60 cycles.

(6) Pressure at receiving end 15,000 volts between wires.

(7) Transformers to have the following average characteristics:

Efficiency	97	per cent.
Copper Loss	1	"
Hysteresis	2	"
Reactance	3½	"
Magnetizing Current	4	"

(8) Distributing mains:

Resistance.	3	per cent.
Inductance.	3	"

The following general characteristics can at once be obtained:

Pressure between any wire and neutral . . . . . $\frac{25000}{\sqrt{3}} = 14430$ volts.

Energy delivered by each branch. . . . . $\frac{3000}{3} = 1000$ kw.

Volt-amperes delivered by each branch. . . . . $\frac{1000000}{.85} = 1176470$

Amperes in each branch. . . . . $\frac{1176470}{14430} = 81.6$, say 82.

Assume drop due to ohmic resistance in each branch to be 10 per cent. of pressure to neutral or $\frac{14430}{10} = 1443$ volts.

As this drop is equal to RI, $R = \frac{1443}{82} = 17.66$ ohms.

As each branch is 15 miles long the resistance per mile is $\frac{17.66}{15} =$ 1.174 ohms.

From TABLE No. 88 this mileage resistance is seen to be between a No. 3 and No. 4 wire. It is preferable to choose the larger commercial size to allow for sag and resistance of joints, etc.

From TABLE No. 88 the following mileage constants are obtained for a No. 3 wire:

Resistance.	1.067	ohms
Inductance.	1.708	henrys
Capacity.	.01761	mf. for one wire
Reactance.	.644	ohms

Charging current $= 6.28 \times 60 \times .0176 \times 14430 \times 10^{-6} = .0956$ amperes.

Each branch will have the following properties:

Resistance.	$1.067 \times 15 = 16.05$	ohms
Inductance.	$1.709 \times 15 = 25.62$	henrys
Capacity.	$.0176 \times 15 = .264$	mf.
Reactance	$.644 \times 15 = 9.66$	ohms
Charging current	$.0956 \times 15 = 1.35$	amperes

To trace the distribution of the *E.M.F.* current and the phase relations, TABLE No. 89 may be formed. To calculate this table the whole system is to be reduced to a uniform pressure by multiplying each portion by the corresponding ratio of transformation, thus reducing all parts to the line pressure. Obviously current quantities are changed in an inverse ratio. Then the table is calculated by starting with the secondary circuit and calculating the energy and induction components, and current values for each portion successively, and summing as the work progresses. The calculation could be made graphically by the methods of Chapter IX equally well.

TABLE No. 89.

Example of Transmission-line Calculation.

CIRCUIT.	VOLTAGE COMPONENTS.		CURRENT.
Secondary Circuit.	Energy.	Induction.	
Energy component ..... 85% × 14430	12265.5		
Induction ..... 52% × 14430		7503.6	
Current	.....	.....	82.0
STEP-DOWN TRANSFORMERS.			
Resistance loss ..... 1% × 14430	144.3		
Reactance loss ..... 3½% × 14430	.....	505.05	
Hysteresis loss ..... 1½% × 82	.....	.....	1.23
Data at high-tension side of transformers	12409.8	8008.65	83.23
LINE.			
Resistance loss ..... 16.05 × 83.23	1335.84		
Reactance loss ..... 9.66 × 83.23	.....	804.0	
$\sqrt{(13745)^2+(8812)^2}=16329$ volts at terminals of step-up transformers.	13745.64	8812.65	83.23
STEP-UP TRANSFORMERS.			
Resistance loss ..... 1% × 16329	1632.9		
Reactance loss ..... 3½% × 16329	.....	571.5	
Hysteresis loss ..... 1½% × 83.23	.....	.....	1.25
$\sqrt{(15318)^2+(9384)^2}=18015$ volts at generator terminals.	15378.54	9384.15	84.48

From this analysis it is seen that the pressure between the leads at the generator must be $18015\sqrt{3}=31202$ volts.

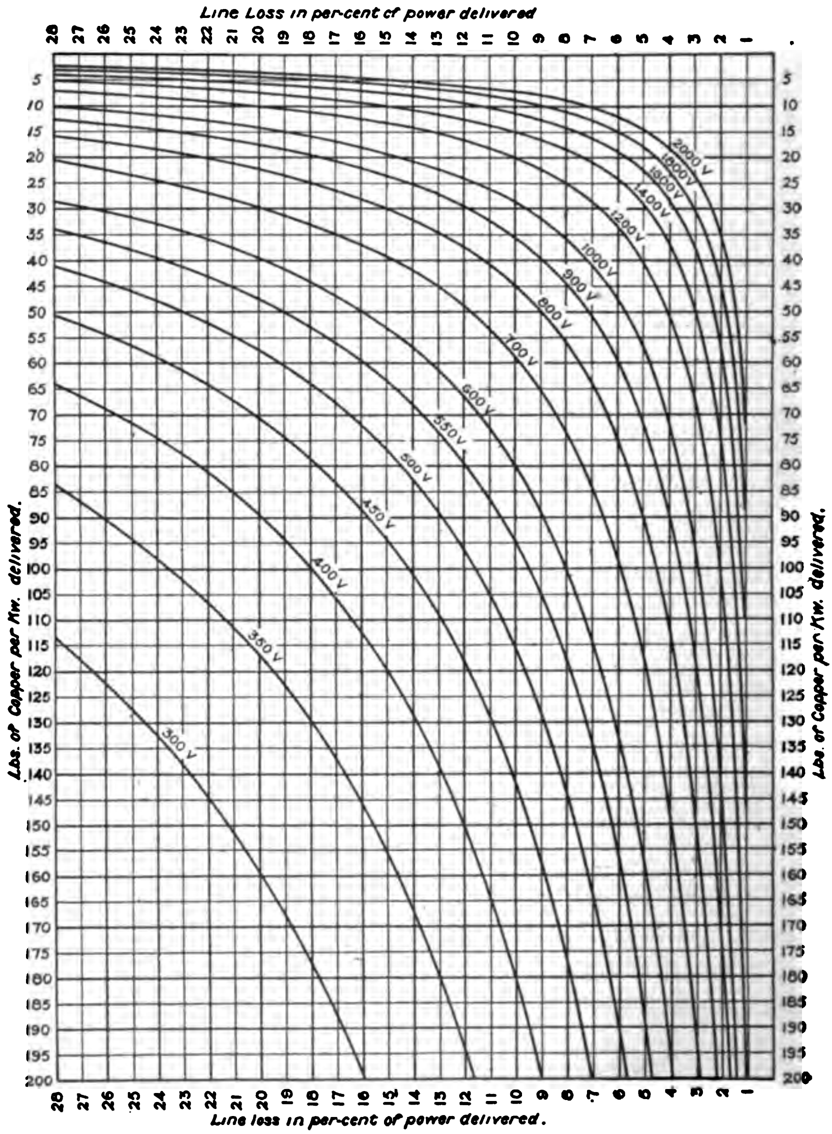
The energy delivered by the generator is $15378 \times 84.5 \times 3 = 3898.3$ kw. The energy in the secondary circuit is $12265.5 \times 82 \times 3 = 3017.3$ kw.

$$\text{Power efficiency} = \frac{3017.3}{3898.3} = 77.4 \text{ per cent.}$$

$$\text{Power factor} = \frac{12265.5 \times 82}{18015 \times 84.5} = 66.00 \text{ per cent.}$$

736. The following method of rapidly arriving at the quantity of conductor material for a given line, and determining the size of wire from the weight, has been developed by the General Electric Co. As

TABLE No. 90.



the weight of electric conductors decreases as the square of the potential employed and increases as the square of the distance, one may divide the potential by the distance, and obtain a convenient figure which can be used for all potentials and distances. The curves of TABLE No. 90 furnish a ready means of obtaining the amount of copper required for a given power transmission. The figures on the curves indicate volts per mile, i.e., potential of line at generator divided by distance in miles. The weight of copper, potential and line loss, are in terms of the power delivered at the end of the line, and not of generated power. The curves are only for three-phase current with 100 per cent. power factor. Two-phase, single-phase, or continuous current transmission requires one third more copper. Five per cent. has been allowed for sag, in weights of copper given.

EXAMPLE.—Assume that 1,000 kw. at 10,000 volts are to be delivered over a three-phase line 10 miles long with 5 per cent. loss, $\frac{10{,}000 \text{ volts}}{10 \text{ miles}} = 1{,}000$ volts per mile. Looking for the 1,000-volt curve we find 5 per cent. line loss corresponds to 57 lbs. of copper per kilowatt delivered.

1,000 kw.$\times$57=57,000 lbs. If copper costs 20 cents a pound, the cost will be 57,000$\times$\$.20=\$11,400.

The proper size of wire is determined by dividing total weight by total wire mileage, and ascertaining the size from TABLE No. 5 as follows:

$$57{,}000 \div 3 \times 10 = 1{,}900 \text{ lbs. per mile.}$$

This is a little larger than No. 0 wire and a little smaller than No. 00 wire.

CHAPTER XVIII.

THE COST OF PRODUCTION AND DISTRIBUTION.

737. The problem of determining the cost, either of an installation for the distribution of energy, or the price of producing that energy, is one containing so many factors, each of which are variables within so wide limits, and are so modified and controlled by local circumstances, that a general solution is an impossibility. Yet to afford some assistance toward an approximate general solution under conditions which are likely to be frequently realized, and to enable the designer to obtain figures necessary for the application of the

TABLE No. 91.

Cost of Conductors.

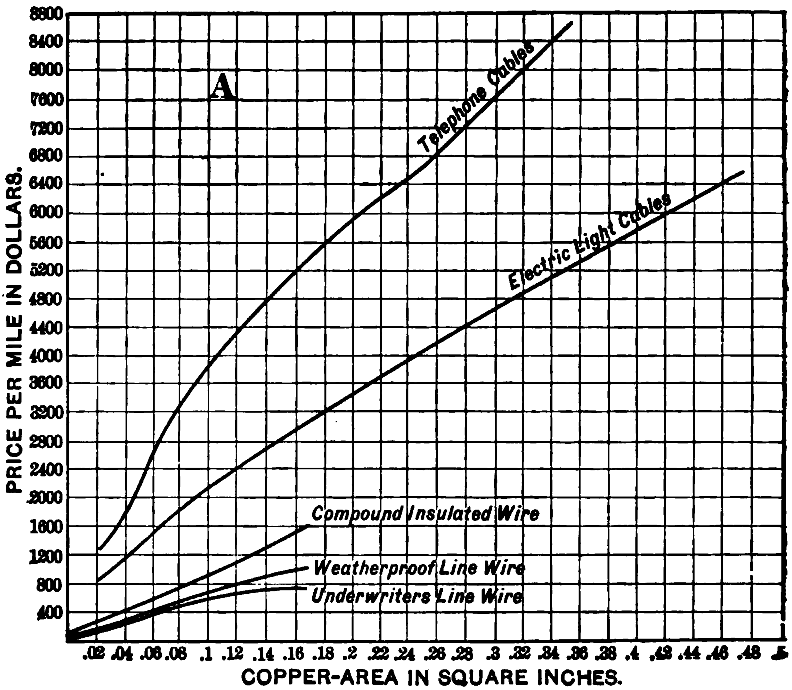
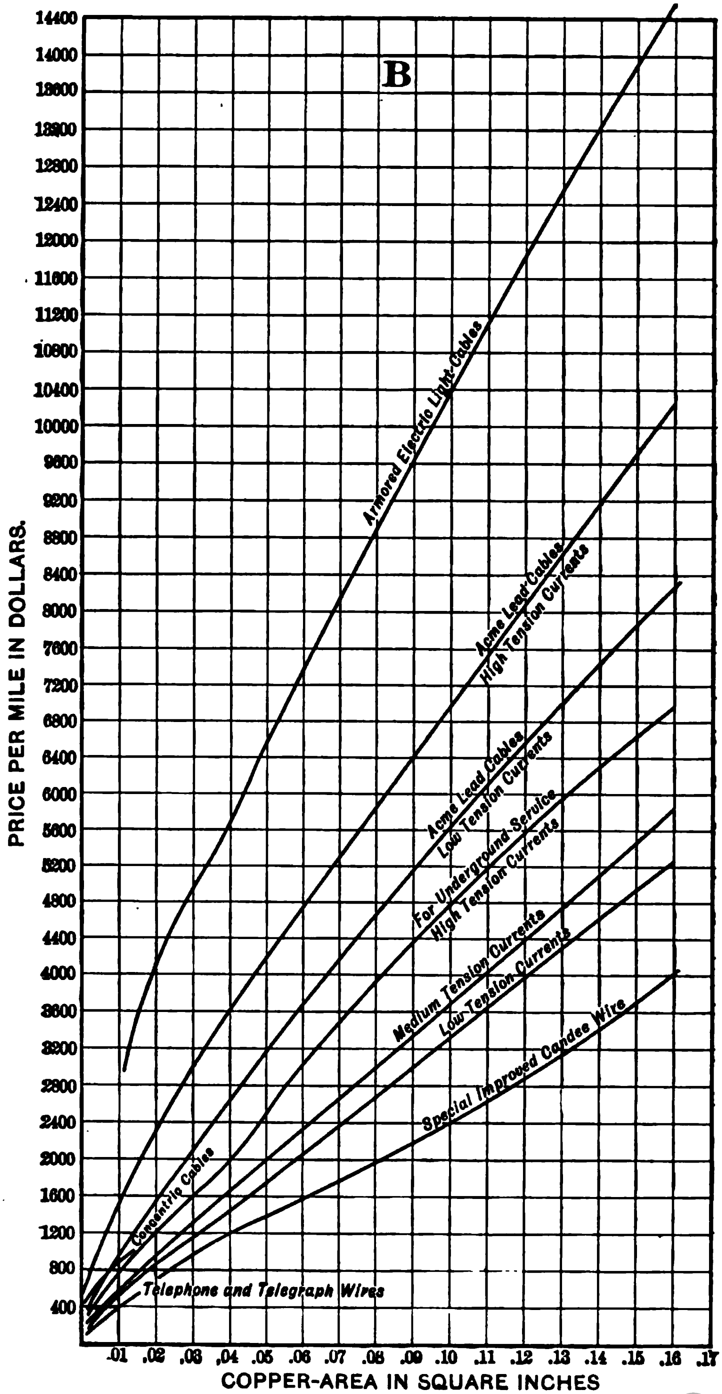


TABLE No. 91 (*Continued*). Cost of Conductors.

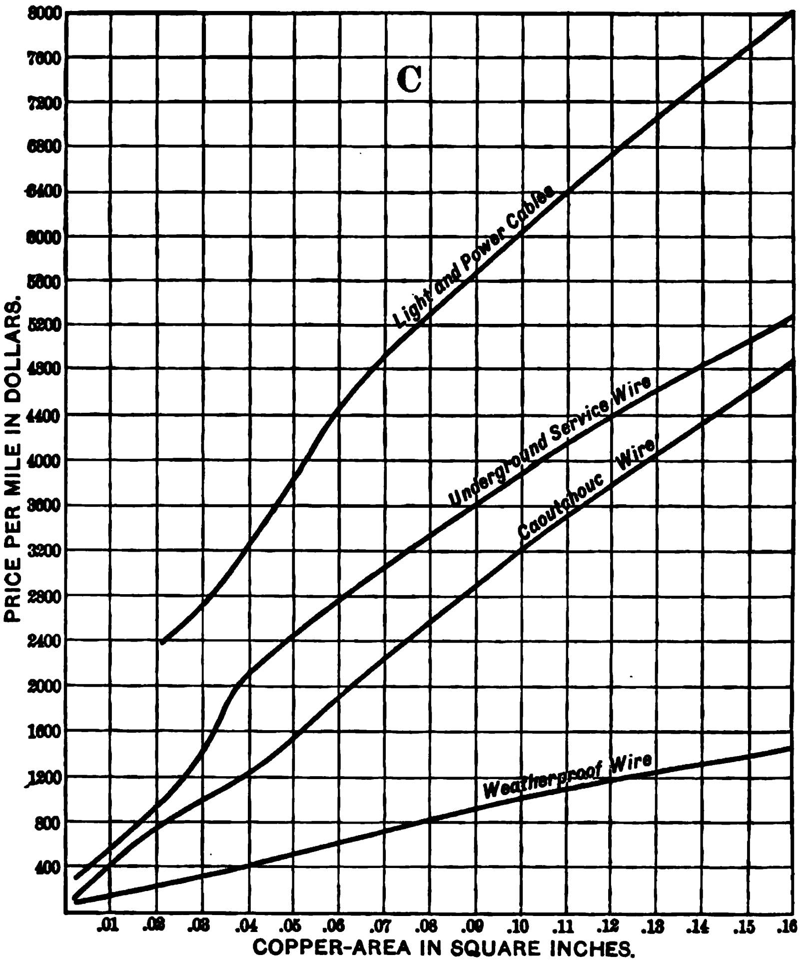


economical formulæ given in Chapters XIV. and XV., the following data are presented.

738. Cost of Conductors. — In Chapter XIV., it has been shown

TABLE No. 91 (*Continued*).

Cost of Conductors.



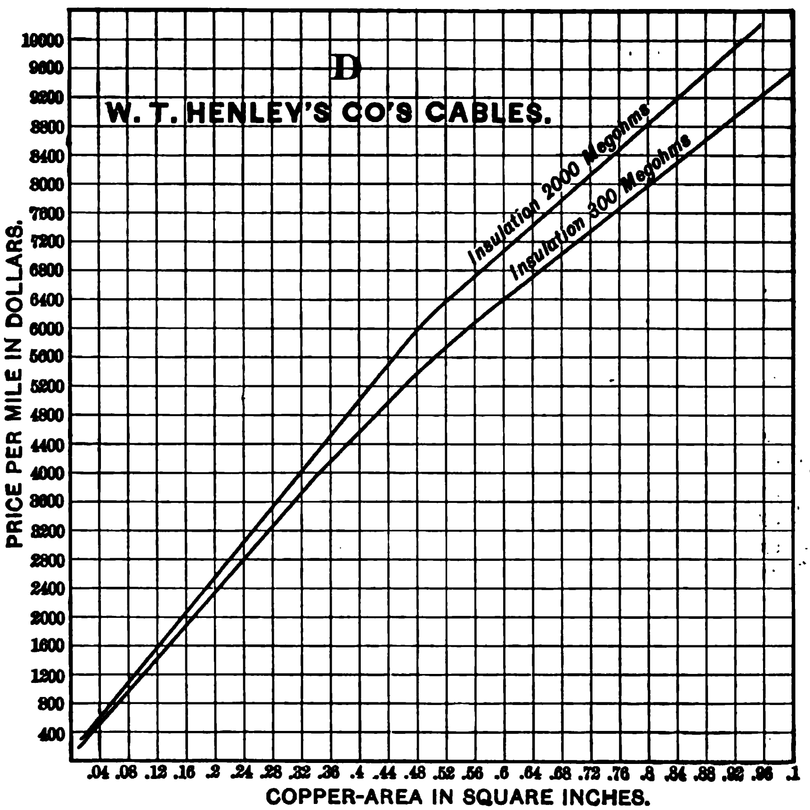
that the cost of conducting mains may be expressed by an equation of the form of $y=a+bx$.

In sheets A, B, C, D, E, and F, TABLE No. 91, are given the

curves calculated by the foregoing equation, for a number of the more common electrical conductors. In determining these curves, the cost of copper has been estimated at seventeen cents per pound, and the cost of the various kinds of insulation determined from the manufacturer's current price-list, without any attempt at the inclu-

TABLE NO. 91 (*Continued*).

Cost of Conductors.

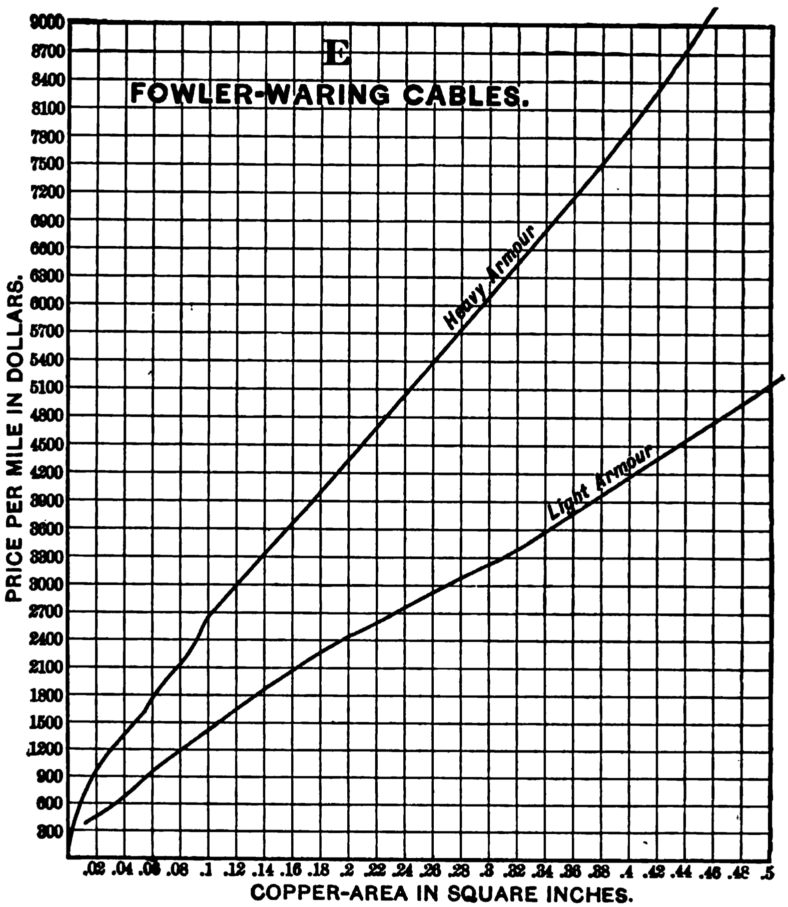


sion of the various trade discounts, which are factors too uncertain to be embraced in tabular values of this kind. All of the curves are plotted with the axis of x as the function of conductor area in square inches, while the axis of y indicates the cost in dollars, per mile, for the corresponding areas. On sheet A will be found curves for

underwriters' line wire, weather-proof wire, compound insulated wire, electric-light cables, and telephone cables. In the case of the first four curves, the copper areas are square inches of conductor section for the whole cable. In the case of the telephone cables, the copper

TABLE No. 91 (*Continued*).

Cost of Conductors.

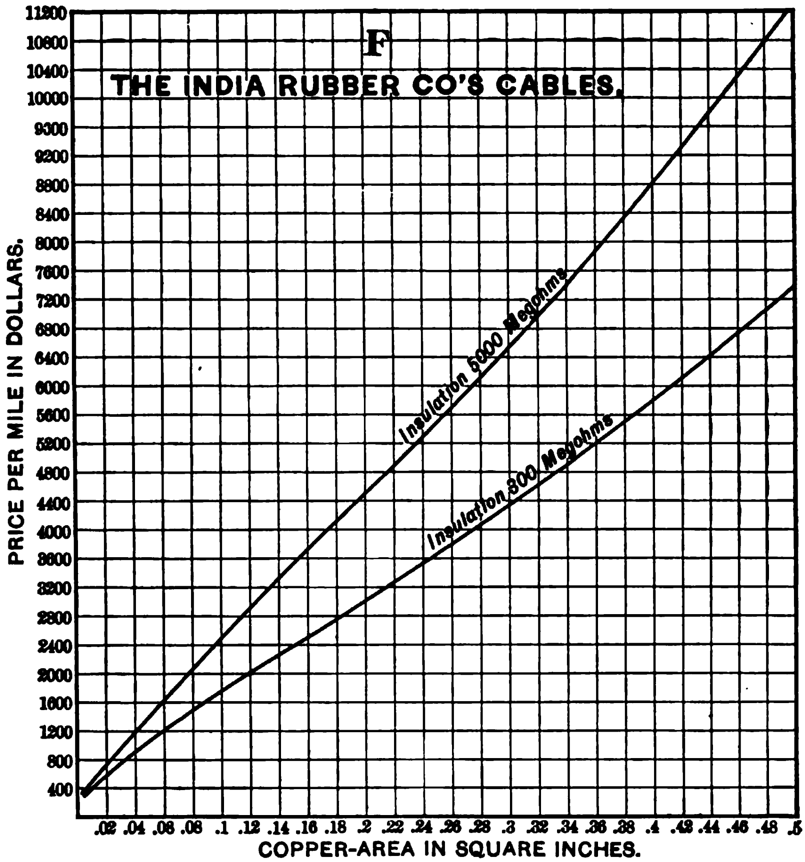


area is the sum of the areas of all of the conductors included in the cable, it being hardly necessary to state that the individual conductors are insulated from each other, in order to render them applicable to telephone service. In sheets B and C, two other series of curves

are given. In sheets D, F, and G will be found some of the more prominent cables made by English manufacturers; the prices, however, correspond very closely to those of American make.

TABLE No. 91 (*Continued*).

Cost of Conductors.

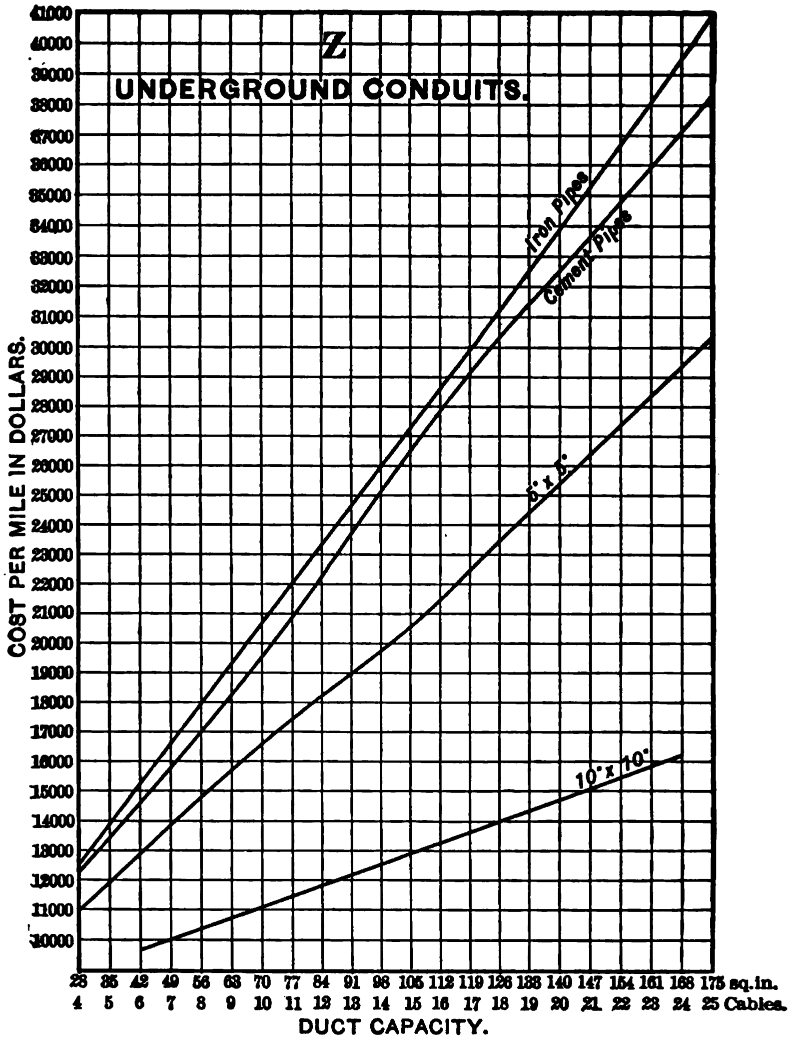


739. Cost of Conduits.—In Chapter XIV. it was shown that the cost of conduit systems could be expressed by an equation similar to that of copper conductors of the form $y = a' + b'x$.

On sheets W and Z, TABLE No. 92, curves for the cost of underground conduits more commonly used in this country, consisting of $10'' \times 10''$ terra-cotta pipe, $5'' \times 5''$ single-duct terra-cotta pipe, cement-lined

TABLE No. 92.

Cost of Conduit.



pipe, and iron pipe, are indicated in a similar manner. In calculating the cost of conduit, the prices of ducts, per foot, are assumed as they would average at any one of the larger cities, either along the Atlantic Coast, or east of the Mississippi River. Prices of labor are estimated

at $2.00 per day; the cost of repaving for macadamized streets at 60 cents per square yard, wooden paved streets at $1.00 per square yard, and granite paved streets at $1.50 per square yard, 10 per cent. allowance being made for waste and loss in paving materials. The manholes have been estimated at intervals of 300 ft., and are supposed to be 5 ft. wide, 7 ft. long, and 5 ft. in depth, with 8″ common brick walls laid in hydraulic cement. The conduits have been estimated at an average depth of 3 ft. below the surface of the ground, with no special allowance for the removal or replacement of complicated underground structures.

740. While the data given on sheets W and Z are average conduit cost, it is impracticable to express the cost of conduit as a simple linear function of the number of ducts installed, because a very slight consideration will show there are a number of items entering into conduit expense which do not vary with the number of ducts, or at least not in any direct proportion. The cost of manholes, engineering expenses, watching, and lighting streets, will be entirely independent of the number of cable spaces provided in the conduit. To afford sufficient room for the workmen to perform their avocations, a trench at least 18 inches in width must be opened and corresponding strips of pavement removed. This is irrespective of whether one or many ducts are to be installed therein. So an accurate estimate of conduit cost can only be obtained by taking the separate items and combining them for each case by itself.

741. Cost of Manholes.—What may be termed the standard manhole is a chamber about 7 ft. long, 5 ft. wide, and 5 ft. high. If built of brick the minimum price will be $50.00, the average price $78.00, and the maximum price $109.00, varying with the locality and cost of labor and material. If constructed of concrete the minimum price will be $44.00, the average price $72.00, and the maximum $104.00. To install a sewer connection in the bottom of a manhole will for minimum price cost $13.00, average $26.00, and for maximum $42.00. Based on the preceding prices the cost per street foot for manholes will depend upon the distance allowed between them, as shown in TABLE No. 93.

TABLE No. 93.

742. **Cost per Street Foot for Manholes in Dollars.**

Items.	Price.	Distance between Manholes in Feet.				
		250	300	350	400	500
Brick manholes	Minimum	.238	.196	.170	.148	.118
	Average	.372	.310	.248	.236	.186
	Maximum	.536	.427	.384	.335	.268
Concrete manholes	Minimum	.176	.148	.127	.111	.089
	Average	.278	.242	.209	.180	.144
	Maximum	.416	.347	.298	.260	.208
Sewer connection	Minimum	.051	.043	.038	.032	.025
	Average	.104	.086	.074	.064	.051
	Maximum	.170	.142	.121	.105	.084

743. **Cost of Street Work.**—Experience has shown that the cost of removing and replacing pavements, the necessary excavations, engineering expense, and removal of obstacles is sensibly the same for all conduits containing from 1 to 9 ducts. For from 10 to 16 ducts the cost of the above items is increased, because a wider and deeper trench is needed, but cost is sensibly constant for subways between these limits. Similarly for from 17 to 25 ducts expense is again augmented. TABLE No. 94 shows the cost of these items per street foot.

TABLE No. 94.

744. **Cost per Street Foot for Exc vation, Paving, Engineering, Removal of Obstacles, etc.**

Item.	Minimum Price.	Average Price.	Maximum Price
1-9 ducts......	.33	.67	1.61
10-16 "	.41	.82	1.87
17-25 "	.52	1.03	2.34

745. **Cost of Duct Material.**—The cost of duct material in place, including labor of laying, necessary concrete, etc., is found to be as in TABLE No. 95.

TABLE No. 95.

746. **Cost per Duct Foot of Duct Material in Place.**

Item.	Minimum.	Average.	Maximum.
Single-duct conduit.....	.046	.095	.145
Multiple duct..........	.061	.10	.139
Creosoted wood	.04	.051	.063

747. Cost of Pole-Lines. — In TABLE NO. 96. the average cost of the construction of pole-lines such as would be suitable, of cross-arms complete with insulators, and the cost of stringing bare wire, not over No. 8 gauge, either iron or copper, is indicated. From the figures here given, it is reasonable to assume that an average estimate of the cost of line construction may be made when the number of wires to be strung is determined upon.

748. Railway Lines. — The cost of building electrical railway lines may be divided into two parts, the cost of the trolley line proper, and the cost for the necessary feeds, and the erection of the same. A trolley line for a double-track road with wooden poles, using

TABLE NO. 96.

Cost of Pole Lines.

Cost per mile of Line Poles set, but *without* arms.
Cost of one *ten*-pin arm, complete with insulators on pole, $1.30, common.
Cost of one *ten*-pin arm, complete with insulators on pole, $1.90, yellow pine.
Cost of stringing wire per mile, $2.50 to $10.00.
The cost of "*Anchor Poles*" is the cost for *each* pole complete.

LOCATION.	HEIGHT OF POLES.							
	30 Ft.		40 Ft.		45 Ft.		50 Ft.	
	Line.	Anchor.	Line.	Anchor.	Line.	Anchor.	Line.	Anchor.
City Line, Favorable Circumstances . .	$500	$135	$675	$150	$800	$160	$1050	$175
City Line, Unfavorable Circumstances . .	550	160	725	180	875	190	1200	200
Country Line, Favorable Circumstances .	250	. .	400	. .	500	. .	775	. .
Country Line, Unfavorable Circumstances	300	. .	450	. .	575	. .	900	. .

one of the cheaper style brackets, may be built for from $800.00 to $1,200.00 per mile. If iron poles are used, the cost will be from $1,200.00 to $1,600.00 per mile. These figures presuppose center-pole construction. If side-pole construction is used, the cost for wooden poles will be increased by about $200.00 per mile, while the cost for iron poles will be augmented by from $400.00 to $600.00 per mile. The cost for stringing feed-wires, including insulators, pins, arms, etc., will vary from $50.00 to $100.00 per mile, depending upon the number of feeds and the size of the wire. A detailed estimate of the cost of overhead-line work will be found in TABLE NO. 97.

TABLE NO. 97.

Estimate of Cost for One Mile of Street Railway Line.

Items.	Unit Prices.	SINGLE TRACK.				DOUBLE TRACK.			
		Ordinary Span Construction.		Bracket Construction.		Ordinary Span Construction.		Bracket Construction.	
		Quantity.	Amount.	Quantity.	Amount.	Quantity.	Amount.	Quantity.	Amount.
Wooden Poles, set	$6.00	Num. 88	$528.00	Num. 44	$264.00	Num. 88	$528.00	Num. 88	$528.00
Pins and Insulators	.07	Num. 44	3.08	Num. 44	3.08	Num. 44	3.08	Num. 44	3.08
Span Wire	.01	1,800 ft.	18.00	...	...	1,800 ft.	18.00	...	...
Trolley Wire, No. 0, B. and S.	.14	1,700 lbs.	238.00	1,700 lbs.	238.00	3,400 lbs.	476.00	3,400 lbs.	476.00
Feed Wire, No. 00, B. & S.	.13½	2,400 lbs.	324.00	2,400 lbs.	324.00	2,400 lbs.	324.00	2,400 lbs.	324.00
Eye Bolts, ordinary	.12	Num. 88	10.56	...	...	Num. 88	10.56	...	...
Eye Bolts, special	.50	...	...	...	...	...	...	...	...
Brackets erected	3.50	...	...	Num. 44	154.00	...	...	Num. 88	308.00
Line Insulators	.75	Num. 44	33.00	...	...	Num. 88	66.00	...	...
Line Insulator Brackets	1.00	...	...	Num. 44	44.00	...	...	Num. 88	88.00
Labor Running Trolley Wire	...	...	75.00	...	75.00	...	150.00	...	150.00
Labor Running Feed Wire	...	...	35.00	...	35.00	...	50.00	...	50.00
Labor Running Span Wire	...	...	50.00	...	...	...	50.00	...	...
Totals for Wooden Pole Construction	...	...	1,314.64	...	1,137.08	...	1,675.64	...	1,927.08
Totals for Iron Pole Construction, including Insulators	22.00	...	2,722.64	...	1,841.08	...	3,083.64	...	3,335.08
Totals for Lattice Iron Pole Construction, including Insulators	31.00	...	3,514.64	...	2,237.08	...	3,875.64	...	4,127.08

749. Electric-Railway Expenses.—The cost of operating electric railways depends upon the road, the kind of service and care and

Table No. 98.

Electric-Railway Operation.

Item.	Cost per Mile of Track.		Cost per Car Mile.	
Transportation:				
Superintendence	$112.00		$.00236	
Labor on cars	2107.00		.04435	
Other labor	251.00		.00528	
Sundries	229.00		.00483	
Total		$2699.00		$.05682
Power Expense:				
Wages, cost	$199.00		$.00418	
Fuel, cost	555.00		.01167	
Miscellaneous cost	75.00		.00200	
Total		829.00		.01785
Maintenance:				
Way	$353.00		$.00742	
Electric line	128.00		.00269	
Steam-plant	59.00		.00123	
Electric generating plant	35.70		.00075	
Car body and truck	331.00		.00696	
Car motors	230.00		.00484	
Buildings	43.00		.00091	
Miscellaneous	65.00		.00137	
Total		1244.70		.02617
General Expense		1116.00		.02348
Total cost		$5888.70		$.12432
Income:	*Per Mile of Track.*		*Per Car Mile.*	
Passengers	$10,650.00		$.21300	
Freight	47.20		.00094	
Mail	19.64		.00039	
Express	18.24		.00036	
Sale of current	350.00		.00702	
Miscellaneous	175.00		.00350	
Total		$11,260.08		$.22521

Ratios:

Passenger income to total income	94.6%
Income from other transportation business to total income	2.3%
Income from sale of current to total income	3.1%
Operating expenses to income	57.5%

Power Cost:	*Per K.W.H.*	
Wages	$.00203	
Fuel	.00568	
Miscellaneous	.00081	
Total		$.00852

Income per mile of track	$11,260.08	Per car mile	$.22521
Operating expenses per mile of track	5,888.70	" " "	.12432
Income above operating expenses	$5,371.38	" " "	$.10089

skill exercised in administration, and also varies very greatly with the type of the road, whether it be an urban, suburban, or interurban line. The U. S. Census Office has just completed an analysis of all the electric railways in the United States, from which the data in TABLE No. 98 are compiled, giving averages of the operating data from all the electric railways in this country.

750. Third-Rail System.—According to present practice it is customary for electric railways using the third-rail system to establish rotary sub-stations at intervals of from 10 to 20 miles and to feed from each sub-station direct current in both directions over a distance of from 5 to 10 miles, so that the third rail becomes at once the conductor and distributor for all electricity delivered to the cars. To install the third rail it is necessary to provide for each mile about 500 extra-long ties, a corresponding number of insulators, the third rail, its fastenings and bonds, and the necessary labor of installing the same. The cost of the ties, insulators, bonds, and labor will be approximately the same whatever size or form of rail is used. Theoretically the conducting power required in the rail will depend upon the number, weight, and speed of the cars to be operated on each section of road, and the number of stops and rate of acceleration. The selection of the rail is very often governed by local market conditions, which may enable the railway to purchase second-hand rails of perhaps a considerably greater section than is indicated by the actual quantity of conductor material required for the particular service. Many third-rail systems have purchased so-called "relayer rails," namely those which have previously served a reasonable life as running-rails, and while possibly too much worn for service in the main road-bed, are amply sufficient to serve as conductors and distributors for current to the cars. It has been shown in Chapter III that the conductivity of steel varies very greatly with its constitution, and this factor must not be forgotten in designing the proper section to be given to a third rail.

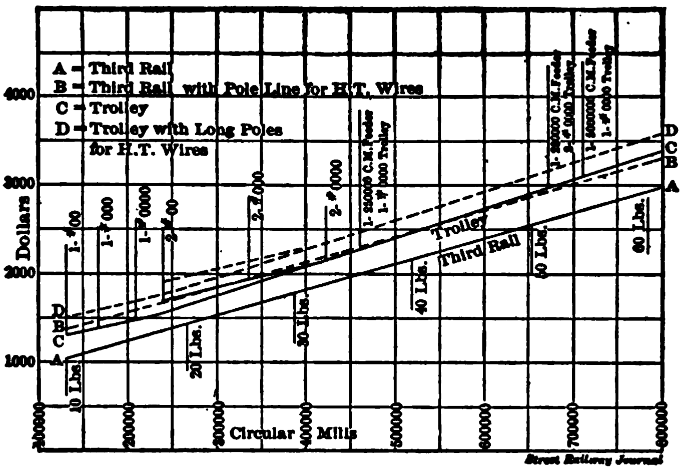
751. TABLE No. 99, from a paper prepared by Mr. Gonzenbach, exhibits the relative and actual cost of the third rail and trolley lines of equal electrical conductivity.* There are four curves in this table, curve *A* showing the cost of the third rail alone, curve *B* that of the

* Street Railway Journal, vol. 21, p. 878.

third rail including pole line for high-tension wires, curve *C* gives the cost of the ordinary trolley line, while curve *D* includes the expense of tall poles to carry the high-tension lines.

TABLE No. 99.

Relative Cost of Third-Rail and Trolley Systems.



COMPARISON OF COST PER MILE OF OVERHEAD TROLLEY AND THIRD RAIL

752. Cost of Power-Stations.—The cost of power-stations, *excluding* the expense for real estate and buildings, varies greatly with the type of prime mover employed, with the conditions necessary to obtain good foundations, and with other local circumstances. The following figures are fair averages:

TABLE No. 100.

Cost of Power-Stations.

High-speed simple engines	$10.00 to $15.00 per H. P.
High-speed compound engines	12.00 to 20.00 per H. P.
Condensing engines	15.00 to 25.00 per H. P.
Triple-expansion engines	20.00 to 30.00 per H. P.
Boilers, horizontal iron tubular	9.00 to 12.00 per H. P.
Boilers, vertical iron tubular	11.00 to 14.00 per H. P.
Boilers, water-tube safety	15.00 to 25.00 per H. P.
Dynamos	20.00 to 30.00 per H. P.
Sundries	10.00 to 20.00 per H. P.

753. Averages taken from a large number of stations throughout the country show that a minimum cost of about $90.00 per kilowatt of electrical output is the least which can be expected, an average cost of $110.00 per kilowatt is frequent, while it is only the exceptional station which costs over $130.00 per kilowatt for station machinery. From statistics compiled by the United States Census the average cost of the electric-lighting plants in the United States is found to be $296.00 per horse-power of plant. This includes all the investment required to cover the entire construction of the generating plant, the distributing system, and the installation of lamps. Operating statistics are as shown in TABLE NO. 101.

TABLE NO. 101.

Average Statistics from United States Electric Plants.

Gross receipts per kilowatt hour	3.494 cents
Total expense per kilowatt hour	2.768 "
Labor expense	37.4 per cent
Fuel and supplies	41.4 "
Miscellaneous expense	21.2 "
Ratio of operating expenses to receipts	79.3 "

These costs include all expenses necessary to the production and distribution of electricity, excepting taxes.

754. The United States Commission of Labor has compiled statistics from over 950 electric-lighting plants of all descriptions. These statistics show installation costs in detail, and from them TABLE NO. 102 is compiled. Upon this table curve *A* shows the installation cost per H. P. of capacity. This installation cost covers land, buildings, and steam-plant, including engines and boilers; electric plant, covering dynamos, switchboards, and transformers, and the cost of the distributing system. The left-hand vertical axis of the table shows the average cost per H. P. for the entire plant, while the horizontal scale shows the rated capacity of the engines employed. Curve *A* shows that the cost of plant for engines of 50 H. P. is about $175.00 per H. P. The cost then drops to $120.00 with engines of 150 H. P., and thence the curve rises to a cost of about $290.00 with engines of 3000 H. P. There are two factors which effect the variation in cost with the size of plant and size of engine. Small plants are proportionally more expensive per unit of rated capacity than medium-sized ones, because

Table No. 100a. (See page 639.)

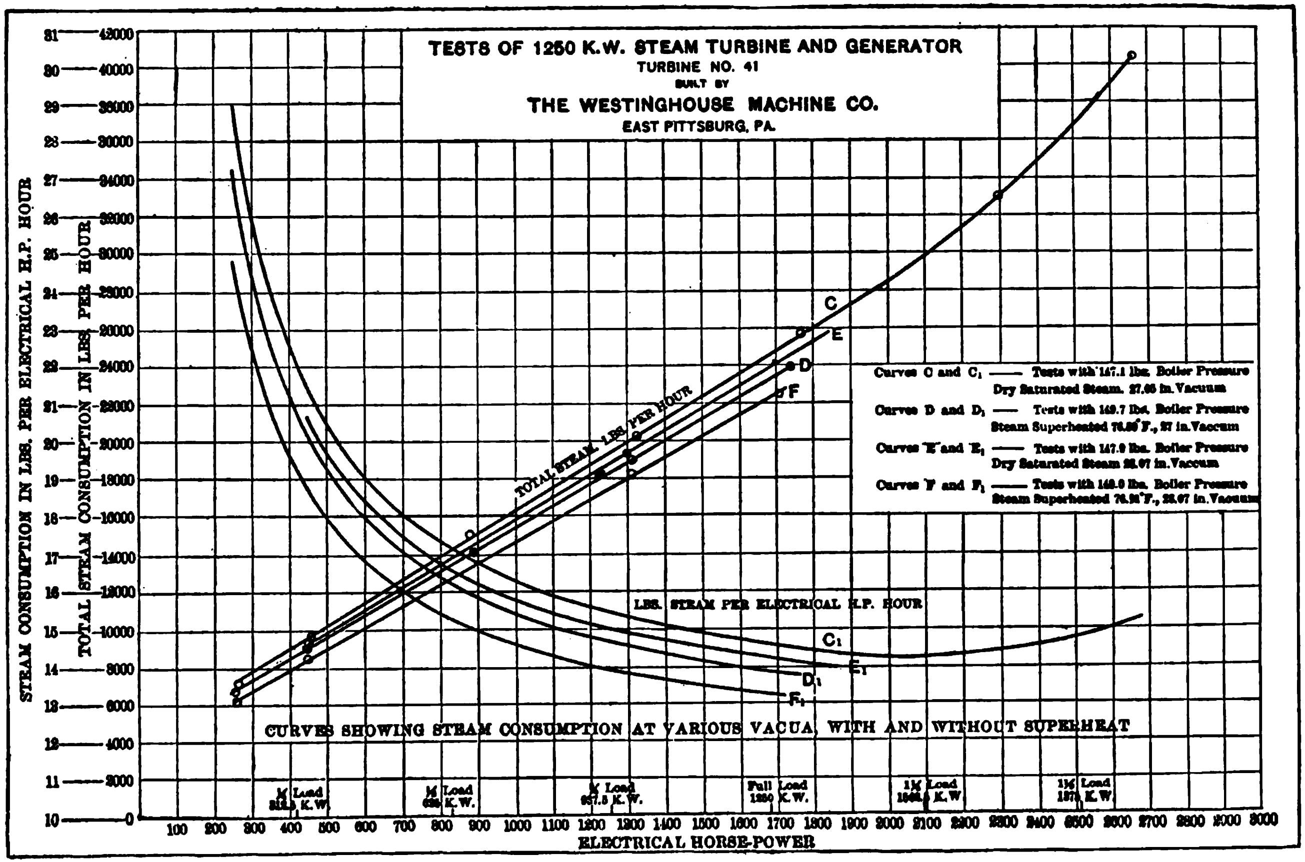
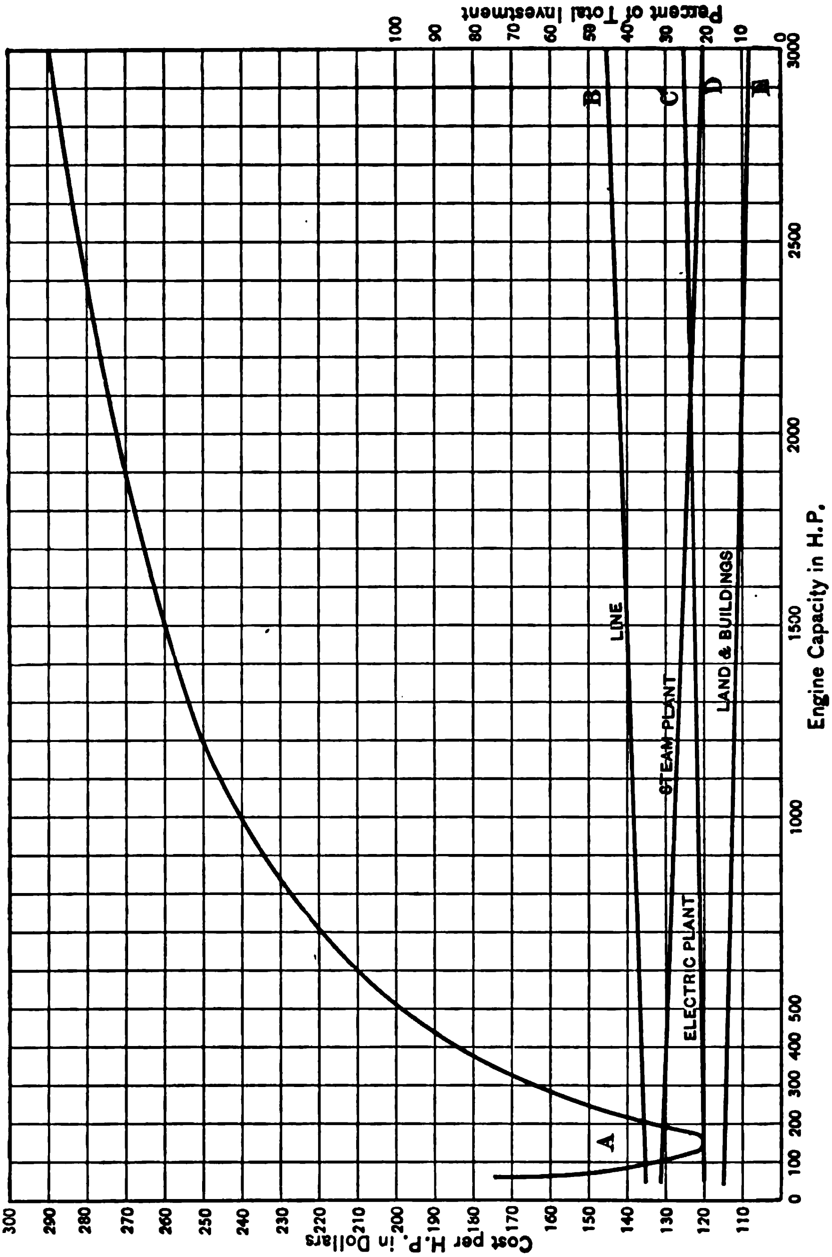


TABLE No. 102.


Percent of Total Investment
100
90
80
70
60
50
40
30
20
10
0
Cost per H.P. in Dollars
300
290
280
270
260
250
240
230
220
210
200
190
180
170
160
150
140
130
120
110
Engine Capacity in H.P.
0
100
200
300
400
500
1000
1500
2000
2500
3000
A
B
C
D
E
LINE
STEAM PLANT
ELECTRIC PLANT
LAND & BUILDINGS

the cost of machinery is greater for small sizes than for larger ones, and the corresponding expense of both buildings and real estate is greater; but as plants increase in size, while the cost per unit of capacity for central-station machinery, land, and buildings decreases, stations become more complicated and a greater variety of machinery is required, and that of a better grade is used. Also the distributing system becomes much more expensive for a large station than for a small one. This increase in expense is partly due to the necessity of employing a greater amount of conductor material to cover a large territory, but is also attributable to the necessity for a more expensive type of construction in a large city than in a small country town and to the greater complexity where large territories are covered; to the employment of an alternating-current system necessitating either static or dynamic sub-stations and corresponding increase in initial investment. TABLE No. 102 (see p. 639) contains, in addition, four other curves, denominated *B*, *C*, *D*, and *E*. The horizontal scale for these curves is the same as that for curve *A*, while the vertical scale, given on the right-hand side of the sheet, is written in percentage. The object of these four curves is to show what proportion of each size of plant may be attributed respectively to land and buildings, central-station steam machinery, central-station electric machinery, and distributing system.

755. Cost of Installing and Operating Arc-Lighting Plants.—Plants for supplying arc lighting form a special class of electric-lighting central stations: but the load line of arc plants differs from that of an incandescent station in showing little or no peak. City arc plants usually show nearly constant line from one hour after sunset to one hour before sunrise. Suburban plants often cut off commercial arcs at from 9 to 11 o'clock and often run on moonlight schedule or moonlight and sunset to midnight schedule, but even under such handicaps the load factor of arc plants is usually rather better than that of an incandescent plant in a similar location.

The cost of installing and operating an arc-lighting plant upon the various systems which are now in current use has been carefully studied by Mr. H. H. Waite.* TABLES Nos. 103 and 104 are compiled from Waite's investigations.

756. The Steam Turbine. — The steam turbine as a prime mover in place of the reciprocating steam-engine is attracting much atten-

* Trans. A. M. Inst. E. E., vol. xvi., p. 555.

TABLE No. 103.—Cost of Installing Arc Lamps of Various Systems. Cost per Lamp in Dollars.

Systems.	Real Estate.	Buildings.	Boilers.	Engines.	Generators.	Switchboard.	Transformers.	Arc Switchboard.	Line.	Lamp.	Machine Cost.	Total.
Alternating Constant Current Transformer, 400 Watt inclosed arc. . . .	$1.90	$5.60	$18.60	$14.90	A. C. $0.15	A. C. $0.60	$15.00	$0.20	$54.70	$20.00	....	$146.50
Alternating Constant Current Transformer, 450 Watt inclosed arc. . . .	2.05	6.16	20.50	16.45	16.60	.78	15.00	.20	57.50	20.00	....	155.24
A. C. Series Inductive Regulator System, 400 Watt inclosed arc. . . .	1.80	5.30	17.70	14.20	12.50	.50	4.90	.20	54.70	19.00	....	139.00
D. C. Arc Machines driven by A. C. Motors, 300 Watt inclosed arc. . . .	1.72	5.20	15.70	12.60	10.00	.50	A. C. motors 5.80	.20	51.10	18.00	Arc dynamos $10.60	131.42
Direct Current Belted Arc, 300 Watt inclosed arc	1.80	5.40	14.90	12.10	....	....	Transmission system 2.20	.20	51.10	18.00	Arc dynamos 12.20	117.90
Direct Current Arc Machines driven by D. C. Motors, 300 Watt inclosed arcs. . . .	1.75	5.30	16.10	12.80	8.10	.40	Constant current motors 5.10	.90	51.10	18.00	Arc dynamos 10.60	129.45
D. C. Belted Arc, 450 Watt inclosed arc. . . .	2.70	8.10	22.40	18.25	....	....	Transmission system 2.95	.20	57.50	18.00	Arc dynamos 16.90	147.00
Direct Current Belted Open Arc, 430 Watt arc. . . .	2.70	8.10	20.80	16.65	....	....	Transmission 3.30	.20	57.50	15.00	Arc dynamos 18.35	142.50
D. C. Direct Driven Arc, 450 Watt open arc. No Increase in Generating-plant. . . .	2.00	6.10	20.20	16.20	....	....	....	.20	57.50	18.00	Arc dynamos 21.80	142.00
A. C. Series Inductive Regulator System, 450 Watt inclosed arc. No Increase in Series Plant	....	....	....	....	....	....	5.40	.24	57.50	19.00	Regulator 9.00	91.14
D. C. Machines driven by A. C. Motors, 450 Watt inclosed arc. . . .	....	....	....	....	....	....	A. C. motors 8.80	.20	57.50	18.00	Arc dynamos 15.90	100.40
Rectifier System, 450 Watt inclosed arc. . . .	2.00	6.10	20.40	16.30	16.00	.80	15.00	.20	57.50	18.00	Rectifier 2.00	154.30
Incandescent High-tension Systems, Group of three 50 C. P. Lamps, 450 Watts	1.90	5.75	19.20	15.35	12.10	.60	....	....	53.00	3 lamps 2.25	Extra copper 11.60	121.75

TABLE No. 104.—Cost of Operating Arc Lamps per Year of 3,800 Hours in Dollars.

Systems.	Coal and Water.	Wages.	Supt. and Office.	Int., Taxes & Ins., 7% Depreciation, 6% Total = 13%.	Extra Dep. Act. Arc Lamp, 4%.	Repairs, Renewals, Supplies at $2.50 per 1 H. P.	Extra Repairs A. C. Trans., $20 per 1 H. P.	Trimming Lamps.	Carbons.	Globes.	Inspection.	Extra Repairs on Rectifier, $.40 per 1 H. P.	Total Operating Expenses.
Alternating Constant Current Transformer, 400 Watt inclosed arc.	$11.85	$8.50	$2.00	$19.00	$0.80	$1.86	$0.15	$5.20	$1.90	$1.60	....	....	$52.86
Alternating Constant Current Transformer, 450 Watt inclosed arc.	13.10	9.35	2.00	20.20	.80	2.26	.18	5.86	2.30	1.80	....	....	57.85
A. C. Series Inductive Regulator System, 400 Watt inclosed arc.	11.30	8.07	2.00	18.15	.76	1.77	....	5.20	1.90	1.60	....	....	50.75
D. C. Arc Machines driven by A. C. Motors, 300 Watt inclosed arc.	10.06	8.85	2.00	17.10	.42	1.57	.32	2.60	1.40	1.00	....	....	46.03
Direct Current Belted Arc, 300 Watt inclosed arc.	8.80	7.78	2.00	15.30	.49	1.36	.28	2.60	1.40	1.00	....	....	41.75
Direct Current Arc Machines driven by D. C. Motors, 300 Watt inclosed arc	10.20	9.00	2.00	16.50	.42	1.60	.32	2.60	1.40	1.00	....	....	46.06
D. C. Belted Arc, 450 Watt inclosed arc.	13.25	11.68	2.00	19.10	.68	2.07	.42	3.15	1.70	1.30	....	....	56.07
D. C. Belted Open Arc, 430 Watt arc.	13.30	11.70	2.00	18.50	.73	2.08	.42	(daily) 11.00	7.50	.70	....	....	68.53
D. C. Direct Driven Arc, 450 Watt open arc. No Increase in Generating-plant.	12.90	11.40	2.00	18.50	.83	2.02	.41	6.50	6.00	.50	....	....	62.58
A. C. Series Inductive Regulator System, 450 Watt inclosed arc. No Increase in Series Plant.	12.80	9.15	2.00	11.90	.76	2.03	....	3.15	2.30	1.80	....	....	45.89
D. C. Machines driven by A. C. Motors, 450 Watt inclosed arc.	15.10	13.30	2.00	13.20	.63	2.36	.47	3.15	1.70	1.30	....	....	53.93
Rectifier System, 450 Watt inclosed arc.	13.00	9.30	2.00	20.00	.72	2.04	.16	3.15	1.70	1.30	....	.32	53.69
Incandescent High-tension Systems, Group of three 50 C. P. Lamps, 450 Watts.	12.25	10.76	2.00	15.60	....	1.92	....	....	....	....	2.00	....	60.85

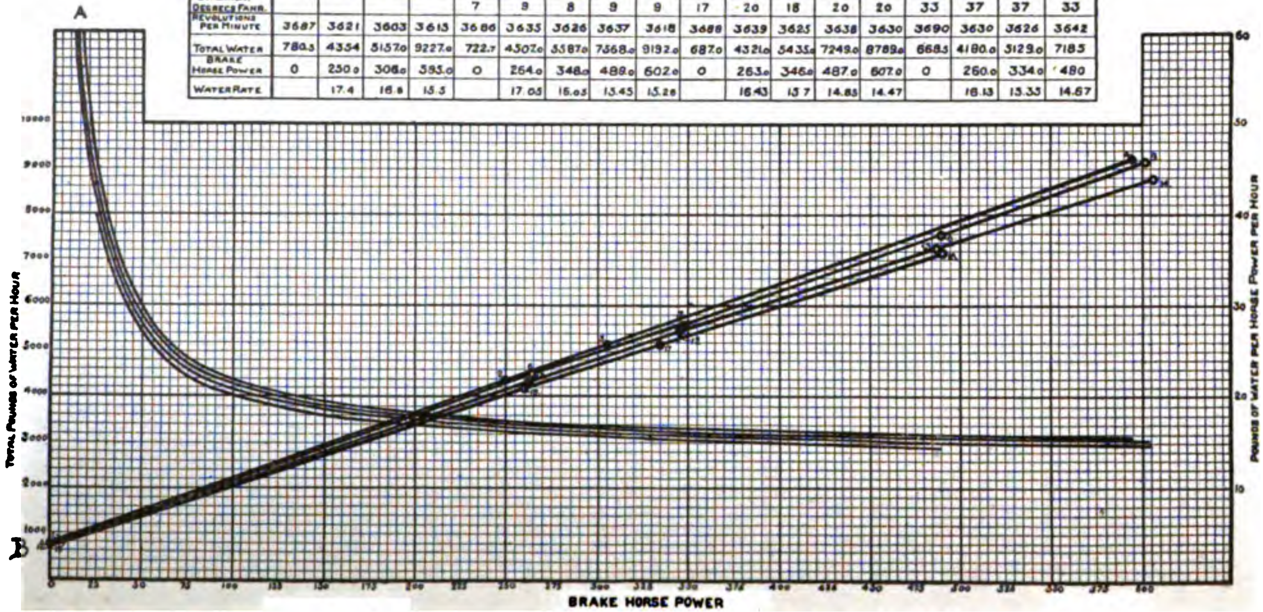
tion, but there are as yet not enough turbine installations from which extended experience can be drawn. The best obtainable information appears to indicate that the steam turbine may show a saving over the reciprocating machine of from 30 to 35 per cent. of coal, and that its efficiency extends over considerably wider ranges of load than has as yet been obtained in the reciprocating engine. TABLE No. 105 (see p. 644) is from a paper by Mr. E. H. Sniffin, presented at the meeting of the American Street Railway Association, October 8, 1892, and gives the result of a number of tests upon steam turbines, curve *A* showing the brake H. P. and relative steam consumption per hour, while the curve *B* gives the total steam consumption in proportion to the total power. Another marked advantage of the turbine is its economy in floor space and in cost of foundations. It is found that the turbine does not require more than 80 per cent. of corresponding vertical space and not over 40 per cent. of that of horizontal space taken by a reciprocating engine, while the cost of a turbine foundation is not more than one tenth of the cost of a similar engine foundation. To offset these advantages, however, the cost of the turbine itself is somewhat greater than that of the reciprocating machine, but it is probable that increased experience in the manufacture, together with inevitable competition, will sooner or later effect a reduction in installation prices.

757. The Cost of Producing Energy. — The cost of producing energy in the various stations throughout the country varies greatly with the price of labor and the cost of fuel. The most accurate information upon this subject has been calculated and compiled by Dr. C. E. Emery, and presented to Electrical Engineers in the Transactions of April, 1893. In this paper Dr. Emery summarizes all of the cost entering into production and maintenance of power for engines which are reasonably well loaded, and for stations of medium capacity, say 500 H. P. Taking Dr. Emery's figures as a basis, TABLE No. 106 has been calculated, showing the cost of delivering electrical energy at the terminals of the generators in the supply-station. Two sets of values are here given: one for 3,080 kilowatt hours per annum, equivalent to the operation of the station for 308 days of ten hours each; the other is the cost of the production of 7,300 kilowatt hours per annum, or equivalent to the operation of the station for 365 days of 20 hours each. In this latter table such a sufficient margin in capitalization is introduced as will provide for about 50 per cent. extra machinery

TABLE No. 105.

TEST OF WESTINGHOUSE-PARSONS STEAM TURBINE

No. of Test	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Boiler Pressure	148	147	147	152	149	149	148	149	148.5	150	155	148	149.5	153	150	150	148	152
Vacuum referred to 30" Barometer	27.4	28.04	27.85	27.06	27.38	27.4	27.1	26.7	26.7	27.37	27.54	27.3	27.3	26.94	27.35	27.3	27.3	2"3
% Moisture in the Steam	2.3	2.3	2.35	1.6														
Superheat Degrees Fahr.					7	9	8	9	9	17	20	18	20	20	33	37	37	33
Revolutions per Minute	3687	3621	3603	3613	3686	3635	3626	3637	3618	3688	3639	3625	3638	3630	3690	3630	3626	3642
Total Water	780.5	4354	5157.0	9227.0	722.7	4307.0	5587.0	7568.0	9192.0	687.0	4321.0	5435.0	7249.0	8789.0	668.5	4180.0	5129.0	7185
Brake Horse Power	0	250.0	308.0	595.0	0	264.0	348.0	488.0	602.0	0	263.0	346.0	487.0	607.0	0	260.0	334.0	480
Water Rate		17.4	16.8	15.5		17.05	16.05	15.45	15.26		16.43	15.7	14.85	14.47		16.13	15.35	14.67



to meet cases of break-downs, and to provide for special station loads. In the lower part of this table these costs have been further computed, showing, under each circumstance, and for each of the varying fuel prices, the cost of the production of one kilowatt hour.

TABLE No. 106.

Cost of Producing Electrical Energy per Kilowatt for 308 Days of 10 Hours, and 365 Days of 20 Hours.

Type of Engine.		308 DAYS OF 10 HOURS EACH. Cost of Coal per 2240 lbs.				365 DAYS OF 20 HOURS EACH. Cost of Coal per 2240 lbs.			
		$2.00	$3.00	$4.00	$5.00	$2.00	$3.00	$4.00	$5.00
Non-condensing.	Simple High Speed . .	$55.80	$65.28	$74.78	$84.25	$95.55	$117.00	$138.43	$159.90
	Compound High Speed	49.33	55.80	65.23	71.74	82.72	99.61	116.52	133.44
	Simple Low Speed . .	57.46	62.99	71.55	89.01	90.35	109.70	129.01	148.36
Condensing.	Simple High Speed . .	44.85	51.14	57.46	63.79	75.57	88.86	103.16	117.45
	Compound High Speed	43.93	49.68	55.41	61.16	71.88	84.87	97.88	110.87
	Simple Low Speed . .	44.88	50.77	56.67	62.58	72.40	86.40	99.07	112.40
	Compound Low Speed	43.75	49.06	54.36	59.64	69.43	81.44	93.44	105.34
Cost per Kilowatt Hour.									
Non-condensing.	Simple High Speed . .	1.8117	2.119	2.428	2.7355	1.3089	1.6028	1.8963	2.1905
	Compound High Speed	1.6015	1.8117	2.118	2.3294	1.1335	1.3647	1.5962	1.8278
	Simple Low Speed . .	1.8655	2.0450	2.323	2.890	1.2375	1.503	1.7685	2.0322
Condensing.	Simple High Speed . .	1.4560	1.6603	1.8655	2.071	1.0353	1.2172	1.4192	1.600
	Compound High Speed	1.4262	1.6130	1.7993	1.9858	0.9847	1.1628	1.341	1.519
	Simple Low Speed . .	1.4570	1.6475	1.8400	2.032	0.9918	1.1836	1.357	1.5398
	Compound Low Speed	1.4203	1.5928	1.7649	1.9364	0.9514	1.1156	1.2800	1.444

758. From TABLE No. 106 it is seen that the cost of producing electrical energy per kilowatt hour varies from a maximum of 2.89 cents to a minimum of .95 cents. The data here given seems at first sight somewhat at variance with that shown for power cost in TABLE No. 98, for in the latter table the cost is given as .85 cents. It must be recollected

that the power cost of TABLE No. 98 merely includes wages, fuel, and miscellaneous items, and does not include interest and depreciation upon central-station machinery, which are noted in TABLE No. 106. It is recorded that one power-station in Chicago has produced electricity on the switchboard for 45 cents per kilowatt hour, for wages, fuel, miscellaneous items, and current maintenance, but for average practice it is safe to assume that electricity will cost 1 cent per kilowatt hour at the switchboard.

759. Water-Power. — The value of electricity as a means of distributing energy has been especially extolled in connection with the utilization of water-powers. In many cases the claims made for this means of distribution have been fully substantiated. It should, however, be recollected that, in most cases where available water-powers exist in proximity to centers where there is a demand for power, the water-powers have been already made thoroughly available, as, for example, in many of the rivers upon the Atlantic Coast, especially in the Northeastern States. In the far West, where water-powers are more plenty, settlement is, as yet, so sparsely distributed that, even were all the necessary machinery in place, the power could not be sold for lack of customers. Some special cases will, however, undoubtedly from time to time appear, as, for example, in the present undertaking to utilize a portion of the water-power of the Niagara, and transmit the same to the neighboring cities, in which the availability of electricity will prove itself of exceeding value. Dr. Emery has shown, by careful computation, that in many cases the cost of improvement required for the utilization of water-power reaches a sum so large that the interest and depreciation upon the same, in the end, aggregate more than the cost of power production by means of steam; and that it is only under very exceptional circumstances that a cost of more than $140.00 per H. P. obtained is justified as an improvement expense.

So far as station equipment expenses are concerned, it is found that the cost of water-wheels, penstocks, shafting, etc., required to deliver the power to the dynamo, is very nearly as great as that incurred in steam machinery. A fair average, perhaps, may be taken at from $40.00 to $75.00 per H. P. rendered available. The actual cost of water-power throughout this country is quite an uncertain quantity.

In the New England States the cost averages nearly $20.00 per H. P. per year, thus approximating to the cost of steam-power.

760. In some of the Southern States, where water-powers are less fully in demand, the cost is lower, varying from $12.00 to $15.00 per annum per H. P. So, while electricity lends itself most readily to the transmission of power over comparatively long distances, the financial outcome of the utilization of any water-power should be closely scrutinized, both from the standpoint of the probable expense of the necessary improvement required, and also from the standpoint of the ability to find customers for the power obtained when the development shall be complete.

761. **The Gas-Engine.** — The internal-combustion engine, operated either by gas or some of the petroleum derivatives, is winning an enviable reputation as a central-station prime mover. The usefulness of petroleum-engines is so far chiefly confined to stations requiring but small power in locations where the price of coal is so great as to enable gasoline or naphtha to become a formidable competitor to coal, so that the gas-engine is that form of internal-combustion engine which is chiefly interesting. As the gas-engine requires no boiler, the expense for land and buildings is decidedly decreased. The labor to operate the central station is economized by the omission of wages for firemen, and cost of handling ashes and coal. As the gas-engine may be started and stopped in the course of a minute or two, it is not necessary to bank fires or to keep idle boilers under steam. On the other hand fuel in the form of gas is usually much more expensive than the corresponding cost of coal. Under certain circumstances, however, the gas-engine is a distinct economy. Thus, the cost per 1,000 ft. of gas decreases very rapidly with the increase in output, and a gas company may, by installing a gas-engine and thus becoming its own largest customer, effect a marked reduction in the cost of producing gas and at the same time advantageously sell electricity. It is usual to consider that the sale of the by-products from the gas-works will largely offset the cost of manufacture, and where this can be accomplished the operation of the gas-engine shows distinct advantages. The amount of gas consumed will depend partly upon the size of the engine and partly upon the composition of the gas. The gas consumption per brake H. P. developed is shown by TABLE NO. 107.

TABLE No. 107.

Gas Consumption in Cu. Ft. per Brake H. P.—Westinghouse Engines.

Note.—Calorific Power of Gas = 1,000 B.T.U.

H. P. Developed.	2-Cylinder Engine.			
	6×8	8×10	9×11	11×12
10	10	16	19	25
20	....	11	12	15
30	....	....	10	12
40	....	....	10	11
50	....	....	....	10
60	....	....	....	10

H. P. Developed.	3-Cylinder Engine.			
	8×10	9×11	11×12	13×14
10	21	26	37	52
20	14	11	22	29
30	11	13	17	21
40	....	11	14	18
50				
60	....	....	11	14
70				

H. P. Developed.	3 Cylinder Engine.		
	15×22	18×22	25×30
100	15	16	27
200	11	11	17
300	....	10	13
400	....	10	12
500	....	....	10
600	....	....	10

762. The Cost of Electrical Energy as Developed by Wind-Power. — In some localities, notably in the Western portion of this country, where fuel is high priced, it is feasible to utilize wind-power by means of windmills for the purpose of generating electricity. Mr. G. H. Morse has compiled some valuable statistics regarding the cost of developing power in this way, which are abstracted in TABLE No. 108. Owing to the uncertainty of the wind, it is necessary to provide a very large margin in the battery plant, or in times of calm weather there will not be a sufficient reserve.

TABLE No. 108.

Showing Cost of Electric Lighting by Wind-Power.

Diameter of Wind-wheel in Feet.	Cost of Geared Windmill, Shafting, and Tower.	Useful Horse-Power Developed, Wind 16 Miles per Hour.	Av. No. of Hours this H.-P. will be Developed per Day.	Expense of Power, Cents per Hour.				Watts Recovered from Dynamo, 50 per cent Efficiency.	Cost of Dynamo.	Expense of Generating Electricity.			Watt Hours Recovered from Accumulators per Day, Efficiency 45 per cent.
				Interest on First Cost at 5 per cent per annum.	Depreciation at 5 per cent.	Attendance.	Oil.			Interest on First Cost.	Depreciation at 5 per cent.	Oil.	
8½	$153	0.04	8	0.26	0.26	0.06	0.04	14.9	$15	$0.03	$0.03	$0.05	53.7
10	164	0.12	8	0.28	0.28	0.06	0.04	44.8	20	0.03	0.03	0.05	161.1
12	177	0.21	8	0.30	0.30	0.06	0.04	78.3	25	0.04	0.04	0.06	279.6
13	181	0.25	8	0.31	0.31	0.06	0.07	93.2	30	0.05	0.05	0.06	335.7
14	227	0.28	8	0.39	0.39	0.06	0.07	104.4	35	0.05	0.05	0.06	376.0
16	301	0.41	8	0.51	0.51	0.06	0.07	152.9	40	0.07	0.07	0.10	550.5
18	350	0.61	8	0.60	0.60	0.06	0.07	227.5	50	0.08	0.08	0.10	1,433.4
20	373	0.79	8	0.64	0.64	0.06	0.10	279.7	60	0.10	0.10	0.30	1,761.9
22	544	1.23	8	0.93	0.93	0.06	0.10	458.8	130	0.22	0.22	0.30	2,890.4
25	584	1.34	8	1.00	1.00	0.06	0.10	499.8	140	0.24	0.24	0.40	3,148.9
30	679	2.40	8	1.16	1.16	0.06	0.13	895.2	175	0.30	0.30	0.40	5,639.8
36	743	2.95	8	1.27	1.27	0.06	0.13	1,100.3	210	0.36	0.36	0.50	6,932.2
40	842	4.42	8	1.44	1.44	0.06	0.13	1,648.7	270	0.46	0.46	0.50	10,386.6
50	1,592	6.88	8	2.73	2.73	0.06	0.16	2,566.2	300	0.51	0.51	0.60	16,167.3
60	1,902	10.00	8	3.26	3.26	0.06	0.16	3,730.0	400	0.68	0.68	0.80	23,499.0

Diameter of Wind-wheel in Feet.	Number of Lamps 16 C. P. 110 Volts.	Number of Hours Lamps will run per Day.	Required Capacity of Accumulators in Ampere Hours.	Cost of 58 Accumulator Cells.	Cost of Automatic Battery Regulator.	Expense of Storing the Electricity.			Total Expense of Obtaining Electricity per Hour.	Total Cost of the 8 Hours Daily Storage.	Equivalent Number of Lamp-Hours.	Average Cost of Lamp-Hour from Wind-Power in Cents.
						Interest on First Cost.	Depreciation at 20 per cent.	Attendance.				
8½	1	0.84	4	$21	$20.00	$0.07	$0.28	$0.24	$1.32	$10.56	0.84	12.57
10	1	2.52	4	21	20.00	0.07	0.28	0.24	1.36	10.88	2.52	4.32
12	1	4.38	5	27	20.00	0.08	0.32	0.24	1.48	11.84	4.38	2.70
13	1	5.26	6	32	20.00	0.09	0.36	0.24	1.60	12.80	5.26	2.43
14	1	5.89	7	37	20.00	0.10	0.40	0.24	1.81	14.48	5.89	2.46
16	2	4.31	10	54	20.00	0.12	0.48	0.24	2.23	17.84	8.62	2.08
18	5	4.49	26	124	20.00	0.25	1.00	0.24	3.08	24.64	22.45	1.10
20	6	4.60	32	153	20.00	0.30	1.20	0.24	3.68	29.44	27.60	1.07
22	10	4.53	52	249	20.00	0.46	1.84	0.24	5.30	42.40	45.30	0.94
25	12	4.11	57	273	20.00	0.50	2.00	0.24	5.78	46.24	49.32	0.94
30	20	4.42	103	470	20.00	0.84	3.36	0.24	7.95	63.60	88.40	0.72
36	25	4.34	125	571	20.00	1.01	4.04	0.24	9.24	73.92	108.50	0.68
40	38	4.30	190	868	20.00	1.52	6.08	0.24	12.23	98.64	163.40	0.60
50	60	4.22	294	1,278	20.00	2.22	8.88	0.24	18.64	149.10	253.20	0.59
60	85	4.33	427	1,857	20.00	3.21	12.84	0.24	25.19	201.50	368.00	0.55

763. Commercial Consideration of Transmission Problems.— Every long-distance installation, from a constructive standpoint, must be regarded as made up of three factors.

First. A generating-station, including prime movers for utilizing the source of energy, and dynamo machinery for transforming mechanical energy into electrical energy at proper potentials for economical transmission.

Second. The necessary line for transferring this energy from the generating-station to the receiving-station.

Third. The receiving-station, embracing such dynamo machinery as is necessary to reduce the high potentials used in the line to convenient voltage for distribution and use by customers.

The vital consideration in all such installations then becomes the cost of energy delivered at the receiving-station by the transmission plant, in comparison with the cost of manufacturing a similar amount of energy by some other means at this station.

764. To determine the cost of energy delivered by the transmission plant, the following items must be taken as affecting the total expense:—

First. The interest and depreciation on the necessary capital invested in improving the water-power, or other source of energy, and in the purchase of the necessary machinery, engines, dynamos, water-wheels, etc., and in the acquisition of real estate and erection of buildings required for the generating-station; in other words, the total cost of the generating-station.

Second. The cost of obtaining power at the generating-station. This item will include rent paid for water-power, or interest on the necessary capital invested in the purchase of water-right. A similar expense would be the cost of purchase of fuel at a location where such a low price for coal could be obtained as would seemingly warrant the installation of a transmission plant.

Third. The expense of energy at the generating-station is further augmented by the cost of such labor and superintendence as may be necessary to operate and maintain the plant.

Fourth. The interest and depreciation on the cost of erecting the line between the generating-station and the receiving-station.

Fifth. The cost of energy lost in transmission between the generating- and the receiving-station.

Interest and depreciation on the cost of the machinery, buildings, etc., for the *receiving-station,* do not enter into the expense of delivering energy at the receiving-station, for the reason that, were any different arrangements made for obtaining electrical energy at the receiving-station than that of the transmission plant, a station essentially similar, so far as this cost is concerned, would be necessary.

Summarized: The cost of energy at the receiving-station, then, stands as follows:—

First. Interest and depreciation upon the capital invested in the generating-station.

Second. Cost of obtaining energy at the generating-station.

Third. Labor at the generating-station.

Fourth. Interest and depreciation upon cost of transmitting line.

Fifth. Losses in line transmission.

765. To determine the advisability of the installation of a long-distance plant, it is necessary to compare the probable cost of energy delivered to the receiving-station by the long-distance plant, with the cost of a corresponding amount, as obtained *at this location,* by any other means. Should the figure obtained for the cost of energy by a long-distance plant be equal to that required by the manufacture of energy in any other way, it is evident that the long-distance plant will stand on precisely equal footing with any other installation. Should the amount for production of energy by a long-distance plant be less than that by other installations, the long-distance plant will be profitable in that proportion.

766. The ability to produce energy at the receiving-station will be limited to the power derived by means of a steam-engine. Cases where wind, tidal power, or other methods would be available, are so infrequent that they may be discarded without seriously affecting the result, and attention confined solely to the production of energy at the receiving-station, by means of a steam-engine, in contradistinction to that obtained by the transmission plant. The cost of the production of energy by means of steam-power is tolerably well ascertained. The cost of energy will vary with the kind of engine, the price of coal, the rate of interest and depreciation upon the capital invested in the plant, and cost of necessary labor. In Table No. 109 a series of curves with necessary data is given, for determining the capital to be invested in a steam-plant, the cost of perpetual

maintenance of the same, and the production of power. The Table is divided into four parts.

First. A schedule giving the cost of steam-plant per horse-power at the engine and per kilowatt of energy delivered at the terminals of the generator. In this latter column the values are assumed for engines working at a reasonably steady full load, with an efficiency of 90 per cent in the generator. The figures are those which would apply to fairly large installations, say from 250 horse-power upwards, and would prevail for most locations east of the Mississippi River, in this country. Special charges necessitated by locations out of the ordinary have not been considered. The schedule is arranged to embrace seven different styles of engines, considered to be those which are more likely to be used.

767. The second division of the Table embraces a set of curves arranged for the purpose of calculating the interest and depreciation to be allowed upon the steam-plant. A separate line is given for each type of engine, the horizontal axis being scaled for interest and depreciation, while the vertical axis gives the amount to be assessed per horse-power per annum, for varying rates on the interest and depreciation scale.

The third part of the Table is devoted to the cost of fuel and supplies in dollars per horse-power per year.

The horizontal axis here embraces the cost of coal in tons, of 2,240 pounds, from $2.00 to $10.00 per ton, while the vertical axis indicates the corresponding cost per horse-power per annum. It should be here noted that the lines are so drawn as to *include* the cost of the ordinary amount of oil and other minor supplies which would be naturally required in a steam-plant. While these values are not absolutely correct, as a slight variation in the cost of the minor supplies, in comparison with the cost of coal, would make slight changes, it is considered that it is sufficiently accurate for ordinary purpose of estimate.

The fourth division of the Table applies in a similar manner to rates of wages for engineer and fireman. On the horizontal axis will be found the rates per day for engineer and fireman, two scales, one for each class of labor, being indicated. The vertical axis gives the wages cost per horse-power per annum. In each of the divisions a separate line will be found for each kind of engine, which may be

readily identified on the schedule by means of a corresponding initial letter, used in each of the divisions. To use the Table, select on the horizontal axis the value required; follow a vertical line to its intersection with the line indicating the kind of engine proposed to install, and then follow a horizontal line to the left to the vertical axis, finding the value desired. This Table forms a convenient means whereby the engineer may rapidly determine the probable cost of energy per horse-power per annum, as developed by a steam-plant erected at the receiving-station. It is now necessary to ascertain the cost of energy delivered at the receiving-station, when obtained through the medium of long-distance transmission, and compare this with the cost of energy as obtained by means of the steam-plant. Sheet 5 gives a number of cost items (Pocket).

768. The factors composing the cost of energy at the *receiving-station*, as delivered by the transmission plant, are as follows:—

First. Interest and depreciation on cost of generating-station.

Second. Cost of power at generating-station.

Third. Cost of labor at generating-station.

Fourth. Interest and depreciation on cost of line.

Fifth. Cost of energy lost in the line.

For this purpose, it is convenient to refer to TABLE NO. 110. The use of this table, as it is slightly complex, will be best comprehended by means of an example.

Assume the following data:—

a.	Cost of generating-station per horse-power . .	$150.00.
1.	Interest and depreciation on generating-station per annum	10 per cent.
2.	Cost of water per horse-power per annum . .	$15.00
3.	Cost of labor	$2.50 per day.
b.	Cost of line per mile	$1800.00.
c.	Length of line	5 miles.
4.	Interest and depreciation on cost of line per annum	15 per cent
5.	Loss in line	20 per cent.
d.	Power transmitted	400 H. P.

To find cost of energy at receiving-station.

769. Refer to TABLE No. 110, finding, on the left-hand side, along the vertical axis, two scales, one labeled "Cost of Generating-Plant

TABLE No. 109.

Cost of Installing and Maintaining Steam-Plant.

Sheet 1.

Designation.	TYPE OF ENGINE.	COST PER H.P. OF Engine.	COST PER H.P. OF Boiler.	COST PER H.P. OF Stack and Buildings.	Total Cost per H.P. with 5% for Inspection and Supervision during Installation.	Total Cost per Kilowatt. 85% Efficiency.
A	Simple High-Speed Non-Condensing,	$17.00	$26.00	$16.00	$61.50	$97.00
B	Simple Low-Speed Non-Condensing,	25.00	24.00	15.00	67.20	106.30
C	Compound High-Speed Non-Condensing,	22.00	21.00	14.00	58.80	92.65
D	Simple High-Speed Condensing,	21.00	18.00	12.00	53.50	84.30
E	Simple Low-Speed Condensing,	27.00	17.00	11.50	58.37	92.00
F	Compound High-Speed Condensing,	25.00	16.00	11.00	54.60	85.30
G	Compound Low-Speed Condensing,	30.00	15.00	11.00	58.80	92.65

NOTE. — For detailed information, see Sheet of Curves in pocket, marked Table No. 74, Sheet 1.

TABLE No. 109 (*Continued*).

Cost of Installing and Maintaining Steam Plant

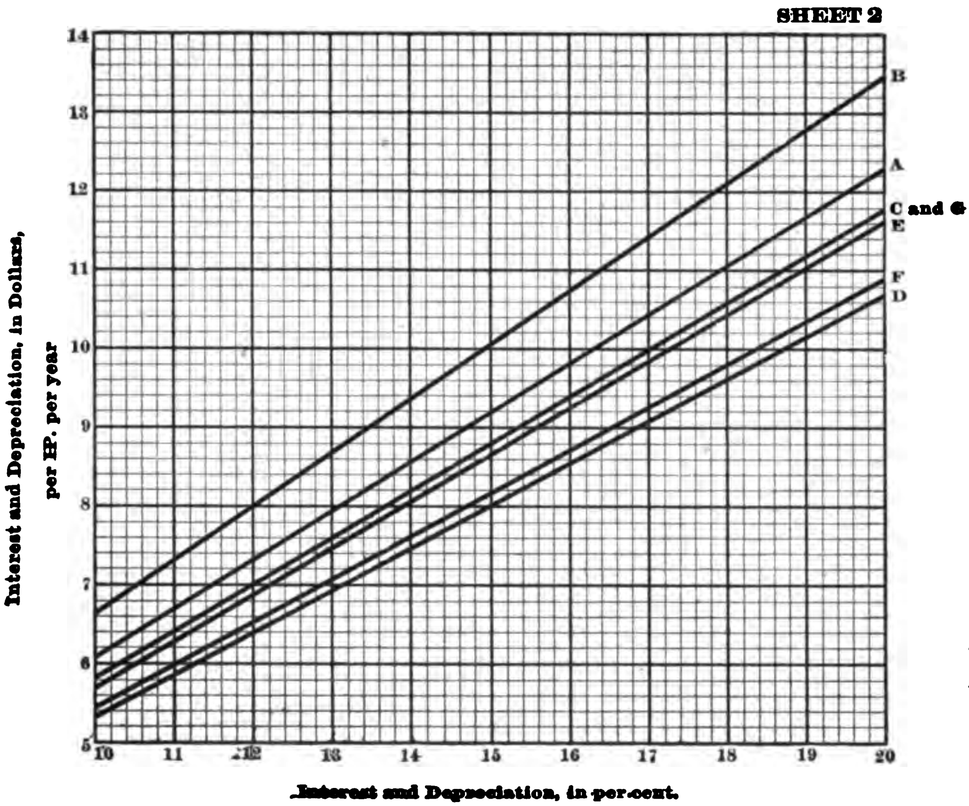


TABLE No. 109 *Continued*).

Cost of Installing and Maintaining Steam-Plant.

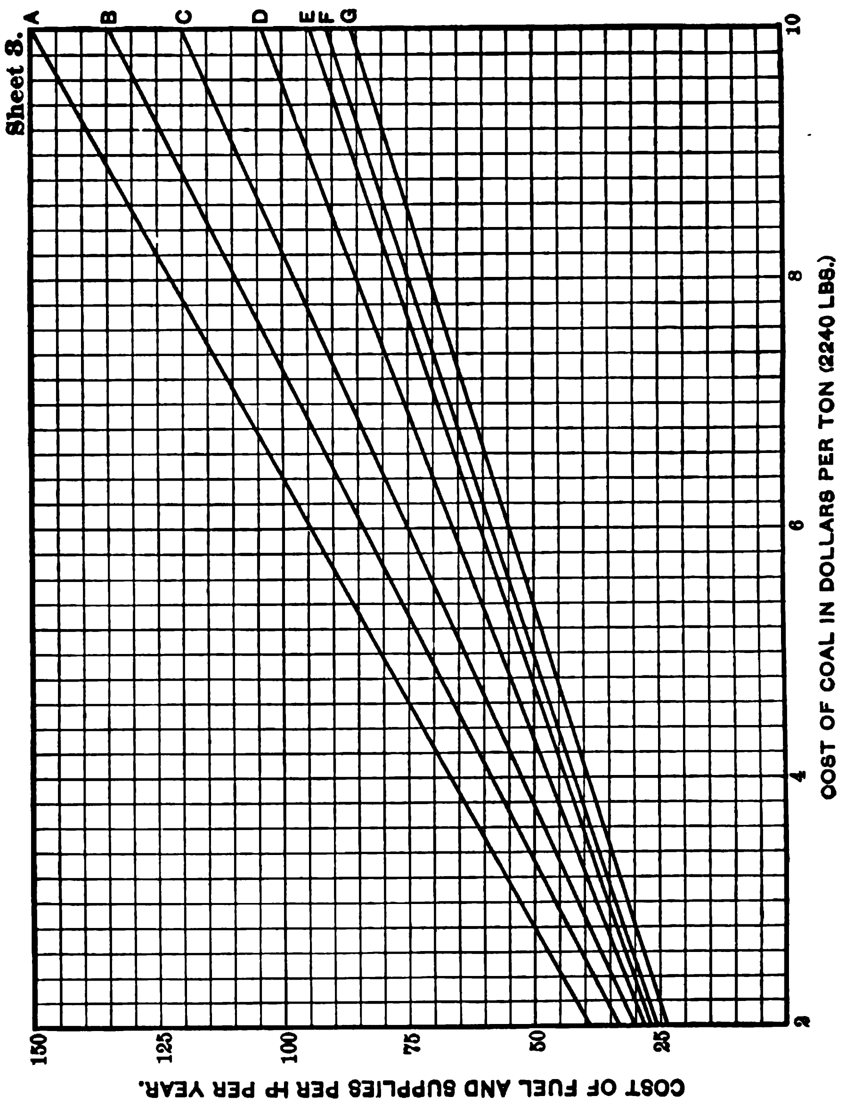
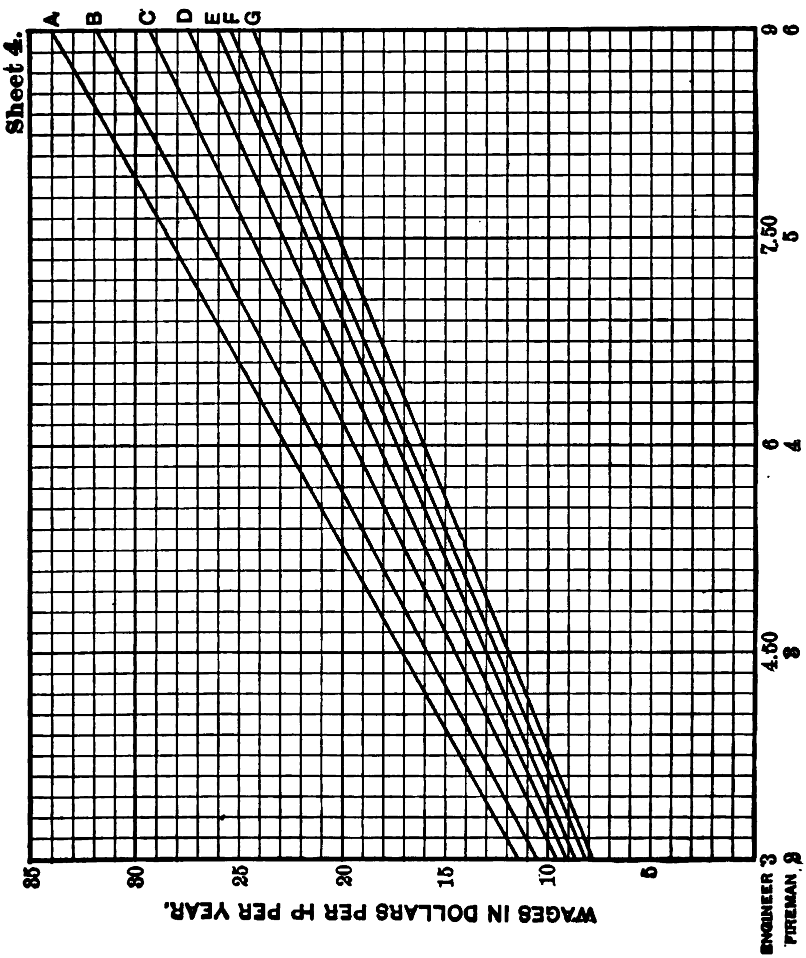


TABLE No. 109 (*Continued*).

Cost of Installing and Maintaining Steam-Plant.



per H. P." Taking this scale, proceed to $150.00 (*a*), the assumed cost of the plant per horse-power. From $150.00 follow a horizontal line (this example may be readily traced by following the dotted lines upon the diagram, which have no reference to any calculations excepting the particular example now under consideration) to the point of intersection of the horizontal with the diagonal line marked "Interest and Depreciation" (1), and labeled "10 per cent," the assumed value. The amount of interest and depreciation is then found by following a vertical line downward to the lower scale, marked "Interest and Depreciation on Generating-Plant per H. P.," giving $15.00 as the interest and depreciation per horse-power of plant capacity per annum. It is now necessary to take into consideration the cost of water per annum, which is assumed to be $15.00 (2).

770. From the point of intersection of the horizontal through $150.00, with the interest diagonal 10 per cent follow a vertical line upwards to the intersection of the diagonal under "Cost of Water per H. P.," labeled "$15.00." Then follow a horizontal line to the right to the left-hand scale on the right-hand side of the diagram, — the scale labeled "Interest and Depreciation on the Generating-Plant plus the cost of Water per H. P." The value here given, $30.00, is the sum of the first and second items. To include the cost of labor (3), return to the intersection with the "Cost of Water" diagonal through $15.00, follow the horizontal line to the right to the intersection of the "Cost of Labor per Day" diagonal, labeled $2.50. At this point follow a vertical line to the extreme upper scale of diagram, labeled "Interest and Depreciation plus Cost of Water, plus Cost of Labor, or total Cost at Generating-Station per H. P.," finding the total value to be $32.50 as the cost per horse-power per annum at the generating-station. It now remains to find and add to this amount the cost of the energy lost in transmission (5) between the generator and the receiving-station, and the interest and depreciation on the cost of the line, in order to get the total expense of energy at the receiving-station.

771. From the intersection of the previous horizontal line with the diagonal for the "Cost of Labor per Day," marked "$2.50," follow a vertical line downward to intersection with the diagonal under the heading "Losses in Line," labeled "20 per cent." From this point follow a horizontal line to the right, to the left-hand scale on

the right-hand side of the diagram, headed "Interest and Depreciation plus Cost of Water, plus Cost of Labor, plus Losses in Line." Here the value of $39.00 will be found as the cost per horse-power per annum of energy delivered at the receiving-station by the transmission plant, exclusive of interest and depreciation on the cost of the line, which figure it is now necessary to ascertain. The previous amount, $39.00, must be carefully noted, as it is necessary to add the amount of the interest and depreciation on the line to it. To find this latter figure, return to the extreme left-hand vertical scale of the diagram headed "Cost of Line per Mile in Dollars" (*b*). Selecting the figure $1800.00, the assumed cost of the line per mile, follow a horizontal line to the right to the intersection of the diagonal marked (15 per cent) (4), the assumed rate of interest and depreciation on the line, under head "Interest and Depreciation." From this intersection follow a vertical line upwards to the lower top scale marked "Interest and Depreciation on Line per Mile," obtaining the value $270.00 as interest and depreciation on the line per mile of length. This figure is, evidently, not the total amount necessary to obtain, as the length of the line is not included. From the upper left-hand corner of the diagram will be seen the number of radiating diagonal lines headed "Length of Line in Miles" (*c*). From the intersection of the previously mentioned vertical with the diagonal headed "5 Miles," follow a horizontal line to the right, obtaining, on the scale marked "Interest and Depreciation on the Whole Line," the figure $1350.00 as the amount of this charge. As the entire calculation is made per unit of power, it is evident that the charge for interest and depreciation on the line must be divided by the total amount of power transmitted in order to obtain the proper proportional charge per unit of energy supplied. To secure this result, the last obtained amount must be divided by the amount of power transmitted, namely, 400 horse-power (*d*). To accomplish this, return on the horizontal line through $1350.00, until an intersection is obtained with the diagonal marked "400 H. P." Here follow a vertical line to the lower top scale of the diagram marked "Interest and Depreciation on the Whole Line per Horse-Power," finding a value of $3.37 as the amount for the interest and depreciation on the line per unit of power, to be added to the previously obtained cost of

$39.00, making a total cost of $42.37 per annum per horse-power, delivered at the receiving-station.

772. In this example the process has been given *in extenso* step by step, in order to familiarize the reader thoroughly with the workings of the diagram, and to show that the process may be stopped at any desired step, and used to obtain the value of any successive set of items. If it is wished to complete the entire calculations without any reference to the intermediate steps, the process is as follows:—

773. Find the cost of the generating-station per horse-power on the left-hand scale of diagram. In this example, start at $150.00, follow a horizontal line to intersection with the "Interest" diagonal; from this point follow a vertical line to the intersection with "Cost of Water per H. P." diagonal, then a horizontal line to the "Cost of Labor per Day" diagonal, then vertical line to the intersection with diagonal headed "Losses of Line," then horizontal line to the right to the left-hand scale on the right-hand side of diagram, finding the figure $39.00 as the "Cost of Power at the Receiving-Station."

774. In a similar manner, to obtain the value for "Interest and Depreciation on Cost of Line per H. P.," start at $1,800.00 on left-hand scale on diagram, follow a horizontal to intersection with the "Interest" diagonal, then the vertical to intersection of diagonal giving "Length of Line," then vertical to lower scale on top of the diagram, giving $3.37 as the total cost per unit of power, for "Interest and Depreciation on the Line." Add these two figures to obtain the desired result.

From slight consideration it is evident, by these tables, problems involving the commercial aspect of long-distance transmission may be rapidly solved, providing the necessary data for obtaining the constants are at hand.

775. Economical Conductor Section.—In long distance transmission, the cost of the line rises to be one of the most important factors, if not the principal one, in the installation.

To determine the most economical area for the conductor, the principles given in Chapter XIV. should be used, and may be directly applied to the greatest advantage. The equation there given for finding the proper conductor area is—

$$U = \alpha + \beta S + \frac{\lambda}{S}.$$

In this equation three coefficients must be considered; namely, α, β, and λ.

By the process of differentiation α disappears; so to determine the value of S, the quantities β and λ only need enter into consideration. The term β is substituted for the expression $L\,[\,b\,(i + d_l) + b'\,(i + d_c)\,]$, involving the interest and depreciation to be allowed upon the cost of the conductor, and upon the structure used for supporting or protecting it. Two rates of interest and depreciation were allowed, as in the most refined calculations, especially those involving the cost of conduit structures, the interest and depreciation assessed upon the conduit would be different from that on the conductor. For ordinary purposes of calculation, as an abridgment of the process, the two rates may be assumed the same, and the value of β considered to be $L\,(b + b')\,(i + d_c)$. The term λ involves the amount of current transmitted, the resistance and length of the line, the interest and depreciation allowed upon the cost of the station per unit of output, and the cost of producing the energy lost in the line; adopting the notation of Chapter XIV., $\lambda = I^2 \rho L\,[FK + K'\,(i + d_s)\,]$.

776. For the purpose of facilitating calculations of the most economical conductor cross-section, TABLES Nos. 111, 112, and 113 are presented for determining the values of the above coefficients, giving a solution directly of the equation $x = \sqrt{\lambda/\beta}$.

TABLES Nos. 112 and 113 are arranged in two parts—part B of each Table being laid out to a reduced scale, as compared with part A. As the scales in all of the Tables are decimal, the range of the Tables may be extended in any direction by multiplying or dividing by any power of 10. By means of the decimal arrangement and the double sets of values given, all problems within ordinary ranges may be readily solved. As the use of the Tables is a little complicated, an example will perhaps best elucidate their application.

Returning to the data given on p. 657 used to exemplify the use of TABLE No. 111, and adding to the constants there assumed, the amount of current to be transmitted through the line, 200 amperes, and the length of time this current flows through the line, 3,000 hours per annum, let it be required to find the most economical cross-section for the conductor.

777. The Tables have been calculated by assuming the length

of the line to be one mile of double circuit; that is, a mile away from the station and a mile back, making the total actual length of the line two miles. It will also appear that the most economical conductor cross-section, as determined for a mile of double circuit, will equally apply to a line of any length, for reason that, as the resistance increases directly in proportion to the length of the line, the amount of energy wasted and the interest and depreciation on the cost of the line will correspondingly increase in the same direct proportion.

778. TABLE No. 111 serves to determine the two constants inside the brackets; namely, FK, and $K'(i + d_s)$. To determine the value of this latter quantity, look for the cost of the generating-station along the top scale of diagram labeled "Cost of the Generating-Station per H. P.," or K'. In the example under consideration, \$150.00 is assumed for the cost of the station, while the interest and depreciation $(i + d_s)$ is given on the diagonals running downward from the right-hand upper corner. From \$150.00 or "Cost of Station" follow a vertical downward to the intersection of the diagonal marked "$(i + d_s)$" for the assumed rate of 10 per cent, then follow a horizontal line to the right, to the left-hand scale marked "$K'(i + d_s)$," here finding the value of "\$15.00" as the amount of this expression. *Note this value.* Now, to determine FK, having the annual cost of producing energy per horse-power, as obtained from TABLE No. 110. It must be recollected that this cost per horse-power, as given by TABLE No. 110, is based upon operating the station 3,000 hours per annum. In order to find the cost *per horse-power hour*, select on the lower horizontal scale, labeled "Cost of Energy at the Generating-Station per H. P.," the cost gathered from TABLE No. 110, follow a vertical line upward to intersection with diagonal line marked "3,000 hours," then follow a horizontal to the right-hand scale on the right-hand side of the diagram, finding the desired amount on the scale marked "Cost of Energy at Generating-Station per H.-P. *hour.*" In the example under consideration, from TABLE No. 110, a cost of \$32.50 was obtained as the "Cost of 1 H.P. for 3,000 hours." Select this point on the lower scale, follow a vertical line upward to intersection with diagonal labeled "3,000 hours," then follow a horizontal to the right, to the right-hand scale, the value of \$.0108 is found as the "Cost of one H.-P. hour." FK is the cost per horse-

power, multiplied by the time of operation, and is obtained from the Table by following a horizontal line to the *left* from the cost per horse-power hour on the right-hand scale till the horizontal intersects the diagonal marked with the number corresponding to the annual time of operation. Thus, supposing the plant to operate for 5,000 hours, following diagonal from the right-hand scale through .0108 to the intersection of the diagonal labeled "5,000 hours," then a vertical downward to the horizontal scale, the value of $54.16 is found for FK. Continuing, however, the original example on the supposition that the plant operates for 3,000 hours, the value of $32.50 is found for FK. *Note this value.* The Table thus gives the values of the two quantities inside the brackets; namely, FK and $K'(i + d_s)$. These two values must now be added, giving $47.50 as the total of the quantity inside the brackets.

779. Now, turning to TABLE No. 112, the top scale is labeled value of $FK + K'(i + d_s)$. The left-hand scale gives the values of $I^2\rho L$, while the lower scale gives the values of $I^2\rho L \times [FK + K'(i + d_s)]$, or the value of λ. On the top scale of the diagram find the value of $FK + K'(i + d_s)$, as obtained from TABLE No. 111. *Connect* this point with the origin at the lower left-hand corner by a diagonal line (see dotted line). The value of $I^2\rho$ must now be obtained. The conductor in this example is assumed to be soft copper, to operate at a temperature not exceeding 30° C. Find upon the lower scale of the diagram the temperature of the conductor, then follow a vertical line upwards to the intersection with the diagonal labeled "200 amperes S. C." (soft copper). Follow a horizontal line to the left from this intersection to the left-hand scale, from which the value of $I^2\rho L$ is obtained as 5. From the intersection of this horizontal line with the diagonal to the origin drawn from the value of $FK + K'(i + d_s)$, on the top scale, follow a vertical line downwards to the lower scale labeled "Value of $I^2\rho L\,[FK + K'(i + d_s)]$," obtaining here the value of this expression as 250, or the value of λ. The dotted lines on the Tables serve to show the course followed in the solution of this particular example, but have *no reference* to the solution of any other. The dotted lines have been drawn on both parts of each diagram, in order to show that the same result is obtained on each. The operator should use that section of diagram which will give the most advantageous scale. Now, turning to TABLE No. 113, find upon the

right-hand vertical scale, headed "Cost of Line per Mile," the amount of capital invested in the line, recollecting that the mile here referred to is two actual miles of circuit. In this case the cost of the circuit mile is $1,800. Follow a horizontal from this figure to intersection with the diagonal giving the determined rate of "Interest and Depreciation," in this example 15 per cent being selected. From this intersection follow a vertical line upward to the top scale of the diagram, here finding the value of $L\,[b\,(i+d_l) + b'\,(i + d_c)]$, or β. The value here obtained is $270.00. Connect this point by a diagonal with the origin at the lower right-hand corner. From TABLE No. 112, the value of λ was found to be 250. On the lower horizontal line, marked "Value of λ," find 250. At this point erect a perpendicular until it intersects the diagonal previously drawn from the point on the top scale, giving the value of β to the origin. The point of intersection of the vertical and this diagonal is, evidently, the value of λ / β. From this intersection follow a horizontal line to the left, until the curve C is intersected, then follow a vertical downward to the lower horizontal scale marked "Value of S," here finding $\frac{8.5}{100}$ of a sq. in. as the most economical value of the cross-section of the conductor.

Though the process of using the Tables, as here described, may seem somewhat complicated, experience gained from the solution of half-a-dozen examples will enable the operator to determine the most economical cross-section of conductor in one-quarter the time that is required to read the description.

By means of the graphical methods thus outlined, the designer may rapidly determine the best cross-section for the conductors of a transmission plant under any of the usual limiting conditions. A careful comparison should always be instituted between the section thus ascertained and that indicated by each of the various other governing factors that enter into every distributing problem; for the most economical conductor section is by no means always the most advisable one to employ.

In the distribution of energy by means of electricity, the principles outlined form a ground-work sufficient to enable the designer to so utilize materials and energy as to attain the desired result. Facts and laws are, however, like tools, the value of the product depending largely on the skill of the workman.

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